In this paper, a complete inverse kinematic model for a 18 DOF robot is proposed. The authors have the objective of modeling only the lower train of the robots, either using commercial robots such as Nao with 21 DOF, or robots with 10 DOF and NWPUBR. All of these research papers calculate the geometric parameters of the robot. In Section 3, forward kinematic model is obtained. The equations of inverse kinematics of the robot are computed in Section 2. The main motivation in this paper is to develop a methodology based on the Denavit-Hartenberg method to represent the position and orientation of the end-effector.

In the present work, the authors propose to obtain the complete kinematic model of the Bioloid robot, considering four open kinematic chains, where the initial frames are the support feet, and we have the left and right pelvis as end-effector frames; the pelvis is also proposed as another initial frame to have the left and right hand as the other end-effector frames.

The paper is organized as follows. In Section 2 the Denavit-Hartenberg method is applied to calculate the geometric parameters of the robot. In Section 3, forward kinematic model is obtained. The equations of inverse kinematics of the robot are computed in Section 4. Finally, the conclusions are given in Section 5.
Table 1. Denavit-Hartenberg parameters of the legs

<table>
<thead>
<tr>
<th>Link</th>
<th>$\alpha_i$</th>
<th>$l_i$</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/2$</td>
<td>0</td>
<td>0</td>
<td>$d_1$</td>
</tr>
<tr>
<td>2</td>
<td>$-\pi/2$</td>
<td>0</td>
<td>$\theta_{b1}$</td>
<td>$d_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$l_1$</td>
<td>$\theta_{b2}$</td>
<td>$d_3$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$l_2$</td>
<td>$\theta_{b3}$</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\theta_{b4}$</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$\theta_{b5}$</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0</td>
<td>$\theta_{b6}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. Denavit-Hartenberg parameters of the right arm

<table>
<thead>
<tr>
<th>Link</th>
<th>$\alpha_i$</th>
<th>$l_i$</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\pi/2$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$d_2$</td>
</tr>
<tr>
<td>2</td>
<td>$-\pi/2$</td>
<td>0</td>
<td>$\theta_{b1}$</td>
<td>$d_3$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$l_1$</td>
<td>$\theta_{b2}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$l_2$</td>
<td>$\theta_{b3}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. Denavit-Hartenberg parameters of the left arm

<table>
<thead>
<tr>
<th>Link</th>
<th>$\alpha_i$</th>
<th>$l_i$</th>
<th>$\theta_i$</th>
<th>$d_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$-\pi/2$</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>$d_2$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi/2$</td>
<td>$l_3$</td>
<td>$\theta_{b4}$</td>
<td>$d_3$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$l_4$</td>
<td>$\theta_{b5}$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$l_5$</td>
<td>$\theta_{b6}$</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Value of the joints corresponding to the home position of the robot legs

<table>
<thead>
<tr>
<th>$\theta_{b1}$</th>
<th>$\theta_{b2}$</th>
<th>$\theta_{b3}$</th>
<th>$\theta_{b4}$</th>
<th>$\theta_{b5}$</th>
<th>$\theta_{b6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi/2$</td>
<td>0</td>
<td>0</td>
<td>$-\pi/2$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5. Value of the joints corresponding to the home position of the robot arms

<table>
<thead>
<tr>
<th>$\theta_{b1}$</th>
<th>$\theta_{b2}$</th>
<th>$\theta_{b3}$</th>
<th>$\theta_{b4}$</th>
<th>$\theta_{b5}$</th>
<th>$\theta_{b6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\pi/2$</td>
<td>0</td>
<td>0</td>
<td>$\pi/2$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

$d_1 = 33$ mm, $d_2 = 118$ mm, $d_3 = 73$ mm,
$l_1 = l_2 = 76$ mm, $l_3 = 16$ mm,
$l_4 = 66$ mm, $l_5 = 108$ mm.

