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Inverse Kinematics Model For a 18 Degrees of Freedom Robot

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Abstract:
The study of humanoid robots is still a challenge for the scientific community, although there are several related works in this area, several limitations have been found in the literature that drive the need to develop an inverse kinematic modeling of biped robots. This paper presents a research proposal for the Bioloid Premium robot. The objective is to propose a complete solution to the inverse kinematics model for a 18 DOF (Degrees Of Freedom) biped robot. This model will serve as a starting point to obtain the dynamic model of the robot in a subsequent work. The proposed methodology can be extended to other biped robots.

Keywords: *bioloid premium robot, forward kinematics, inverse kinematic, kinematic chain.*

1. Introduction

The problem of study related to the kinematics of biped robots has been widely studied in the scientific community; in the literature have been found several limitations in the models of biped robot kinematics. This drives the need to develop an inverse kinematics model for the Bioloid robot of 18 DOF. Specifically, the problem of the lack of study of the kinematics of the upper train in biped robots arises [1–13, 19–26]. Due to the high number of degrees of freedom and the complexity involved in the calculation of the inverse and forward kinematics equations, most authors have the objective of modeling only the lower train of the robots, either using commercial robots such as Nao with 12 DOF legs [1], HYDROïD which has 8 active DOF per leg [2], Scout [3] and NWPUBR-1 [4] with 12 DOF legs, Ostrich Bionic with 13 DOF legs [5], Cassie with 20 DOF legs [6], or robots wich are author's design with 12 DOF [7–9] 10 DOF [10–12] and 9 DOF [13]. All of these research papers calculate the forward kinematics model by taking one of the robot's feet as supporting foot.

In other works, it is possible to obtain the forward and inverse kinematics solution for both legs and arms, using the HRP-2 robot with 12 DOF legs [14], DARwIn-OP with 6 DOF per leg [15], AXIS with 12 DOF legs [16], NAO with 21 DOF [17], Digit robot with 20 DOF [18], but these models propose the torso or pelvis of the robot as the initial frame.

The Bioloid robot has been used by the scientific community to perform several studies related to kinematics, dynamics and control. Most of the works obtain the kinematic model of the legs, taking into account only one foot as the initial frame [19–24]; proposing two different cases where the supporting foot is either the right or the left foot [25], in [26] the torso is taken as the initial frame. In [27] the kinematic model of the robot legs and arms is obtained but uses the torso and pelvis as initial frames.

All the works mentioned previously calculate the kinematic modeling considering the Denavit–Hartenberg method to represent the position and orientation of the end-effector.

On the other hand, the authors have not established a complete inverse kinematic model for a 18 DOF bipedal robot. Therefore, the Bioloid Premium robot with 18 DOF is proposed as a study target. The main motivation in this paper is to develop a methodology based on the Denavit-Hartenberg method to obtain the forward and inverse kinematic model for a 18 DOF Bioloid Premium robot.

In the present work we propose to obtain the complete kinematic model of the Bioloid robot, considering four open kinematic chains, where the initial frames are the support feet, and we have the left and right pelvis as end-effector frames; the pelvis is also proposed as another initial frame to have the left and right hand as the other end-effector frames.

The paper is organized as follows. In Section 2 the Denavit-Hartenberg method is applied to calculate the geometric parameters of the robot. In Section 3, forward kinematic model is obtained. The equations of inverse kinematics of the robot are computed in Section 4. Finally, the conclusions are given in Section 5.

2. Denavit-Hartenberg Parameters

The key idea is to generate four open kinematic chains to describe the position and orientation of each link of the Bioloid Premium robot. Using the Denavit-Hartenberg method, the frames and parameters of each link, as well as the position and orientation of each joint of the robot are presented in Figure 1.

We can observe that the supporting right and left feet are proposed as the initial frames $\Sigma_{d0}(\mathrm{x}_{d0}, \mathrm{y}_{d0}, \mathrm{z}_{d0})$ and $\Sigma_{d0}(\mathrm{x}_{d0}, \mathrm{y}_{d0}, \mathrm{z}_{d0})$, then the first two kinematic chains goes up to the pelvis frame, from this point three open kinematic chains can be considered, one of them has the left foot end-effector frame $\sum_{12}(x_{12}, y_{12}, z_{12})$, while the second chain takes into account the right hand end-effector frame $\Sigma_{d3}(\mathrm{x}_{d3}, \mathrm{y}_{d3}, \mathrm{z}_{d3})$ and finally, the third chain considers the left hand end-effector frame $\Sigma_{i3}(\mathrm{x}_{i3}, \mathrm{y}_{i3}, \mathrm{z}_{i3})$.

