Robustness of Model-Reference Variable-Structure Robot Control Methods

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Abstract:

Different variations of one simple model-reference variable-structure control (MRVSC) method are presented and used for planar robot control. The quality of trajectory tracking was tested by simulation with respect to robot parameter changes, different complex sinusoidal external disturbances and measurement noise. The results proved satisfactory robustness of all presented modified MRVSC methods.

Keywords: *robustness, reference model, variable-structure control, robot, simulation*

1. Introduction

A robot is a nonlinear, multivariable, coupled dynamic system with a high-precision performance demand. Therefore, it is very difficult to design effective robot controllers which will assure accurate robot trajectory tracking, especially in the presence of different system uncertainties. It is shown in [2], [4], [7], [11] that a proper combination of Model Reference Adaptive Control (MRAC) and Variable-Structure Control (VSC) with sliding-mode can provide good transient behaviour and maintain robustness with respect to bounded disturbances, nonlinearities, parameter variations and unmodelled dynamics. In MRAC a reference model is used to specify the ideal response of an adaptive control system to the input signal, while the aim of an adaptation mechanism is to keep the difference between the model and the plant states as small as possible [1], [17].

On the other hand, a very fast and computationally simple variable-structure (VS) control method, with a sliding mode as the main operation mode, can be used to deal with robot parameters variations and unmodelled robot dynamics for obtaining good tracking control [5], [9]. Although sliding-mode VSC provides immediate reaction to the smallest deviation of aforementioned causes, it also brings a serious chattering problem in the controlled system [15]. To avoid the chattering, some approaches are proposed in [5], [8], [10], [13], [14], [15]. The main idea is to add a boundary layer along the switching surface to filter out the chattering behaviour, so the system output error converges into the boundary layer in finite time. However, the ultimate accuracy of the sliding mode is partially lost [10], [15], [17].

2. MRVSC method with modifications

One of the simplest model-reference variable-structure (MRVS) robot control methods [10], [13] is given in Fig. 1. This MRVSC method consists of the inner and the outer control loop.

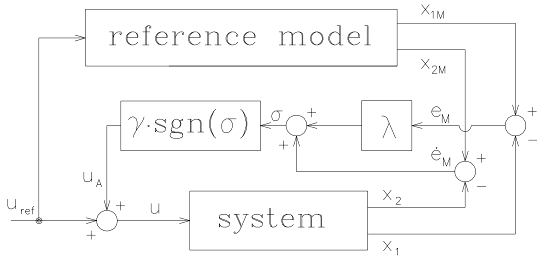


Fig. 1. Scheme of model-reference variable-structure control.

The inner loop is an ordinary servo control loop composed of a process and a feedback controller. The goal is to model this inner control loop as simple as possible, e.g. as a 2nd–order system with the following closed-loop transfer function:

$$G_{cs}(s)=\frac{K_s}{1+2\cdot\xi_s\cdot T_s\cdot s+T_s^2\cdot s^2}. \tag{1}$$

The inner servo control loop consists of a PD-controller with a proportional gain P and a derivative gain D, an amplifier with gain K_{AM}, a robot joint with a Direct-Current (DC) motor and sensors for measurement of system states, which are a motor shaft angle x_1 and a motor shaft velocity x_2. The parameters of DC motor are: resistance R_a and inductance L_a of the armature winding (where $K_a=1/R_a$, $T_a=L_a/R_a$), torque constant K, moment of inertia J_m, viscous motor friction coefficient b_{vm}. The coefficients of the PD-controller are set according to the demand to eliminate the smaller time constant of the open-loop system and to avoid overshoot in the closed-loop system response ($\xi_s=1$):

$$P=\frac{\left(b_{vm}+K_a\cdot K^2\right)^2}{2\cdot K_{AM}\cdot K_a\cdot K}\cdot\frac{1}{T_a\cdot b_{vm}+J_m+\sqrt{\left(T_a\cdot b_{vm}+J_m\right)^2-4\cdot T_a\cdot J_m\cdot\left(b_{vm}+K_a\cdot K^2\right)}}, \tag{2}$$

