SPEED ANALYSIS OF A DIGITAL CONTROLLER IN TIME CRITICAL APPLICATIONS

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Abstract:

Traditionally, control algorithms are designed without a consideration of their real-time implementation details. The performance of a digital control system, besides the sampling period, depends on many variables, such as the control loop execution time, jitter, complexity of the control algorithm etc. In this paper attention is focused on the interaction of the parameters of the scheduled tasks and on the performance of control loops closed with digital controller. A design approach that is based on the relative speed classification of the control system has been proposed. The approach is illustrated by the analysis of control systems developed for laboratory magnetic levitation process.

Keywords: *digital control, real-time control, time-critical systems, FPGA, magnetic levitation, timing model.*

1. Introduction

Computer-based digital controllers typically have the ability to monitor a number of discrete and analog inputs, execute complex control algorithms, and drive several outputs, all at defined, often very high, speeds. In general, computer-based digital controllers must detect external events and respond to them by taking appropriate actions. It is required that all above operations and calculations take place within the required time. This imposes timing requirements on hardware and software of the computer-based control systems. More precisely, computer-based digital controllers must have sufficient processing power, sufficient high-speed input/output hardware peripherals and operating system, fulfilling more or less hard timing requirements and handling error conditions in a predefined way.

Limited processing power combined with a non-optimised hardware and software components introduce delays and non-deterministic behaviour of the real-time system. Digital control theory normally assumes evenly spaced sampling intervals and a negligible (or constant) control delay between sampling and actuation [1]. However, this can seldom be practically achieved in a real, resource-constrained system. Time delays and timing variations in control loop execution degrade the control performance and may, in extreme cases, lead to instability.

Control theory does not very often advise how to design controllers to take that limitation into account [2]. Usually, control algorithms are designed without consideration of their real-time implementation details. Designers usually try to separate the real-time aspects and the dynamics of the control system. They develop controllers that guarantee all tasks deadlines under worst-case load and external interrupt occurrence scenario. The design of safety-critical controllers is based on this approach. Plant can be suitably controlled, but at the cost of poor computer resource utilization. However, if sampling rate bound determined by the speed of the control computer is close to the minimal required by the plant, then the sampling rate of the control system becomes time critical. For such a system the performance of the real-time operating system is essential for correct operation of the control system.

It has been stated in previous work (see for example [10], [12]), that integrated approaches combining two disciplines real-time computation and control theory, results in better quality for digital control systems. This is also true for networked or multirate systems [4], [6], [9]. This problem is analysed in this paper. The notion of relative speed of the control system is introduced and illustrated, on the example of magnetic levitation (MagLev) real-time control. Control system design approach based on the relative speed system classification is proposed.

2. Relative speed of a digital-controlled plant

The general scheme of a digital control system is given in Fig. 1 [11], [3]. The operation of the closed-loop system can be split into three main tasks: sampling, control algorithm computation and actuation. Models and methods used by discrete-time control theory implicitly impose the timing of the tasks in the computer implementation.

The tasks are associated with the events, including timer event, termination of a data frame transmission, signals that data are ready to be read from the input devices, fault detection, etc. The tasks usually share the same processor and exchange data with each other.

Although a great variety of scheduling methods is available, in this work periodic task scheduling method is assumed.

The following timing assumptions are made in relation to the three main tasks of a digital control loop operation:

1. Sampling is performed at equidistant time instants given by T_0, but some variation of T_0 are allowed.
2. The actuation is performed instantly when the control signal $u(kT_0)$ is delivered to the D/A converter.
3. The control algorithm computation is executed as soon as the input data are available.
4. The control algorithm design is based on correctly identified models of the process and the disturbances (referred to "nominal models").