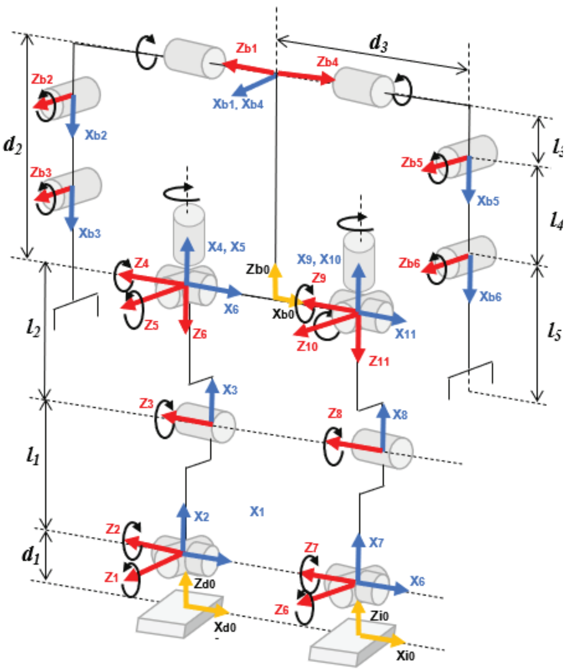

Figure 1. Frames assigned to the joints of the Bioloid robot

Table 1. Denavit-Hartenberg parameters of the legs

Link	α_i	l_i	θ_i	d_i
1	$\pi/2$	0	0	d_1
2	$-\pi/2$	0	θ_1	0
3	0	l_1	θ_2	0
4	0	l_2	θ_3	0
5	$\pi/2$	0	θ_4	0
6	$\pi/2$	0	θ_5	0
7	0	0	θ_6	0

Table 2. Denavit-Hartenberg parameters of the right arm

Link	α_i	l_i	θ_i	d_i
1	$\pi/2$	0	$-\pi/2$	d_2
2	$-\pi/2$	l_3	θ_{b1}	d_3
3	0	l_4	θ_{b2}	0
4	0	l_5	θ_{b3}	0

Table 1 presents the Denavit-Hartenberg parameters for the kinematic chain corresponding to the robot legs, which relates the frame $\sum_1 (x_1, y_1, z_1)$ and frame $\sum_{12} (x_{12}, y_{12}, z_{12})$.

Table 2 shows the Denavit-Hartenberg parameters for the kinematic chain corresponding to the right arm of the robot, which relates the frame $\sum_{d1} (x_{d1}, y_{d1}, z_{d1})$ and frame $\sum_{d3} (x_{d3}, y_{d3}, z_{d3})$.

Table 3 has the Denavit-Hartenberg parameters for the kinematic chain corresponding to the left arm of the robot, which relates the frame $\sum_{i1} (x_{i1}, y_{i1}, z_{i1})$ and frame $\sum_{i3} (x_{i3}, y_{i3}, z_{i3})$.

The robot's home position is given by the angles shown in Tables 4 and 5.

To define the value of the variables corresponding to the leg links, the real measurements of the Bioloid Premium robot leg joints were used:

Table 3. Denavit-Hartenberg parameters of the left arm

Link	α_i	l_i	θ_i	d_i
1	$-\pi/2$	0	$-\pi/2$	d_2
2	$\pi/2$	l_3	θ_{b4}	d_3
3	0	l_4	θ_{b5}	0
4	0	l_5	θ_{b6}	0

Table 4. Value of the joints corresponding to the home position of the robot legs

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6
$\pi/2$	0	0	0	$-\pi/2$	0

Table 5. Value of the joints corresponding to the home position of the robot arms

θ_{b1}	θ_{b2}	θ_{b3}	θ_{b4}	θ_{b5}	θ_{b6}
$-\pi/2$	0	0	$\pi/2$	0	0

$$d_1 = 33 \text{ mm}, \; d_2 = 118 \text{ mm}, \; d_3 = 73 \text{ mm},$$
$$l_1 = l_2 = 76 \text{ mm}, \; l_3 = 16 \text{ mm},$$
$$l_4 = 66 \text{ mm}, \; l_5 = 108 \text{ mm}.$$

3. Forward Kinematics

To calculate the forward kinematics of the robot, the transformation matrix defined in Equation (1) was used.

$$H_{i-1}^{i} = \begin{pmatrix} cos(\theta_i) & -sin(\theta_i)cos(\alpha_i) & sin(\theta_i)sin(\alpha_i) & l_i cos(\theta_i) \\ sin(\theta_i) & cos(\theta_i)cos(\alpha_i) & -cos(\theta_i)sin(\alpha_i) & l_i sin(\theta_i) \\ 0 & sin(\alpha_i) & cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

Where the superscript i represents the number of the current joint and the subscript $i-1$ indicates the number of the previous joint. Therefore, H_{i-1}^{i} is the homogeneous transformation matrix representing the rotation and translation of joint i with respect to joint $i-1$.