3. Forward Kinematics

To calculate the forward kinematics of the robot, the transformation matrix defined in Equation (1) was used.

$$H_{i-1}^i = \begin{pmatrix}
\cos(\theta_i) - \sin(\theta_i)\cos(\alpha_i) & \sin(\theta_i)\sin(\alpha_i) & 1\cos(\theta_i)
\sin(\theta_i) \cos(\alpha_i) & \cos(\theta_i) \sin(\alpha_i) & 1\sin(\theta_i)
0 & -\sin(\alpha_i) & 0
0 & \cos(\alpha_i) & 0
\end{pmatrix}
$$

Where the superscript $i$ represents the number of the current joint and the subscript $i-1$ indicates the number of the previous joint. Therefore, $H_{i-1}^i$ is the homogeneous transformation matrix representing the rotation and translation of joint $i$ with respect to joint $i-1$.

To simplify the results obtained, the following compact notation is used:

$$
\sin(\theta_i) = S_i, \cos(\theta_i) = C_i,
\sin(\theta_i + \theta_j) = S_{i,j}, \cos(\theta_i + \theta_j) = C_{i,j}
$$

where $i, j$ denote the joint number.

The transformation matrix $H_0^i$ relating the frame $\sum_0 (x_0, y_0, z_0)$ to frame $\sum_1 (x_1, y_1, z_1)$ corresponding to the robot’s foot is shown in (2), where $\alpha = \pi/2, \theta = l = 0$.

$$H_0^1 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 1 & 0 & d_1 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

The homogeneous transformation matrices corresponding to the leg joints from the frame $\sum_1 (x_1, y_1, z_1)$ to frame $\sum_6 (x_6, y_6, z_6)$ are as follows:

$$H_1^2 = \begin{pmatrix}
C_1 & 0 & -S_1 & 0 \\
S_1 & 0 & C_1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
$$
The homogenous transformation matrices corresponding to the joints of the left arm, from the frame $\Sigma_b(t_x, y_0, z_0)$ to frame $\Sigma_b(t_x, y_0, z_0)$ are as follows:

\[
H_1 = \begin{pmatrix}
0 & -S_3 & C_3 & 0 \\
C_3 & S_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_2 = \begin{pmatrix}
0 & -S_4 & C_4 & 0 \\
C_4 & S_4 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_3 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & z_0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_4 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & y_0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_5 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

The elements of matrix $H_1$ are as follows:

\[
H_1 = \begin{pmatrix}
0 & -S_3 & C_3 & 0 \\
C_3 & S_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

The forward kinematics relating the right foot frame $\Sigma_b(t_x, y_0, z_0)$ and the right hand effector frame $\Sigma_b(t_x, y_0, z_0)$ is calculated using Eq. (6).

\[
H_1 = \begin{pmatrix}
0 & -S_3 & C_3 & 0 \\
C_3 & S_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_2 = \begin{pmatrix}
0 & -S_4 & C_4 & 0 \\
C_4 & S_4 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_3 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & z_0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_4 = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & y_0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
H_5 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

The forward kinematics relating the right foot frame $\Sigma_b(t_x, y_0, z_0)$ and the right hand effector frame $\Sigma_b(t_x, y_0, z_0)$ is calculated using Eq. (6).
4.1. Inverse Kinematics of Legs

The kinematic decoupling method presented in [28,29] is used to simplify the robot’s legs inverse kinematic model, which consists of the separation of orientation and position in robots with 6 degrees of freedom; Robots usually have three additional degrees of freedom, located at the end of the kinematic chain, and those axes generally intersect at a point informally called the robot’s wrist. Thus, given a desired final position and orientation, the position of the cutting point (robot wrist) is established by calculating the values of \( \theta_1, \theta_2 \) and \( \theta_3 \), and then from the orientation data and those already calculated, the values of the rest of the joint variables \( \theta_1, \theta_5 \) and \( \theta_6 \) are obtained. Similarly, the three hip axes of the robot are considered as the wrist of a robot manipulator, for which reason the position of the cutting point of the three axes of the hip, at this point, the origins of the reference systems of the three coincide. The orientation data and those already calculated, the values of the rest of the joint variables \( \theta_1, \theta_5 \) and \( \theta_6 \) are obtained. Similarly, the three hip axes of the robot are considered as the wrist of a robot manipulator, for which reason the position of the cutting point of the three axes of the hip, at this point, the origins of the reference systems of the three coincide. degrees of freedom of the hip.