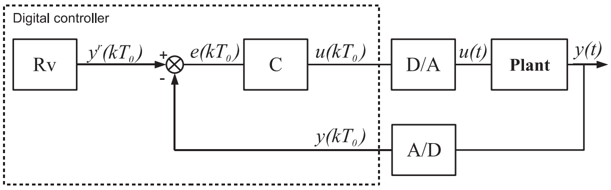


Fig. 1. The general scheme of a computer controlled system. A/D - analog to digital converter, D/A - digital to analog converter, Rv - reference value, C - controller.

5. For the nominal models, it is possible to estimate maximal, admissible sampling period T_0 that would guarantee acceptable control performance. This sampling period can be estimated as:

$$T_0 \in \left[T_0^l, T_0^u\right] \tag{1}$$

where:
T_0^u - is time period for "ideal" control system, where modelling and identification errors as well as time delays and variations of time period T_0 are negligible and control parameters are optimal. For "ideal" control system sampling period can be extended to the upper limit defined e.g. by the Shannon theory.
T_0^l - is the sampling period guaranteeing robust operation of the control system, if it is under the influence of external and internal disturbances. This sampling/control period, besides fulfilling the Shannon theorem, can be selected following one of various "rules of thumb" [1], [3], depending on the desired closed-loop system performance.

6. The performance of the closed-loop control system is a strictly monotonic function of T_0. Any applied sampling period $T_0 < T_0^u$ improves control performance. For $T_0 < T_0^l$ the improvement is not observed.

7. The proposed control platform (processor, peripherals hardware and operating systems) are characterized by minimal (a shortest accessible) closed -loop execution time, estimated as:

$$\tau_s \in \left[\tau_s^l, \tau_s^u\right] \tag{2}$$

where:
τ_s^l - is the control loop execution time for simple control algorithms,
τ_s^u - is the control loop execution time for complex control algorithms.

The control algorithm is classified as a "simple", if the pseudocode of the controller task includes no more than 5-10 operations (loops are excluded). The examples of "simple" algorithms are: incremental PID or state feedback controller.

If the pseudocode of the controller includes more than 10 operations or loops then the algorithm is classified as "complex". The examples of "complex" control algorithms are: time-optimal, model-reference controller, predictive controller.

Fig. 2 illustrates typical timing models one can use for regularly sampled process.

For the model a), sampling and actuation are performed at the same sampling time (time delay is zero). For this model we have: $\tau_s \ll T_0$ and so called *causality rule* is fulfilled.

Model b) from Fig. 2 is more realistic because it takes into account that control task takes time. Control loop execution time is constant and is less than sampling period: $\tau_s < T_0$. Causality rule is not fulfilled in this case. The causality rule can be fulfilled if actuation is performed at the next sampling instant, e.g. one step delay is assumed. It is so called *Strictly Proper Control Law* [5].