To simplify the results obtained, the following compact notation is used:

$$\sin(\theta_i) = S_i, \; \cos(\theta_i) = C_i,$$
$$\sin(\theta_i + \theta_j) = S_{i,j}, \cos(\theta_i + \theta_j) = C_{i,j}$$

where i, j denote the joint number.

The transformation matrix H_0^1 relating the frame $\sum_0 (x_0, y_0, z_0)$ to frame $\sum_1 (x_1, y_1, z_1)$ corresponding to the robot's foot is shown in (2), where $\alpha = \pi/2, \theta = l = 0$.

$$H_0^1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

The homogeneous transformation matrices corresponding to the leg joints from the frame $\sum_1 (x_1, y_1, z_1)$ to frame $\sum_6 (x_6, y_6, z_6)$ are as follows:

$$H_1^2 = \begin{pmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_2^3 = \begin{pmatrix} C_2 & -S_2 & 0 & l_1C_2 \\ S_2 & C_2 & 0 & l_1S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_3^4 = \begin{pmatrix} C_3 & -S_3 & 0 & L_2C_3 \\ S_3 & C_3 & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_4^5 = \begin{pmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_5^6 = \begin{pmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_6^7 = \begin{pmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The transformation matrix H_b^{b1} relating the frame $\sum_b (x_b, y_b, z_b)$ to frame $\sum_{b1} (x_{b1}, y_{b1}, z_{b1})$ corresponding to the right shoulder of the robot is shown in (3). The transformation matrix H_b^{b5} relating the frame $\sum_b (x_b, y_b, z_b)$ to frame $\sum_{b4} (x_{b4}, y_{b4}, z_{b4})$ corresponding to the left shoulder of the robot is shown in (4).

$$H_b^{b1} = \begin{pmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

$$H_b^{b5} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{4}$$

The homogeneous transformation matrices corresponding to the joints of the right arm, from the frame $\sum_b (x_b, y_b, z_b)$ to frame $\sum_{b3} (x_{b3}, y_{b3}, z_{b3})$ are as follows:

$$H_{b1}^{b2} = \begin{pmatrix} C_{b1} & 0 & -S_{b1} & l_3C_{b1} \\ S_{b1} & 0 & C_{b1} & l_3S_{b1} \\ 0 & -1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{b2}^{b3} = \begin{pmatrix} C_{b2} & S_{b2} & 0 & l_4C_{b2} \\ S_{b2} & C_{b2} & 0 & l_4S_{b2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{b3}^{b4} = \begin{pmatrix} C_{b3} & -S_{b3} & 0 & l_5C_{b3} \\ S_{b3} & C_{b3} & 0 & l_5S_{b3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

The homogeneous transformation matrices corresponding to the joints of the left arm, from the frame $\sum_b (x_b, y_b, z_b)$ to frame $\sum_{b6} (x_{b6}, y_{b6}, z_{b6})$ are as follows:

$$H_{b5}^{b6} = \begin{pmatrix} C_{b4} & 0 & S_{b4} & l_3C_{b4} \\ S_{b4} & 0 & -C_{b4} & l_3S_{b4} \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{b6}^{b7} = \begin{pmatrix} C_{b5} & -S_{b5} & 0 & l_4C_{b5} \\ S_{b5} & C_{b5} & 0 & l_4S_{b5} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$H_{b7}^{b8} = \begin{pmatrix} C_{b6} & -S_{b6} & 0 & l_5C_{b6} \\ S_{b6} & C_{b6} & 0 & l_5S_{b6} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Therefore, the forward kinematics relating the right foot frame $\sum_0 (x_0, y_0, z_0)$ and the right pelvis end-effector frame $\sum_6 (x_6, y_6, z_6)$, is calculated employing Eq. (5).

$$H_0^6 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6 H_6^7 \tag{5}$$

The forward kinematics relating the pelvis frame $\sum_b (x_b, y_b, z_b)$ and the right hand end-effector frame $\sum_{b3} (x_{b3}, y_{b3}, z_{b3})$, is calculated employing Eq. (6).

$$H_b^{b4} = H_b^{b1} H_{b1}^{b2} H_{b2}^{b3} H_{b3}^{b4} \tag{6}$$

The forward kinematics relating the right foot frame $\sum_b (x_b, y_b, z_b)$ and the left arm end-effector frame $\sum_{b6} (x_{b6}, y_{b6}, z_{b6})$, is calculated using Eq. (7).