$$D=\frac{b_{vm}+K_a\cdot K^2}{4\cdot K_{AM}\cdot K_a\cdot K}\cdot\frac{T_a\cdot b_{vm}+J_m-\sqrt{\left(T_a\cdot b_{vm}+J_m\right)^2-4\cdot T_a\cdot J_m\cdot\left(b_{vm}+K_a\cdot K^2\right)}}{T_a\cdot b_{vm}+J_m+\sqrt{\left(T_a\cdot b_{vm}+J_m\right)^2-4\cdot T_a\cdot J_m\cdot\left(b_{vm}+K_a\cdot K^2\right)}}. \tag{3}$$

Because of the simplicity of the 2nd–order system model, not all of the robot dynamics and nonlinearities were modelled during the synthesis of the inner PD-controller parameters. This disadvantage can be overcome by the outer control loop. The variable-structure controller in this outer loop drives the system state variables x_1 and x_2 in order to hit a defined sliding surface and then slide along it to approach a reference model [5].

The 2nd–order reference model from Fig. 1 can be described by the following state equations:

$$\begin{bmatrix} \dot{x}_{1M} \\ \dot{x}_{2M} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\dfrac{1}{T_M^2} & -\dfrac{2 \cdot \xi_M}{T_M} \end{bmatrix} \cdot \begin{bmatrix} x_{1M} \\ x_{2M} \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{K_M}{T_M^2} \end{bmatrix} \cdot u_{ref}. \tag{4}$$

As can be seen in Fig. 1, the following common error representing a switching plane can be defined:

$$\sigma(e_M, \dot{e}_M) = \lambda \cdot e_M(t) + \dot{e}_M(t). \tag{5}$$

The purpose of the sliding-mode control is to push the system trajectory to the origin of the switching plane. This can be accomplished by using the following nonlinear control law (the 1st MRVSC method) [3], [10], [12]:

$$u_A(t) = \gamma \cdot \mathrm{sgn}(\sigma(t)). \tag{6}$$

The problem with this MRVSC method is in the chattering of the control signal (6), which results in increased energy dissipation. Therefore, a boundary layer is introduced into the control law (6) to eliminate the chattering problem. The results are the following MRVSC methods with a modified control law (6):

- with a continuous control signal [8], [10] (the 2nd MRVSC method):

$$u_A(t) = \gamma \cdot \frac{\sigma(t)}{|\sigma(t)| + \delta}, \tag{7}$$

where δ denotes the thickness of the boundary layer,

- with a saturation function [5], [10], [13], [14] (the 3rd MRVSC method):

$$u_A(t) = \gamma \cdot sat[\sigma(t)], \tag{8}$$

- with an exponential function [10] (the 4th MRVSC method):

$$u_A(t) = \gamma \cdot \mathrm{sgn}(\sigma(t)) \cdot \left(1 - e^{-\frac{|\sigma|}{\delta}}\right) \tag{9}$$

3. Simulation results

The efficiency and the robustness of all four mentioned MRVSC methods were tested by computer simulations. During the simulations the stepwise input signal $u_{ref}(t)=1*S(t)$ was fed to each robot joint of a three-axis articulated planar robot, driven by a DC motor with viscous, dynamic and static joint and motor frictional forces [16], with parameters given in Table 1.

Table 1. The three-axis robot parameters used in simulations.

	number of robot segment or motor		
	1	2	3
length of robot segment a [m]	0.3	0.2	0.1
mass of robot segment m [kg]	1	0.7	0.3
viscous motor friction coefficient b_{vm} [kg·m^2/s]	0.00003855	0.00003855	0.00003855
viscous joint friction coefficient b_v [kg·m^2/s]	0.2	0.2	0.2
dynamic joint friction coefficient b_d [kg·m^2]	0.1	0.1	0.1
static joint friction coefficient b_s [kg·m^2]	0.3	0.3	0.3
small constant ε	0.1	0.1	0.1
armature motor winding gain K_a [Ω^{-1}]	0.12195	0.12195	0.12195
armature motor time constant T_a [ms]	2.012195	2.012195	2.012195
motor torque constant K [N·m/A]	0.0394	0.0394	0.0394
moment of motor inertia J_m [kg·m^2]	0.00000268	0.00000268	0.00000268
maximal motor armature current I_{am} [A]	0.745	0.745	0.745
maximal output controller voltage U_{Rm} [V]	10	10	10
amplifier coefficient K_{AM}	2.4	2.4	2.4
gear ratio N_r	291	388	582