Then, the first three joints of the leg can be calculated taking into account the matrixes \( H_0^1, H_1^2, H_2^3, H_3^4 \), which were obtained in the direct kinematics model. Therefore, using the inverse matrix, the following matrix equation can be determined:

\[
(H_3^4)^{-1}(H_2^3)^{-1}(H_1^2)^{-1}H_0^1 = H_0^2
\]

For \( n = 1, 2, 3 \),

\[
\begin{pmatrix}
C_3 & S_3 & 0 & l_2 C_3 \\
S_3 & C_3 & 0 & l_2 S_3 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
= \begin{pmatrix}
r_{1,1} & o_y S_2 + o_x C_1 C_2 + o_z C_2 S_1 \\
r_{2,1} & o_y C_2 - o_x C_1 S_2 - o_z S_1 S_2 \\
r_{3,1} & o_x C_1 - o_z S_1 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

where

\[
r_{1,1} = n_y S_2 + n_x C_1 C_2 + n_z C_2 S_1
\]

Analyzing Eq. (9) it is possible to match the 16 terms that a matrix contains, in other words, 16 equations can be proposed and the one that is most friendly to clear the joint variable can be chosen. Therefore, from (9) the angles \( \theta_1, \theta_2, \theta_3 \) can be calculated. First, \( \theta_1 \) is calculated using the (3,4) term on both sides of the equation, as follows:

\[
P_z \cos(\theta_1) - P_z \sin(\theta_1) - d_1 \cos(\theta_1) = 0
\]

\[
\theta_1 = \arctan \left( \frac{P_z - d_1}{P_x} \right)
\]

Then \( \theta_2 \) is calculated using the (2,4) term on both sides of the equation:

\[
o_y \sin(\theta_2) + o_x \cos(\theta_1) \cos(\theta_2) + o_z \cos(\theta_2) \sin(\theta_1) = 0
\]

\[
\theta_2 = \arctan \left( \frac{o_x \cos(\theta_1) + o_z \sin(\theta_1)}{-o_y} \right)
\]

Subsequently, \( \theta_1 \) is calculated using the term (3,3) on both sides of the equation:

\[
A = P_y \cos(\theta_2) - P_z \cos(\theta_1) \sin(\theta_2)
\]

\[
B = P_y \sin(\theta_2) - l_1 + P_x \cos(\theta_1) \cos(\theta_2)
\]

\[
+ P_z \cos(\theta_2) \sin(\theta_1) - d_1 \cos(\theta_2) \sin(\theta_1)
\]

\[
l_2 \sin(\theta_3) = \frac{A}{B}
\]

\[
\theta_3 = \arctan \left( \frac{A}{B} \right)
\]

The next step is to find the joint variables \( \theta_4, \theta_5 \) and \( \theta_6 \) using the matrix equation (8), in which it is not necessary to use the homogeneous transformation matrices because there are no translations, only rotations, for this reason reason you can use only the rotation submatrices.

The rotation matrix from 0 to 6 can be written in a generic way through the noa matrix, which is nothing more than the total rotation matrix that has been carried out with the last coordinate system that corresponds to the hip on the transversal axis.