$$H_b^{b8} = H_b^{b5} H_{b5}^{b6} H_{b6}^{b7} H_{b7}^{b8} \tag{7}$$

4. Inverse Kinematics

The matrix H_0^7 can be computed using de forward kinematic model. Then, by successively multiplying H_0^7 by the inverse matrix of H_{i-1}^i, seven matrixes can be obtained:

$$H_0^7 = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6 H_6^7$$

$$(H_0^1)^{-1} H_0^7 = H_1^2 H_2^3 H_3^4 H_4^5 H_5^6 H_6^7$$

$$(H_1^2)^{-1} (H_0^1)^{-1} H_0^7 = H_2^3 H_3^4 H_4^5 H_5^6 H_6^7$$

$$(H_2^3)^{-1} (H_1^2)^{-1} (H_0^1)^{-1} H_0^7 = H_3^4 H_4^5 H_5^6 H_6^7$$

$$(H_3^4)^{-1} (H_2^3)^{-1} (H_1^2)^{-1} (H_0^1)^{-1} H_0^7 = H_4^5 H_5^6 H_6^7$$

$$(H_4^5)^{-1} (H_3^4)^{-1} (H_2^3)^{-1} (H_1^2)^{-1} (H_0^1)^{-1} H_0^7 = H_5^6 H_6^7$$

$$(H_5^6)^{-1} (H_4^5)^{-1} (H_3^4)^{-1} (H_2^3)^{-1} (H_1^2)^{-1} (H_0^1)^{-1} H_0^7 = H_6^7$$

The elements of matrix H_{i-1}^i are as follows:

$$H_{i-1}^i = \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where the matrix *noa* can be defined as follows:

$$noa = \begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix}$$

4.1. Inverse Kinematics of Legs

The kinematic decoupling method presented in [28, 29] is used to simplify the robot's legs inverse kinematic model, which consists of the separation of orientation and position in robots with 6 degrees of freedom; Robots usually have three additional degrees of freedom, located at the end of the kinematic chain, and those axes generally intersect at a point informally called the robot's wrist. Thus, given a desired final position and orientation, the position of the cutting point (robot wrist) is established by calculating the values of $\theta_1,\ \theta_2$ and θ_3, and then from the orientation data and those already calculated, the values of the rest of the joint variables $\theta_4,\ \theta_5$ and θ_6 are obtained. Similarly, the three hip axes of the robot are considered as the wrist of a robot manipulator, for which reason the position of the cutting point of the three axes of the hip, at this point, the origins of the reference systems of the three coincide. degrees of freedom of the hip.

Then, the first three joints of the leg can be calculated taking into account the matrixes $H_0^1, H_1^2, H_2^3, H_3^4$, which were obtained in the direct kinematics model. Therefore, using the inverse matrix, the following matrix equation can be determined:

$$(H_2^3)^{-1}(H_1^2)^{-1}(H_0^1)^{-1}H_0^4 = H_3^4$$

$$\begin{pmatrix} C_2 & S_2 & 0 & -l_1 \\ -S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_1 & S_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -S_1 & C_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_3 & -S_3 & 0 & l_2C_3 \\ S_3 & C_3 & 0 & l_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

$$\begin{pmatrix} r_{1,1} & o_yS_2 + o_xC_1C_2 + o_zC_2S_1 & a_yS_2 + a_xC_1C_2 + a_zC_2S_1 & P_yS_2 - l_1 + P_xC_1C_2 + P_zC_2S_1 - d_1C_2S_1 \\ r_{2,1} & o_yC_2 - o_xC_1S_2 - o_zS_1S_2 & a_yC_2 - a_xC_1S_2 - a_zS_1S_2 & P_yC_2 - P_xC_1C_2 - P_zS_1S_2 + d_1S_1S_2 \\ r_{3,1} & o_zC_1 - o_xS_1 & a_zC_1 - a_xS_1 & P_zC_1 - P_xS_1 - d_1C_1 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} C_3 & -S_3 & 0 & l_2C_3 \\ S_3 & C_3 & 0 & l_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{9}$$

where

$$r_{1,1} = n_yS_2 + n_xC_1C_2 + n_zC_2S_1$$

$$r_{2,1} = n_yC_2 - n_xC_1S_2 - n_zS_1S_2$$

$$r_{3,1} = n_zC_1 - n_xS_1$$

Analyzing Eq. (9) it is possible to match the 16 terms that a matrix contains, in other words, 16 equations can be proposed and the one that is most friendly to clear the joint variable can be chosen. Therefore, from (9) the angles $\theta_1,\ \theta_2,\ \theta_3$ can be calculated. First, θ_3 is calculated using the (3,4) term on both sides of the equation, as follows:

$$P_zcos(\theta_1) - P_xsin(\theta_1) - d_1cos(\theta_1) = 0$$

$$\theta_1 = arctan\left(\frac{P_z - d_1}{P_x}\right) \tag{10}$$

Then θ_2 is calculated using the (2,4) term on both sides of the equation:

$$o_y\text{sin}(\theta_2) + o_x\text{cos}(\theta_1)\text{cos}(\theta_2) + o_z\text{cos}(\theta_2)\text{sin}(\theta_1) = 0$$

$$\theta_2 = arctan\left(\frac{o_x\text{cos}(\theta_1) + o_z\text{sin}(\theta_1)}{-o_y}\right) \tag{11}$$