PD-controllers gain coefficients P and D are set according to (2) and (3) as: $P_1=P_2=P_3=0.512$ [V/rad] and $D_1=D_2=D_3=0.001256$ [V·s/rad]. Parameters of the reference model are set to be equal to the parameters of the 2nd–order system: $K_{Mi}=K_{si}$, $\xi_{Mi}=\xi_{si}$, $T_{Mi}=T_{si}$, $1\leq i\leq 3$. The criteria for determination of the parameters of the 1st type VS controller were a desired velocity of transient responses and a maximally allowed model tracking error for each robot motor $e_{Mm}=0.04u_{ref}$. This yielded: $\lambda_1=\lambda_2=\lambda_3=100$ [s^{-1}]; $\gamma_1=14.11$, $\gamma_2=1.029$, $\gamma_3=0.1221$ [10].

During simulations, an accumulated error and a total energy consumed by all robot motors were calculated [6]:

$$e_{M\Sigma} = \sum_{i=1}^{3}\left(\int_{0}^{T_s}\left|e_M(t)\right| \cdot dt\right), \quad (10)$$

$$E = \sum_{i=1}^{3}\left(\int_{0}^{T_s} U_{a_i} \cdot I_{a_i} \cdot dt, \quad U_{a_i} \cdot I_{a_i} > 0\right), \quad (11)$$

where T_s denotes a simulation time, U_{ai} is the armature voltage and I_{ai} is the i^{th} motor armature current ($1 \le i \le 3$).

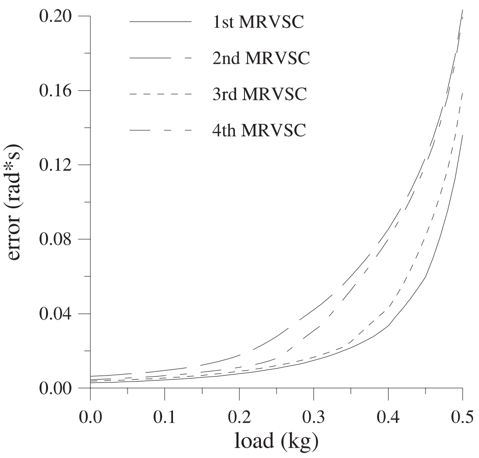
Due to the inevitable chattering in the control signal (6), the energy consumption calculated during the simulation time $T_s=0.5$ [s] was the highest, $E=3.58$ [J], while $e_{M\Sigma}$ was the smallest, $e_{M\Sigma}=0.00279$ [rad·s]. The energy consumption could be reduced by using modified MRVSC methods with properly defined δ. It was shown in [10], [17] that the increase of δ also increased the steady-state error, i.e. it worsened the system accuracy. Therefore, δ must be as small as possible, for example, $\delta=0.1$. In that case, the results for all modified MRVSC methods were as follows:

- for the 2nd method: $E=1.27$ [J], $e_{M\Sigma}=0.00628$ [rad·s], $\gamma_1=9.79, \gamma_2=0.895, \gamma_3=0.107$;
- for the 3rd method: $E=1.35$ [J], $e_{M\Sigma}=0.00372$ [rad·s], $\gamma_1=14.15, \gamma_2=0.981, \gamma_3=0.105$;
- for the 4th method: $E=1.32$ [J], $e_{M\Sigma}=0.00450$ [rad·s], $\gamma_1=13.05, \gamma_2=0.897, \gamma_3=0.105$.