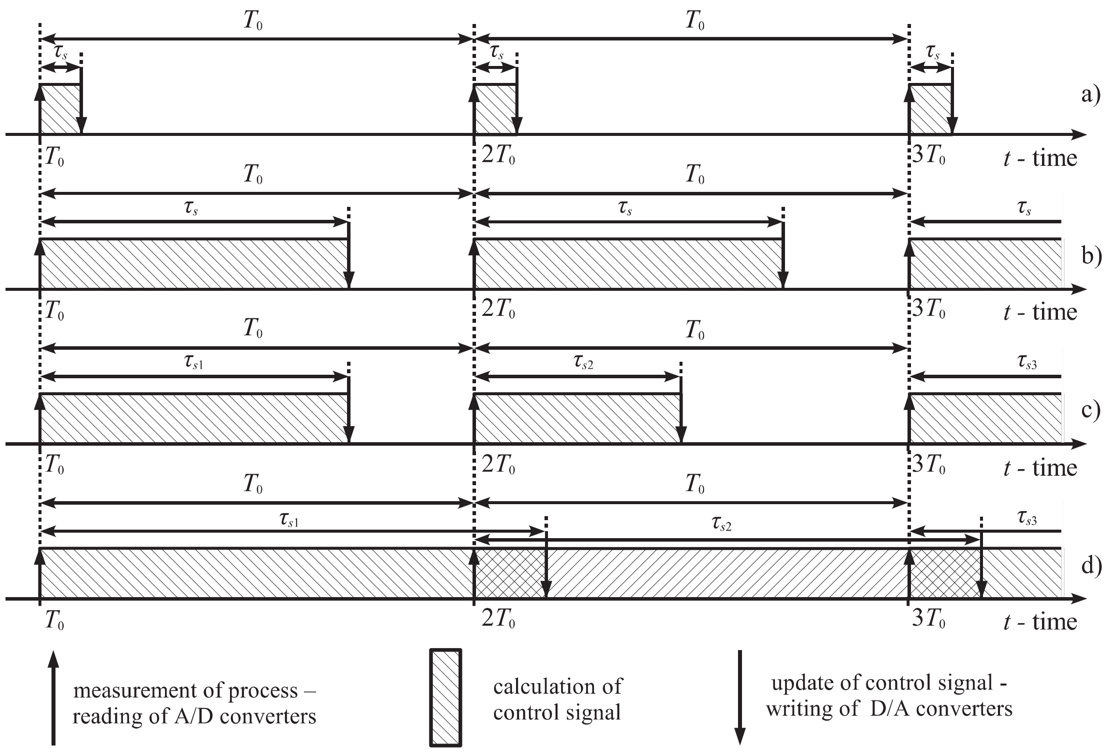


Fig. 2. Timing models that can be used for regularly sampled process.

Model c) also takes into account that control task takes time, but time delay is variable and is less than sampling period: $\tau_s < T_0, \tau_{s1} \neq \tau_{s2}$. The causality rule can be fulfilled if the actuation is performed at the next sampling instant, i.e. one step delay is assumed.

The constant delay can be compensated during the algorithm design. For continuous-time design, Smith predictor can be implemented for discrete-time design, plant model augment method can be applied [3].

For model d), the observed time delays are longer than the sampling period. The control task is to complex for assumed sampling period. Both causality rule and real-time constraints are not fulfilled in this case. The plant cannot be controlled in a proper way.

The relative speed of control system can be characterized by the factor $\frac{\tau_s}{T_0}$.

The following classification of control system is proposed:

1. The control system will be referred to **relatively low-speed** if $\frac{\tau_s}{T_0} \ll 1$ (model from Fig. 2a).
2. The control system will be referred to **relatively medium-speed** if $\frac{\tau_s}{T_0} \approx 1$ (models from Fig. 2b and from Fig. 2c).
3. The control system will be referred to **relatively high-speed** if $\frac{\tau_s}{T_0} > 1$ (model from Fig. 2d).

Note that high-speed execution platform applied for the process described by a slow dynamic is classified as relatively low-speed control system. A digital control system will be considered as **time-critical** if it is classified as relatively high-speed or relatively medium-speed. A low-speed control loop applied for fast dynamic process will be classified as relatively high-speed control system.

3. Adapting control system parameters for time-critical system

For $\frac{\tau_s}{T_0} > 1$ the control loop is relatively high-speed and therefore becomes time-critical. It can be classified as medium- speed system, if (Fig. 3):

a) after assuming $T_0 = T_0^u$ the condition $\tau_s < T_0^u$ is fulfilled. Non-robust control is available in this case,
b) after assuming $\tau_s = \tau_s^l$ the condition $\tau_s^l < T_0$ is fulfilled. Applications of "simple" control algorithms become possible.

Such adaptation of control system parameters is limited and in most cases cannot move the system to low-speed class. This class can be reached by changing the control platform: using the most efficient processor, better operating system, etc. (case B in Fig. 3).

4. Example: real-time control of MagLev system

A laboratory magnetic levitation system presented on Fig. 4 was used as an example of relatively high-speed plant [7], [8]. The MagLev was chosen with regard to its specific nonlinearities and fast dynamics.