Subsequently, θ_1 is calculated using the term (3,3) on both sides of the equation:

$$\begin{aligned} A = {} & P_y\text{cos}(\theta_2) - P_x\text{cos}(\theta_1)\text{sin}(\theta_2) \\ & - P_z\text{sin}(\theta_1)\text{sin}(\theta_2) + d_1\text{sin}(\theta_1)\text{sin}(\theta_2) \end{aligned}$$

$$\begin{aligned} B = {} & P_y\text{sin}(\theta_2) - l_1 + P_x\text{cos}(\theta_1)\text{cos}(\theta_2) \\ & + P_z\text{cos}(\theta_2)\text{sin}(\theta_1) - d_1\text{cos}(\theta_2)\text{sin}(\theta_1) \end{aligned}$$

$$\frac{l_2\text{sin}(\theta_3)}{l_2\text{cos}(\theta_3)} = \frac{A}{B}$$

$$\theta_3 = arctan\left(\frac{A}{B}\right) \tag{12}$$

The next step is to find the joint variables θ_4, θ_5 and θ_6, using the matrix equation (8), in which it is not necessary to use the homogeneous transformation matrices because there are no translations, only rotations, for this reason reason you can use only the rotation submatrices.

The rotation matrix from 0 to 6 can be written in a generic way through the noa matrix, which is nothing more than the total rotation matrix that has been carried out with the last coordinate system that corresponds to the hip on the transversal axis.

Using the Denavit-Hartenberg parameters from Table 3 it is possible to define the rotation matrix R_4^7 as observed in

$$R_4^7 = R_4^5R_5^6R_6^7 \tag{13}$$

where:

$$R_4^5 = \begin{pmatrix} C_4 & 0 & S_4 \\ S_4 & 0 & -C_4 \\ 0 & 1 & 0 \end{pmatrix} \quad R_5^6 = \begin{pmatrix} C_5 & 0 & S_5 \\ S_5 & 0 & -C_5 \\ 0 & 1 & 0 \end{pmatrix}$$

$$R_6^7 = \begin{pmatrix} C_6 & -S_6 & 0 \\ S_6 & C_6 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus:

$$R_4^7 = \begin{pmatrix} S_4S_6 + C_4C_5C_6 & C_6S_4 - C_4C_5S_6 & C_4S_5 \\ C_5C_6S_4 - C_4S_6 & -C_4C_6 - C_5S_4S_6 & S_4S_5 \\ C_6S_5 & -S_5S_6 & -C_5 \end{pmatrix} \tag{14}$$

The rotation matrix from 0 to 3 is found with the parameters α, θ and l from Table 2.

$$R_0^4 = R_0^1 R_1^2 R_2^3 R_3^4 \tag{15}$$

$$R_0^1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \quad R_1^2 = \begin{pmatrix} C_1 & 0 & -S_1 \\ S_1 & 0 & C_1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$R_2^3 = \begin{pmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_3^4 = \begin{pmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Thus:

$$R_0^4 = \begin{pmatrix} C_{2,3}C_1 & -S_{2,3}C_1 & -S_1 \\ S_{2,3} & C_{2,3} & 0 \\ C_{2,3}S_1 & -S_{2,3}S_1 & C_1 \end{pmatrix}$$

$$(R_0^4)^{-1} = (R_0^4)^T = \begin{pmatrix} C_{2,3}C_1 & S_{2,3} & C_{2,3}S_1 \\ -S_{2,3}C_1 & C_{2,3} & -S_{2,3}S_1 \\ -S_1 & 0 & C_1 \end{pmatrix} \tag{16}$$

Substituting Eqs. (15) and (16) and the matrix noa in the equation we have:

$$R_4^7 = (R_0^3)^T R_0^7 \tag{17}$$

$$\begin{pmatrix} S_4S_6 + C_4C_5C_6 & C_6S_4 - C_4C_5S_6 & C_4S_5 \\ C_5C_6S_4 - C_4S_6 & -C_4C_6 - C_5S_4S_6 & S_4S_5 \\ C_6S_5 & -S_5S_6 & -C_5 \end{pmatrix}$$
$$= \begin{pmatrix} C_{2,3}C_1 & S_{2,3} & C_{2,3}S_1 \\ -S_{2,3}C_1 & C_{2,3} & -S_{2,3}S_1 \\ -S_1 & 0 & C_1 \end{pmatrix} \begin{pmatrix} n_x & o_x & a_x \\ n_y & o_y & a_y \\ n_z & o_z & a_z \end{pmatrix} \tag{18}$$