These results indicate that the total energy consumption was reduced from 2.65 to 2.82 times, while the system accuracy, as expected, became worse. The next goal of simulation was to analyse the robustness of studied MRVSC methods. The analysis was performed for changes of robot load, influence of white measurement noise and different types of external disturbances. The influence of robot parameter changes on the robustness of all MRVSC methods was first analysed by adding different loads to the robot tool (from 0 to 0.5 [kg]). The results related to $e_{M\Sigma}$ and E are shown in Figs. 2a and 2b. One can see that the greatest effect of the load change was on the 2nd MRVSC method and the smallest on the 1st one. The values of E were very close for all three modified control methods, while the value of E for the 1st MRVSC method was much higher.

The next simulation had a goal to examine the robustness of studied MRVSC methods in the presence of white measurement noise added to each of the robot motor shaft speed signals (varied in the range from 0% to 25% of the measured speed). The results given in Figs. 2c and 2d show that the added measurement noise slightly deteriorated the accuracy of all MRVSC methods, but having the greatest influence on the 2nd one, also resulting in smallest energy consumption.

a



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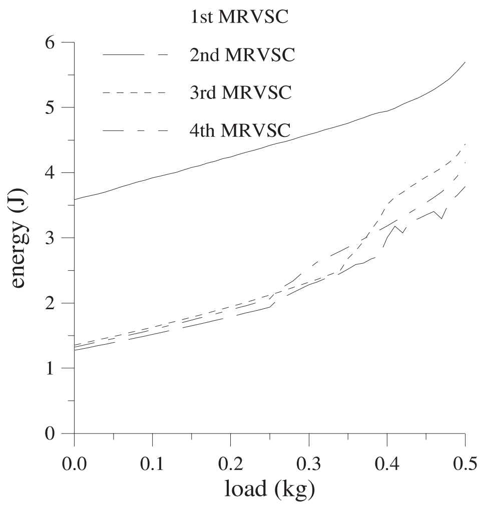

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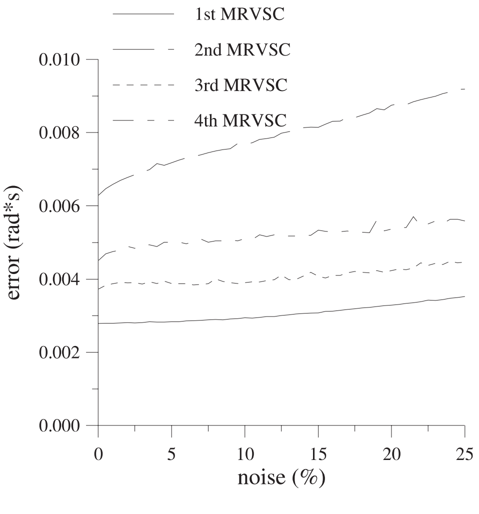

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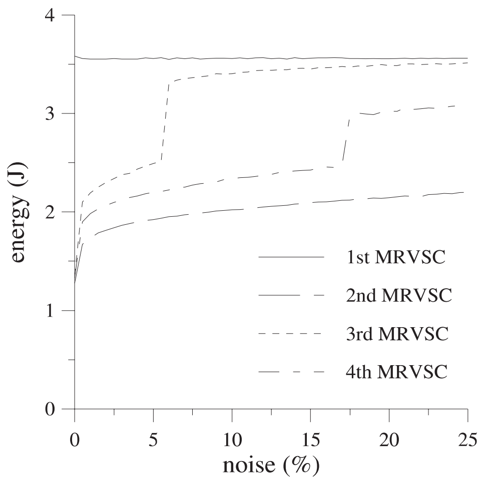