Using the Denavit-Hartenberg parameters from Table 3 it is possible to define the rotation matrix \( R_i^j \) as observed in

\[
R_i^j = R_i^k R_k^l R_l^j
\]

where:

\[
R_i^j = \begin{pmatrix}
C_4 & 0 & S_4 \\
S_4 & 0 & -C_4 \\
0 & 1 & 0
\end{pmatrix}
\]

\[
R_j^k = \begin{pmatrix}
C_5 & 0 & S_5 \\
S_5 & 0 & -C_5 \\
0 & 1 & 0
\end{pmatrix}
\]

\[
R_k^l = \begin{pmatrix}
C_6 & -S_8 & 0 \\
S_6 & C_8 & 0 \\
0 & 0 & 1
\end{pmatrix}
\]
Thus:
\[ R_4^T = \begin{pmatrix} S_4 S_6 + C_4 C_5 C_6 & C_6 S_4 - C_4 C_5 S_6 & C_4 S_5 \\ C_5 C_6 S_4 - C_4 S_6 & -C_4 C_6 - C_5 S_4 S_6 & S_4 S_5 \\ C_6 S_5 & -S_6 S_6 & -C_6 \end{pmatrix} \]
\[ R_3^T = \begin{pmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} R_4^T = \begin{pmatrix} C_3 & -S_3 & 0 \\ S_3 & C_3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

The rotation matrix from 0 to 3 is found with the parameters \( \alpha, \theta \) and \( l \) from Table 2.

\[ R_0^T = R_0^T R_2^T R_1^T R_0^T \]
\[ R_0^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \]
\[ R_1^T = \begin{pmatrix} C_1 & 0 & -S_1 \\ S_1 & 0 & C_1 \\ 0 & -1 & 0 \end{pmatrix} \]
\[ R_2^T = \begin{pmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

Thus:
\[ R_0^T = \begin{pmatrix} C_{2,3} C_1 & S_{2,3} S_1 & C_{2,3} S_1 \\ -S_{2,3} C_1 & C_{2,3} & -S_{2,3} S_1 \\ -S_1 & C_{2,3} & C_2 \end{pmatrix} \]
\[ (R_0^T)^{-1} = (R_0^T)^T \]

Substituting Eqs. (15) and (16) and the matrix \( n_o a \) in the equation we have:
\[ R_3^T = (R_0^T)^T R_7^T \]

\[ \begin{pmatrix} S_4 S_6 + C_4 C_5 C_6 & C_6 S_4 - C_4 C_5 S_6 & C_4 S_5 \\ C_5 C_6 S_4 - C_4 S_6 & -C_4 C_6 - C_5 S_4 S_6 & S_4 S_5 \\ C_6 S_5 & -S_6 S_6 & -C_6 \end{pmatrix} = \begin{pmatrix} C_{2,3} C_1 & S_{2,3} S_1 & C_{2,3} S_1 \\ -S_{2,3} C_1 & C_{2,3} & -S_{2,3} S_1 \\ -S_1 & C_{2,3} & C_2 \end{pmatrix} \]

From Eq. (18) the terms that generate a friendly equation are chosen to clear the joint variables \( \theta_4, \theta_5 \) and \( \theta_6 \). First, \( \theta_4 \) is calculated using the term (3,3) on both sides of Eq. (18), as follows:
\[ C = a_y \cos(\theta_2 + \theta_3) - a_x \sin(\theta_2 + \theta_3) \cos(\theta_1) \]
\[ -a_y \sin(\theta_2 + \theta_3) \sin(\theta_1) \]
\[ D = a_y \sin(\theta_2 + \theta_3) + a_x \cos(\theta_2 + \theta_3) \cos(\theta_1) \]
\[ +a_x \cos(\theta_2 + \theta_3) \sin(\theta_1) \]
\[ \sin(\theta_4) \sin(\theta_5) \]
\[ \cos(\theta_4) \sin(\theta_5) \]
\[ \theta_4 = \arctan \left( \frac{C}{D} \right) \]

Then \( \theta_5 \) is calculated using the term (2,2) as follows:
\[ \frac{\cos(\theta_4) \sin(\theta_5)}{-\cos(\theta_5)} = \frac{D}{a_y \cos(\theta_1) - a_x \sin(\theta_1)} \]