From Eq. (18) the terms that generate a friendly equation are chosen to clear the joint variables θ_4, θ_5 and θ_6. First, θ_4 is calculated using the term (3,3) on both sides of Eq. (18), as follows:

$$C = a_y\cos(\theta_2+\theta_3) - a_x\sin(\theta_2+\theta_3)\cos(\theta_1) - a_z\sin(\theta_2+\theta_3)\sin(\theta_1)$$

$$D = a_y\sin(\theta_2+\theta_3) + a_x\cos(\theta_2+\theta_3)\cos(\theta_1) + a_z\cos(\theta_2+\theta_3)\sin(\theta_1)$$

$$\frac{\sin(\theta_4)\sin(\theta_5)}{\cos(\theta_4)\sin(\theta_5)} = \frac{C}{D}$$

$$\theta_4 = arctan\left(\frac{C}{D}\right) \tag{19}$$

Then θ_5 is calculated using the term (2,2) as follows:

$$\frac{\cos(\theta_4)\sin(\theta_5)}{-\cos(\theta_5)} = \frac{D}{a_z\cos(\theta_1) - a_x\sin(\theta_1)}$$

Table 6. Inverse kinematics equations of the robot's legs

Link	Equation
1	$\theta_1 = arctan\left(\frac{P_z - d_1}{P_x}\right)$
2	$\theta_2 = arctan\left(\frac{o_x\cos(\theta_1)+o_z\sin(\theta_1)}{-o_y}\right)$
3	$\theta_3 = arctan\left(\frac{A}{B}\right)$
4	$\theta_4 = arctan\left(\frac{C}{D}\right)$
5	$\theta_5 = arctan\left(\frac{D}{-(\cos(\theta_4)(a_z\cos(\theta_1)-a_x\sin(\theta_1)))}\right)$
6	$\theta_6 = arctan\left(\frac{-(o_z\cos(\theta_1)-o_x\sin(\theta_1))}{n_z\cos(\theta_1)-n_x\sin(\theta_1)}\right)$

$$\theta_5 = arctan\left(\frac{D}{-(\cos(\theta_4)(a_z\cos(\theta_1) - a_x\sin(\theta_1)))}\right) \tag{20}$$

Then, θ_6 is calculated using the term (1,2), as follows:

$$-\frac{\sin(\theta_5)\sin(\theta_6)}{\cos(\theta_6)\sin(\theta_5)} = \frac{o_z\cos(\theta_1) - o_x\sin(\theta_1)}{n_z\cos(\theta_1) - n_x\sin(\theta_1)}$$

$$\theta_6 = arctan\left(\frac{-(o_z\cos(\theta_1) - o_x\sin(\theta_1))}{n_z\cos(\theta_1) - n_x\sin(\theta_1)}\right) \tag{21}$$

The equations to find the angles of legs are shown in Table 6.

It is important to mention that the previous process is the same to calculate the value of joint positions $\theta_1, \theta_2, \theta_3, \theta_4, \theta_5$ and θ_6 of both legs.

4.2. Inverse Kinematics of Arms

To obtain the inverse kinematics of the left arm, consider the elements of matrix H_d^{d4}, which is shown in Eq. (6):

$$H_d^{d4} = \begin{pmatrix} n_{bx} & o_{bx} & a_{bx} & p_x \\ n_{by} & o_{by} & a_{by} & p_y \\ n_{bz} & o_{bz} & a_{bz} & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{22}$$

Then, from (6) the following matrix equation is defined:

$$(H_{b2}^{b3})^{-1}(H_{b1}^{b2})^{-1}(H_{b0}^{b1})^{-1}H_b^{b4} = H_{b3}^{b4}$$

$$\begin{pmatrix} C_{b2} & S_{b2} & 0 & -l_4 \\ -S_{b2} & C_{b2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{b1} & S_{b1} & 0 & -l_3 \\ 0 & 0 & -1 & d_3 \\ -S_{b1} & C_{b1} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & -d_2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} n_{bx} & o_{bx} & a_{bx} & p_{bx} \\ n_{by} & o_{by} & a_{by} & p_{by} \\ n_{bz} & o_{bz} & a_{bz} & p_{bz} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} C_{b3} & -S_{b3} & 0 & l_5C_{b3} \\ S_{b3} & C_{b3} & 0 & l_5S_{b3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{matrix} n_{bx}S_{b2} - n_{by}C_{b1}C_{b2} + n_{bz}C_{b2}S_{b1} \\ n_{bx}C_{b2} + n_{by}C_{b1}S_{b2} - n_{bz}S_{b1}S_{b2} \\ n_{bz}C_{b1} + n_{by}S_{b1} \\ 0 \end{matrix}\right.$$

$$\begin{matrix} o_{bx}S_{b2} - o_{by}C_{b1}C_{b2} + o_{bz}C_{b2}S_{b1} \\ o_{bx}C_{b2} + o_{by}C_{b1}S_{b2} - o_{bz}S_{b2}S_{b2} \\ o_{bz}C_{b1} + o_{by}S_{b1} \\ 0 \end{matrix}$$