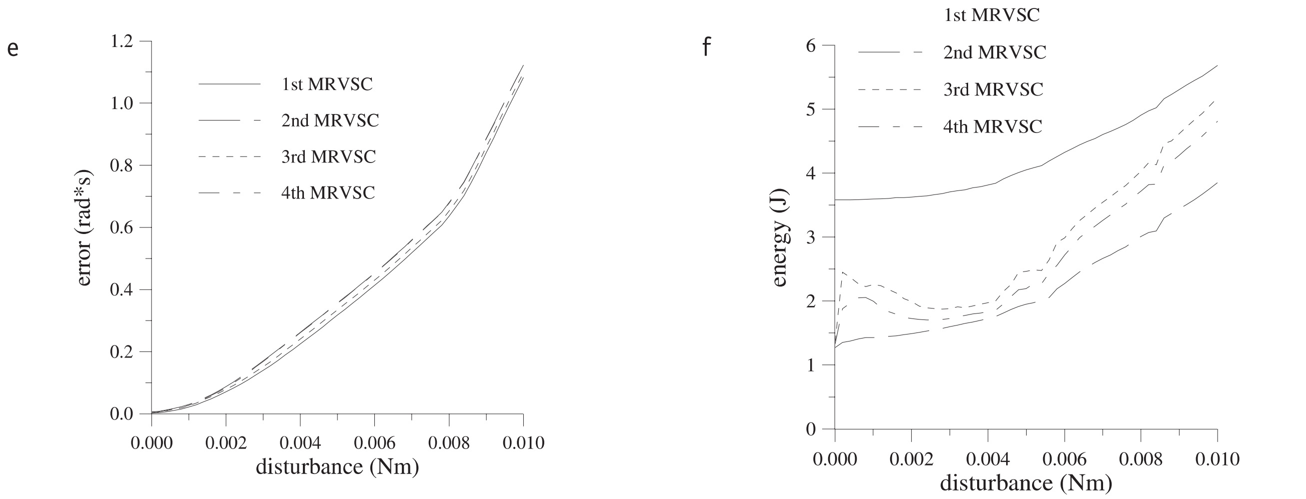
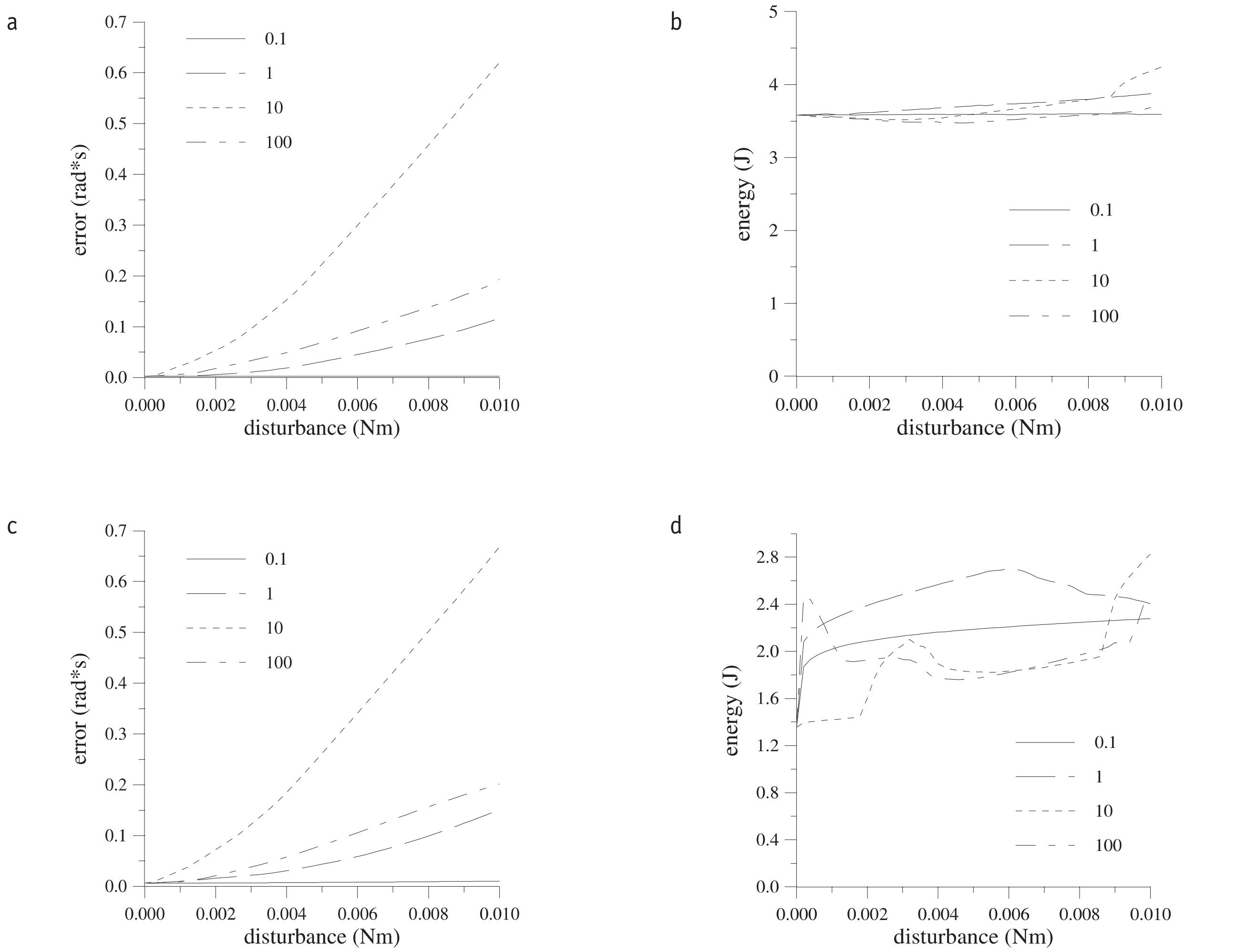