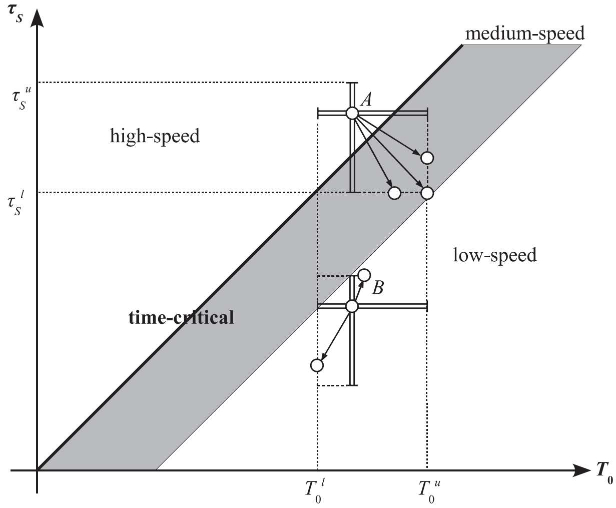


Fig. 3. Moving time critical control system (A) to the medium speed class (adapting the parameters) and to low speed class (B), by changing the control platform.

The identified parameters of MagLev process were: $T_0^l = 40\mu s$, $T_0^u = 2000\mu s$. The parameters of the MagLev digital control system and its relation to the proposed relative-speed definitions are given in Table 1.

This plant, when controlled by a PC or a general-purpose microcontroller, can be classified as a relatively medium-speed system. MagLev controlled by a simple 8-bit microcontroller is classified as relatively high-speed system. It is a representative example to show that:

- usage of more efficient control system with shorter control period T_0 results in better quality of control by changing classification of system from relatively medium-speed to relatively low-speed,
- usage of more efficient control system with shorter control loop execution time τ_s results in better quality of control by better satisfying the causality rule.

Table 1. Control system parameters for MagLev.

	PC	microcontroller	FPGA
τ_s^l	$500\mu s$	$2000\mu s$	$2\mu s$
τ_s^u	$2000\mu s$	n.a.	n.a.

Experiments with MagLev plant controlled by two different control systems were carried out. The first configuration was based on PC computer and the second was based on FPGA circuits. Both of them were developed using an extension board consisting of the FPGA circuit. Control algorithm for PC-based control system was calculated as a controller task by MATLAB/Simulink real-time application. In this case the FPGA circuit was used only for signal generation for analog/digital and digital/analog converters and for providing data to the PC computer. In the case of FPGA-based system the control algorithm was calculated directly by the FPGA circuit. The PC was used only for monitoring the plant and logging the data.

Both control systems are presented in the Fig. 4.

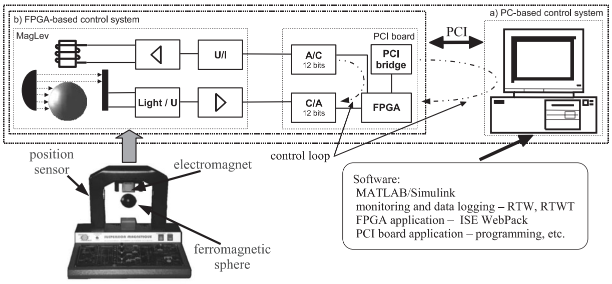


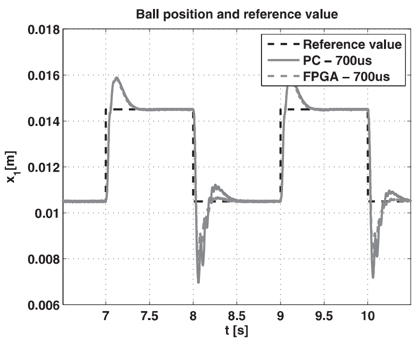
Fig. 4. Block diagram of control systems: a) PC-based, b) FPGA-based.

The ferromagnetic ball has followed the changing reference signal. The square wave with 2s period was applied as the reference value. The desired centre of the ball movement was in the distance of 0.0125m from the electromagnet and the amplitude of movement was equal to 0.002m. Exactly the same parameters were used for both of the tested control systems.