$$\left.\begin{matrix} a_{bx}S_{b2} - a_{by}C_{b1}C_{b2} + a_{bz}C_{b2}S_{b1} & r_{1,4} \\ a_{bx}C_{b2} + a_{by}C_{b1}S_{b2} - a_{bz}S_{b2}S_{b2} & r_{2,4} \\ a_{bz}C_{b1} + a_{by}S_{b1} & r_{3,4} \\ 0 & 1 \end{matrix}\right)$$

$$= \begin{pmatrix} C_{b3} & -S_{b3} & 0 & l_5C_{b3} \\ S_{b3} & C_{b3} & 0 & l_5S_{b3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{23}$$

where,

$$r_{1,4} = P_xS_{b2} - l_4 - l_3C_{b2} + d_3S_{b2} - P_yC_{b1}C_{b2} + P_zC_{b2}S_{b1} - d_2C_{b2}S_{b1}$$

$$r_{2,4} = P_xC_{b2} + d_3C_{b2} + l_3S_{b2} + P_yC_{b1}S_{b2} - P_zS_{b1}S_{b2} + d_2S_{b1}S_{b2}$$

$$r_{3,4} = P_zC_{b1} + P_yS_{b1} - d_2C_{b1}$$

Taking the quotient of the elements (3,4) of both sides of Eq. (23) the angle θ_{b1} is calculated as follows:

$$P_z\cos(\theta_{b1}) + P_y\sin(\theta_{b1}) - d_2\cos(\theta_{b1}) = 0$$

$$\theta_{b1} = arctan\left(\frac{P_z - d_2}{-P_y}\right) \tag{24}$$

Considering the element (1,3) of both sides of Eq. (23) the angle θ_{b2} is calculated as follows:

$$a_x\sin(\theta_{b2}) - a_y\cos(\theta_{b1})\cos(\theta_{b2}) + a_z\cos(\theta_{b2})\sin(\theta_{b1}) = 0$$

$$\theta_{b2} = arctan\left(\frac{a_y\cos(\theta_{b1}) - a_z\sin(\theta_{b1})}{a_x}\right) \tag{25}$$

Using the element (2,4) and (1,4) of both sides of the equation, the angle θ_{b3} is calculated:

$$E = P_x\cos(\theta_{b2}) + d_3\cos(\theta_{b2}) + l_3\sin(\theta_{b2}) + P_y\cos(\theta_{b1})\sin(\theta_{b2}) - P_z\sin(\theta_{b1})\sin(\theta_{b2}) + d_2\sin(\theta_{b1})\sin(\theta_{b2})$$

$$F = P_x\sin(\theta_{b2}) - l_4 - l_3\cos(\theta_{b2}) + d_3\sin(\theta_{b2}) - P_y\cos(\theta_{b1})\cos(\theta_{b2}) + P_z\cos(\theta_{b2})\sin(\theta_{b1}) - d_2\cos(\theta_{b2})\sin(\theta_{b1})$$

$$\frac{l_5\sin(\theta_{b3})}{l_5\cos(\theta_{b3})} = \frac{E}{F}$$

$$\theta_{b3} = arctan(E, F) \tag{26}$$

The equations to find the angles of right arm are shown in Table 7.

To obtain the inverse kinematics of the right arm, consider the equation shown in Eq. (27).

The following matrix equation is defined:

$$(H_{b6}^{b7})^{-1}(H_{b5}^{b6})^{-1}(H_{b}^{b5})^{-1}H_{b}^{b8} = H_{b7}^{b8}$$

Table 7. Inverse kinematics equations of the robot's right arm

Link	Equation
1	$\theta_{b1} = arctan\left(\frac{P_z - d_2}{-P_y}\right)$
2	$\theta_{b2} = arctan\left(\frac{a_y\cos(\theta_{b1}) - a_z\sin(\theta_{b1})}{a_x}\right)$
3	$\theta_{b3} = arctan(E, F)$

$$\begin{pmatrix} C_{b5} & S_{b5} & 0 & -l_4 \\ -S_{b5} & C_{b5} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_{b4} & S_{b4} & 0 & -l_3 \\ 0 & 0 & 1 & -d_3 \\ S_{b4} & -C_{b4} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & d_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} nb_x & ob_x & ab_x & pb_x \\ nb_y & ob_y & ab_y & pb_y \\ nb_z & ob_z & ab_z & pb_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} C_{b6} & -S_{b6} & 0 & l_5C_{b6} \\ S_{b6} & C_{b6} & 0 & l_5S_{b6} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{matrix} n_{bx}S_{b5} - n_{by}C_{b4}C_{b5} - n_{bz}C_{b5}S_{b4} \\ n_{bx}C_{b5} + n_{by}C_{b4}S_{b5} + n_{bz}S_{b4}S_{b5} \\ n_{bz}C_{b4} - n_{by}S_{b4} \\ 0 \end{matrix}\right.$$