Table 6. Inverse kinematics equations of the robot’s legs

<table>
<thead>
<tr>
<th>Link</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \theta_1 = \arctan \left( \frac{R_{3y} - d_1}{R_{3x}} \right) )</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_2 = \arctan \left( \frac{a_y \cos(\theta_1) + a_x \sin(\theta_1)}{-y} \right) )</td>
</tr>
<tr>
<td>3</td>
<td>( \theta_3 = \arctan \left( \frac{z}{l_2 - b_1} \right) )</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_4 = \arctan \left( \frac{D}{D} \right) )</td>
</tr>
<tr>
<td>5</td>
<td>( \theta_5 = \arctan \left( \frac{-a_x \cos(\theta_1) - a_y \sin(\theta_1)}{n_x \cos(\theta_1) - n_y \sin(\theta_1)} \right) )</td>
</tr>
<tr>
<td>6</td>
<td>( \theta_6 = \arctan \left( \frac{D}{D} \right) )</td>
</tr>
</tbody>
</table>

Then, \( \theta_6 \) is calculated using the term (1,2), as follows:
\[ \frac{-\sin(\theta_5) \sin(\theta_6)}{\cos(\theta_0) \sin(\theta_5)} = \frac{a_y \cos(\theta_1) - a_x \sin(\theta_1)}{n_x \cos(\theta_1) - n_y \sin(\theta_1)} \]
\[ \theta_6 = \arctan \left( \frac{-a_x \cos(\theta_1) - a_y \sin(\theta_1)}{n_x \cos(\theta_1) - n_y \sin(\theta_1)} \right) \]

The equations to find the angles of legs are shown in Table 6.

It is important to mention that the previous process is the same to calculate the value of joint positions \( \theta_1, \theta_2, \theta_3, \theta_4, \theta_5 \) and \( \theta_6 \) of both legs.

4.2. Inverse Kinematics of Arms

To obtain the inverse kinematics of the left arm, consider the elements of matrix \( H_4^{24} \), which is shown in Eq. (6):

\[ H_4^{24} = \begin{pmatrix} n_{b_5} & a_{b_5} & a_{b_5} & p_x \\ n_{b_y} & a_{b_y} & a_{b_y} & p_y \\ n_{b_x} & a_{b_x} & a_{b_x} & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \]

Then, from (6) the following matrix equation is defined:

\[ (H_{l1}^{24})^T (H_{l2}^{24})^T (H_{l3}^{24})^T = H_{l4}^{24} \]

\[ \begin{pmatrix} C_{l2} & S_{l2} & 0 & -l_4 \\ -S_{l2} & C_{l2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]
\[ \begin{pmatrix} C_{l1} & S_{l1} & 0 & -l_3 \\ -S_{l1} & C_{l1} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]
The equations to find the angles of right arm are:

\[ a_{x} \sin(\theta_{b2}) - a_{y} \cos(\theta_{b1}) \cos(\theta_{b2}) + a_{z} \cos(\theta_{b2}) \sin(\theta_{b1}) = 0 \]

\[ \theta_{b2} = \arctan\left( \frac{a_{x} \cos(\theta_{b1}) - a_{z} \sin(\theta_{b1})}{a_{x}} \right) \]  

(25)

Where, \( \theta_{b1} \) is calculated:

\[ P_{z} \cos(\theta_{b1}) + P_{y} \sin(\theta_{b1}) - d_{3} \cos(\theta_{b1}) = 0 \]

\[ \theta_{b1} = \arctan\left( \frac{P_{z} - d_{3}}{-P_{y}} \right) \]  

(24)