Digital version of PID algorithm was used for both performed experiments. The parameters of the algorithm were recalculated for the specific hardware architecture i.e. the type of arithmetic or the applied control period. The results of the experiments are presented in Fig. 5, Fig. 6 and Fig. 7.

A comparison of the PC controlled process response with the response of control system based on FPGA circuit is presented in Fig. 5. The control period of 700 μs was used for both control systems. The evident improvement of control quality is observed in the case of MagLev controlled by FPGA-based controller that guarantees shorter control loop execution time τ_s.

a)



b)

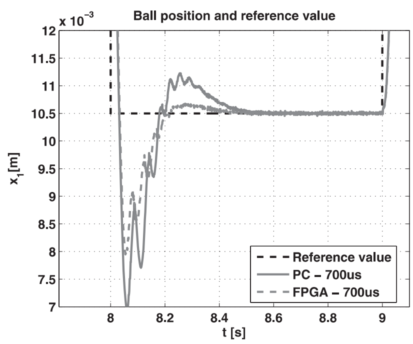
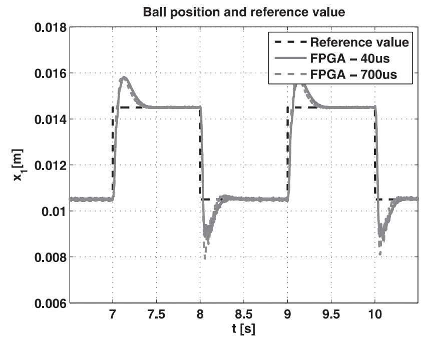


Fig. 5. A comparison of the MagLev process response controlled by PC system (700 μs control period) and the system based on FPGA circuits (700 μs control period): a) complete range of desired ball movement, b) magnification of up-motion part.

A comparison of the process response (FPGA-based control system), for different control periods 700 μs and 40 μs, is presented in Fig. 6. Smaller values of the overshoot and the shorter settling time are observed in the case of MagLev controlled by FPGA-based control system with the shorter control period T_0

a)



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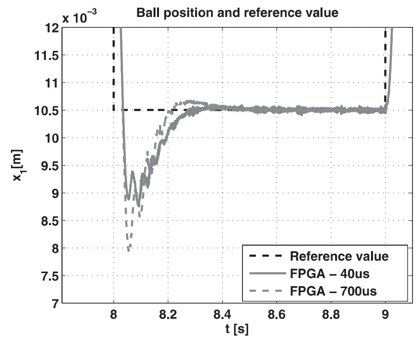
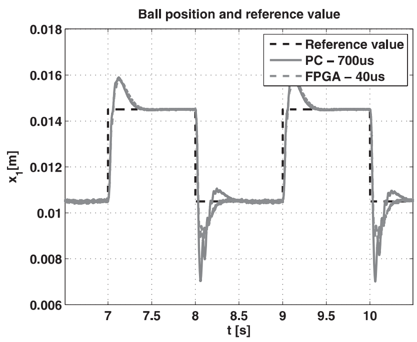


Fig. 6. A comparison of the MagLev process response controlled by the system based on FPGA (700 μs control period) and the system based on FPGA circuits (40 μs control period): a) complete range of desired ball movement, b) magnification of up-motion part.

a)



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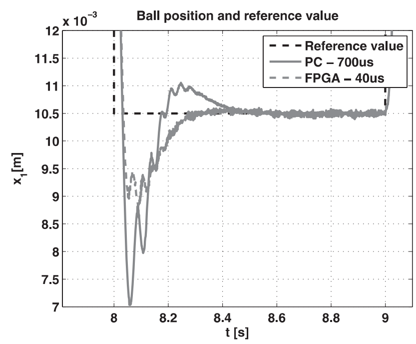

Fig. 7. A comparison of the MagLev process response controlled by the system based on PC (700 μs control period) and the system based on FPGA circuits (40 μs control period): a) complete range of desired ball movement, b) magnification of up-motion part.