Fig. 2. Total error (a, c, e) and energy consumption (b, d, f) for MRVSC methods in the case of load change (a, b), measurement noise (c, d) and complex external disturbance (e, f).

The robustness of MRVSC methods was also tested to the influence of complex sinusoidal external disturbance $T_d=T_{Ad}sin(0.1t)+T_{Ad}sin(t)+T_{Ad}sin(10t)+$ $+T_{Ad}\,sin(100t)$, added to all robot motor shafts, where T_{Ad} changes from 0 to 0.01 [Nm]. From the results displayed in Figs. 2e and 2f, one can see that the influence of such disturbances was very similar for all MRVSC methods, although the 1st one had the greatest value of the consumed energy.

Another test of robustness of the studied MRVSC methods was performed by applying frequency-dependent external disturbances $T_d=T_{Ad}\cdot sin(\omega t)$, $\omega=0.1,1,10,100$ [rad/s], added to all robot motor shafts, where T_{Ad} was changed from 0 to 0.01[Nm]. The results obtained with the 1st (Figs. 3a and 3b), the 2nd (Figs. 3c and 3d), the 3rd (Figs. 3e and 3f) and the 4th (Figs. 3g and 3h) MRVSC method show that the largest influence of the frequency-dependent disturbance on all four methods was with the frequency $\omega=10$ [rad/s], while the smallest influence was with $\omega=0.1$ [rad/s]. The energy consumption was not affected much by different disturbance frequencies; the smallest amount of energy consumption was noticed for the 2nd MRVSC method and the biggest for the 1st one.