$$\begin{matrix} o_{bx}S_{b5} - o_{by}C_{b4}C_{b5} - o_{bz}C_{b5}S_{b4} \\ o_{bx}C_{b5} + o_{by}C_{b4}S_{b5} + o_{bz}S_{b4}S_{b5} \\ o_{bz}C_{b4} + o_{by}S_{b4} \\ 0 \end{matrix}$$

$$\left.\begin{matrix} a_{bx}S_{b5} - a_{by}C_{b4}C_{b5} - a_{bz}C_{b5}S_{b4} & u_{1,4} \\ a_{bx}C_{b5} + a_{by}C_{b4}S_{b5} + a_{bz}S_{b4}S_{b5} & u_{2,4} \\ a_{bz}C_{b4} - a_{by}S_{b4} & u_{3,4} \\ 0 & 1 \end{matrix}\right)$$

$$= \begin{pmatrix} C_{b6} & -S_{b6} & 0 & l_5C_{b6} \\ S_{b6} & C_{b6} & 0 & l_5S_{b6} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{27}$$

where,

$$u_{1,4} = P_xS_{b5} - l_4 - l_3C_{b5} - d_3S_{b5} - P_yC_{b5}C_{b5} - P_zC_{b5}S_{b4} + d_2C_{b5}S_{b4}$$

$$u_{2,4} = P_xC_{b5} - d_3C_{b5} + l_3S_{b5} + P_yC_{b4}S_{b5} + P_zS_{b4}S_{b5} - d_2S_{b4}S_{b5}$$

$$r_{3,4} = P_zC_{b4} - P_yS_{b4} - d_2C_{b4}$$

Taking the quotient of the elements (3,4) of both sides of Eq. (27) the angle θ_{b4} is calculated as follows:

$$P_z\cos(\theta_{b4}) - P_y\sin(\theta_{b4}) - d_2\cos(\theta_{b4}) = 0$$

$$\theta_{b4} = arctan\left(\frac{P_z - d_2}{P_y}\right) \tag{28}$$

Considering the element (1,3) of both sides of Eq. (27) the angle θ_{b5} is calculated as follows:

$$a_x\sin(\theta_{b5}) - a_y\cos(\theta_{b4})\cos(\theta_{b5}) - a_z\cos(\theta_{b5})\sin(\theta_{b4}) = 0$$

Table 8. Inverse kinematics equations of the robot's right arm

Link	Equation
1	$\theta_{b4} = arctan\left(\frac{P_z - d_2}{P_y}\right)$
2	$\theta_{b5} = arctan\left(\frac{a_y \cos(\theta_{b1}) + a_z \sin(\theta_{b1})}{a_x}\right)$
3	$\theta_{b6} = arctan(G, H)$

$$\theta_{b5} = arctan\left(\frac{a_y \cos(\theta_{b1}) + a_z \sin(\theta_{b1})}{a_x}\right) \tag{29}$$

Using the element (2,4) and (1,4) of both sides of the equation, the angle θ_{b6} is calculated:

$$\begin{aligned} G = {} & P_x \cos(\theta_{b5}) - d_3 \cos(\theta_{b5}) + l_3 \sin(\theta_{b5}) \\ & + P_y \cos(\theta_{b4}) \sin(\theta_{b5}) + P_z \sin(\theta_{b4}) \sin(\theta_{b5}) \\ & - d_2 \sin(\theta_{b4}) \sin(\theta_{b5}) \end{aligned}$$

$$\begin{aligned} H = {} & P_x \sin(\theta_{b5}) - l_4 - l_3 \cos(\theta_{b5}) - d_3 \sin(\theta_{b5}) \\ & - P_y \cos(\theta_{b4}) \cos(\theta_{b5}) - P_z \cos(\theta_{b5}) \sin(\theta_{b4}) \\ & + d_2 \cos(\theta_{b5}) \sin(\theta_{b4}) \end{aligned}$$

$$\frac{l_5 \sin(\theta_{b6})}{l_5 \cos(\theta_{b6})} = \frac{G}{H}$$

$$\theta_{b6} = arctan(G, H) \tag{30}$$

The equations to find the angles that correspond to the joints of the left arm are shown in Table 8.

5. Conclusion

This paper presents a complete solution of the inverse kinematics model using the Denavit-Hartenberg methodology for a 18 DOF robot. The forward kinematics model allowed to represent the Bioloid Premium robot.

Unlike the other geometric methods, our research proposal considers the decoupling kinematic method, taking the feet and the pelvis as points of origin, generating 4 open kinematic chains to calculate the joint positions of both arms and legs of the robot in a three-dimensional space (x, y, z), consequently it is possible to determine the final position of each end-effector of the robot, taking the supporting feet as fixed reference frame.

This methodology is an important step forward to obtaining the differential kinematics and subsequently calculating the dynamic model of the robot in a later work.

On the other hand, the proposed methodology can be extended to other biped robots.

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