Using the element (2,4) and (1,4) of both sides of the equation, the angle \( \theta_{b3} \) is calculated:

\[ E = P_{x} \cos(\theta_{b3}) + d_{3} \cos(\theta_{b2}) + l_{5} \sin(\theta_{b2}) \\
+ P_{y} \cos(\theta_{b1}) \sin(\theta_{b2}) - P_{z} \sin(\theta_{b1}) \sin(\theta_{b2}) \\
+ d_{3} \sin(\theta_{b1}) \sin(\theta_{b2}) \]

\[ F = P_{x} \sin(\theta_{b2}) - l_{4} - l_{3} \cos(\theta_{b2}) + d_{3} \sin(\theta_{b2}) \\
- P_{y} \cos(\theta_{b1}) \cos(\theta_{b2}) + P_{z} \cos(\theta_{b2}) \sin(\theta_{b1}) \\
- d_{3} \cos(\theta_{b2}) \sin(\theta_{b1}) \]

\[ l_{5} \sin(\theta_{b3}) = E \\
l_{5} \cos(\theta_{b3}) = F \]

\[ \theta_{b3} = \arctan(E, F) \]  

(26)

The equations to find the angles of right arm are shown in Table 7.

To obtain the inverse kinematics of the right arm, consider the equation shown in Eq. (27).

The following matrix equation is defined:

\[ (H_{b5}^{7})^{-1} (H_{b5}^{6})^{-1} (H_{b}^{5})^{-1} H_{b}^{6} = H_{b7}^{68} \]
Table 8. Inverse kinematics equations of the robot’s right arm

<table>
<thead>
<tr>
<th>Link</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>[ \theta_{b4} = \arctan \left( \frac{P_x - x_l}{P_y - y_l} \right) ]</td>
</tr>
<tr>
<td>2</td>
<td>[ \theta_{b5} = \arctan \left( \frac{a \cos(\theta_{b1}) + a_z \sin(\theta_{b1})}{a_x} \right) ]</td>
</tr>
<tr>
<td>3</td>
<td>[ \theta_{b6} = \arctan(G, H) ]</td>
</tr>
</tbody>
</table>

\( \theta_{b5} = \arctan \left( \frac{a \cos(\theta_{b1}) + a_z \sin(\theta_{b1})}{a_x} \right) \) (29)

Using the element (2,4) and (1,4) of both sides of the equation, the angle \( \theta_{b6} \) is calculated:

\[ G = P_x \cos(\theta_{b5}) - d_4 \cos(\theta_{b5}) + l_3 \sin(\theta_{b5}) \]
\[ + P_y \cos(\theta_{b4}) \sin(\theta_{b5}) + P_z \sin(\theta_{b4}) \sin(\theta_{b5}) \]
\[ - d_2 \sin(\theta_{b4}) \sin(\theta_{b5}) \]
\[ H = P_x \sin(\theta_{b5}) - l_4 - l_3 \cos(\theta_{b5}) - d_3 \sin(\theta_{b5}) \]
\[ - P_y \cos(\theta_{b4}) \cos(\theta_{b5}) - P_z \cos(\theta_{b4}) \sin(\theta_{b4}) \]
\[ + d_2 \cos(\theta_{b4}) \sin(\theta_{b4}) \]
\[ \frac{l_3 \sin(\theta_{b5})}{l_3 \cos(\theta_{b5})} = \frac{G}{H} \]
\[ \theta_{b6} = \arctan(G, H) \] (30)

The equations to find the angles that correspond to the joints of the left arm are shown in Table 8.

5. Conclusion

This paper presents a complete solution of the inverse kinematics model using the Denavit-Hartenberg methodology for a 18 DOF robot. The forward kinematics model allowed to represent the Bioloid Premium robot.

Unlike the other geometric methods, our research proposal considers the decoupling kinematic method, taking the feet and the pelvis as points of origin, generating 4 open kinematic chains to calculate the joint positions of both arms and legs of the robot in a three-dimensional space (x, y, z), consequently it is possible to determine the final position of each end-effector of the robot, taking the supporting feet as fixed reference frame.

This methodology is an important step forward to obtaining the differential kinematics and subsequently calculating the dynamic model of the robot in a later work.

On the other hand, the proposed methodology can be extended to other biped robots.

References