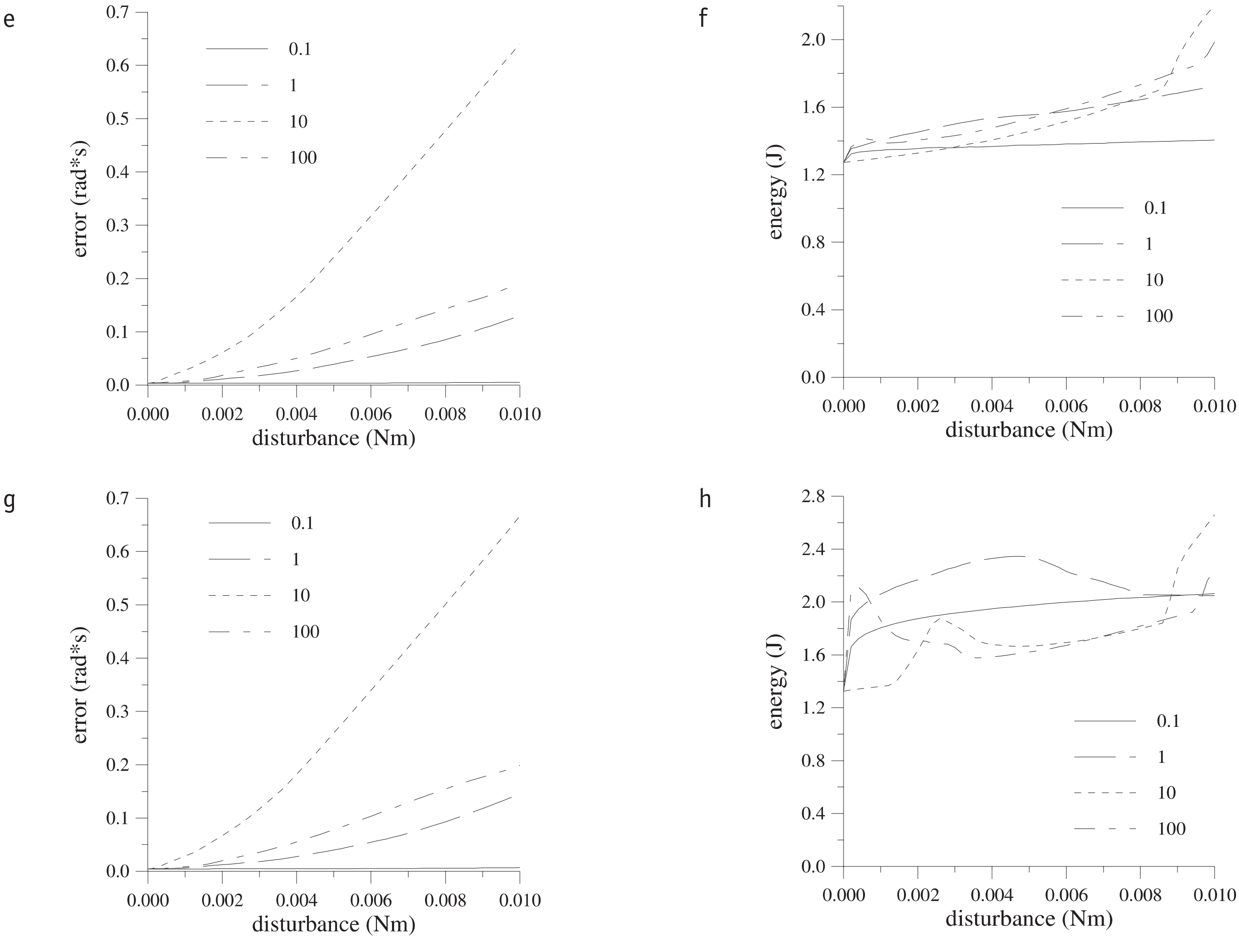


Fig. 3. Total error (a, c, e, g) and energy consumption (b, d, f, h) for the 1st (a, b), the 2nd (c, d), the 3rd (e, f), the 4th (g, h) MRVSC method with different frequency-dependent external disturbances.

4. Conclusion

Servo control of robot joints is significantly affected by different shaft loads having both linear and nonlinear characters (e.g. gravity-dependent loads, friction, external loads). Moreover, robot joint servo loops are subjected to continuous variations of system parameters, mostly position and speed dependent. The type of servo control that can cope very well with the abovementioned sources of system performance deterioration combines good features of two control concepts; model reference based adaptive control (MRAC) and robust sliding mode-based variable structure control (VSC). This hybrid concept, called the model reference variable structure control (MRVSC) has been studied in this paper to find out how parameter and load variations affect its performance. Four MRVCS methods were taken into consideration containing the sign function, a continuous control signal, a saturation function and an exponential function in the control signal. By using the 2nd–order reference model and by comparing the results obtained with these four methods on the studied three degrees-of-freedom planar robot control system, it is shown that all presented modified MRVSC methods provide accurate reference model tracking and significantly contribute to reduction of energy consumption and chattering in the control system. It can be concluded that modified MRVSC methods are satisfactorily robust to robot parameter changes and existence of external disturbances and measurement noise, although the initial robustness of the 1st type of the MRVSC method with the sign function was partially lost.

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