A comparison of the process response controlled by PC computer and the control system based on FPGA circuits is presented in Fig. 7. Smaller values of the overshoot and the shorter settling time are observed in the case of MagLev controlled by FPGA-based control system with the boundary control period T_0^l and the boundary control loop execution time τ_s^l.

5. Conclusions

Digital, computer based control systems are the applications that pose the very sharp timing requirements, because digital control theory usually assumes a highly deterministic sampling. Consequently, the application of the controller performing a number of real-time tasks in the control loop makes the analysis and design more complex.

The performance of a digital control system depends on many variables, such as sampling period, control loop execution time, jitter, and complexity of the control algorithm. The performance of a digital control system depends not only on the performance of its individual components but also on their interaction and cooperation.

The focus of this paper was the interaction of parameters of the scheduled tasks and the performance of control loops closed over digital controller. In particular, we have proposed a new design approach that is based on the relative speed system classification and applies the following paradigm: the application platform should be selected in such a way that closed-loop execution time and process dynamics are balanced. The relative speed of the system should be located just bellow the line separating "time critical" solution, giving optimal utilization of computing system resources.

The results of presented experiments have showed that both factors: reduction of the control period and reduction of the loop execution time improve quality of control for magnetic levitation system. Both solutions have located the relative speed of the MagLev control system below the "time-critical" range. The obtained overshoots and settling times were much better with FPGA-based controller. The maximal improvement of control quality was observed for simultaneously reduced control period and control loop execution time. This effect was obtained for control loop closed directly *via* PID control application embedded into FPGA circuit.

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References

[1] Aström K.J., Wittenmark B., *Computer Controlled Systems*, Prentice Hall, London, 1997.

[2] Arzen K-E., Cervin A., Eker J., Sha L., "An Introduction to Control and Scheduling Co-design". In: *39th IEEE Conference on Decision and Control*, Australia, Sydney 12th-15th December, 2000.

[3] Franklin G. F., Powell J. D., Emami-Naeini A., *Feedback Control of Dynamic Systems*, Addison-Wesley Publishing Company, London, 1994.

[4] Grega W., "Stability of Distributed Control Systems with Uncertain Delays". In: *8th IEEE International Conference on Methods and Models in Automation and Robotics*, Międzyzdroje, 2002, pp. 303-307.

[5] Middleton R. H., Goodwin G. C: *Digital Control and Estimation: A Unified Approach*, Prentice-Hall International, Englewood Cliffs, New Jersey, 1990.

[6] Nilsson J., *Real-time Control Systems with Delays*. Ph.D. Dissertation, Sweden, Lund Institute of Technology, 1998.

[7] Piątek P., *Application of Specialized Hardware Architectures for Realization of Time-critical Control Tasks* - Ph.D. Thesis (in Polish), Grega W. - supervisor. AGH University of Science and Technology, Department of Automatics, Poland, Kraków, 2007.

[8] Piłat A., "Programmable Analog Hardware for Control Systems Exampled by Magnetic Suspension". In: *Proc. Computer Methods and Systems*, Kraków, Poland, 14th-16th November, 2005, pp. 143-148.

[9] Sågfors M., *Optimal Sampled-Data and Multirate Control* - Ph.D. Dissertation, Process Control Laboratory, Faculty of Chemical Engineering, Åbo Akademi University, 1998.

[10] Schinkel G., Rantzer A., "Sample Data with Varying Sampling Time". In: *Proceedings of European Control Conference*, Porto, Portugal, September, 2001, pp. 624-628.

[11] Vaccaro R. J., *Digital Control a State-Space Approach*, McGraw-Hill, Inc., New York, 1995.

[12] Wittenmark B., Bastian B., Nilsson J., "Analysis of Time Delays in Synchronous and Asynchronous Control Loops". In: *Proc. of 37th IEEE Conference on Decision and Control*, Tampa, USA, 12th-15th December, 1998.