

# ADAPTIVE FUZZY-SLIDING MODE CONTROLLER FOR TRAJECTORY TRACKING CONTROL OF QUAD-ROTOR

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**Abstract:** *This paper deals with the design of an adaptive-fuzzy-PD-sliding mode controller to achieve stabilization of a quadrotor aircraft in the presence of wind disturbance. Firstly, the dynamic system modeling is carried out using Euler-Lagrange formalism. Then, an adaptive PD-sliding mode control system with an integral-operation switching surface is investigated for quadrotor desired trajectory tracking. Finally, an adaptive fuzzy-PD-sliding mode controller is proposed to achieve control objectives and system stabilization where the fuzzy logic system used to dynamically control parameters settings of the PD-sliding mode equivalent control law. Effectiveness and robustness of the proposed control scheme is verified through simulation results taking into account external disturbances. The simulation results of a quadrotor aircraft control with the proposed controller demonstrate the high performance during flight such as null tracking error and robustness in the presence of external disturbances.*

**Keywords:** *Quadrotor UAV, Sliding mode control, Adaptive PD-sliding mode controller, Fuzzy PD-sliding mode*

## 1. Introduction

Unmanned aerial vehicles (UAVs) already have a wide area of possible applications in military and civilian purposes such as surveillance, traffic monitoring, inspection, law enforcement, search and rescue, among others [1]. Continuous evolution of robotic vehicles technology also offer a remarkable growth in the market of unmanned helicopters, which nowadays includes vehicles of various types, sizes and operational capabilities. While many possible types of small UAVs exist, one very promising vehicle with respect to size, weight and maneuverability is the so called quadrotor [1-4].

The quadrotor is classified as a powered rotary wing vertical take-off and landing (VTOL) aircraft. It is consisting of fixed-pitch rotors mounted at the four ends of a simple cross configuration, as well as the direction of rotation of the rotors implies that front and the rear motor rotate clockwise, the left and the right motor rotate counter-clockwise (Fig. 1) [1-4].

The quadrotor is an interesting alternative to the classical helicopter because is mechanically simpler than a regular helicopter since it does not require a swash plate or teeter hinges and has several advantages in terms of maneuverability, motion control and cost. Because of the fixed pitch and its symmetric structure, this Omni-directional helicopter is dynamically excellent and its mathematical model is quite simple [4-6].

The dynamics of a quadrotor are a simplified form of helicopter dynamics that exhibit the basic problems of rotorcraft including under actuation, multi-input/multi-output (MIMO) design, and unknown nonlinearities and the states are highly coupled [3-6]. The movement of the quadrotor is caused by the resultant forces and moments of four independent rotors. Therefore the control algorithms designed for a quadrotor could be applied to a helicopter with relatively straightforward modifications, so that the quadrotor serves as a suitable, more tractable, case study for rotorcraft controls design [4-6].

The system we consider is an under actuated and has six degrees of freedom (DOF) with only four control inputs consisting of thrust and the three rotational torque inputs; characteristics which can make the platform difficult to control. To deal with this system, many modelling approaches have been presented [2, 3, 4, 7, 8] and various types of controllers have been proposed such as feedback linearization method [9, 10], backstepping control technique [11, 12], sliding mode control [8, 10, 11, 13], backstepping-sliding mode control [7, 14] and adaptive control [15, 16, 17, 18]. The authors of [17] have proposed an adaptive backstepping sliding mode control algorithm to stabilize the attitude. Considering the under-actuation, strong coupling properties of the aircraft, a nested double-loops control structure is designed in [17] and the adaptive estimation and sliding mode approach are used in the design procedure. In [5, 6], Madani and Benallegue divided the quad-rotor into three interconnected subsystems and developed a backstepping controller based on the Lyapunov stability theory in order that the aircraft tracks the desired trajectories. Bonna and Camino proposed a nonlinear control design based on feedback linearization for the quadrotor that guarantees the convergence trajectory to a given reference trajectory [9]. Zemalache and al. [13] a four rotors helicopter is studied and controlled using cascade-sliding mode control. In [13], the stabiliz-

ing/tracking control problem for the three decoupled displacement of the quadrotor has been considered. Furthermore, much research effort has been directed towards design of intelligent hybrid controllers using fuzzy logic. The authors of [18] studied a trajectory tracking of quadrotor unmanned aerial vehicle using a self-tuning fuzzy proportional integral derivative controller. The main idea of [18] is the design a fuzzy system tuning gains of the proportional-integral-derivative controller to stabilize the quadrotor.

In this paper, a fuzzy PD-sliding mode controller is developed for quadrotor dynamics trajectory control. The proposed controller, which combines the merits of the sliding mode control and the fuzzy inference system, is derived to overcome the drawback of sliding mode control. In this scheme, a fuzzy logic controller is used to dynamically control parameters settings of the PD-sliding mode equivalent control law. The new adaptive fuzzy PD-sliding mode controller has been achieved, fulfilling the robustness criteria specified in the sliding mode control and yielding a high performance in implementation to Control of Quad-Rotor. The rest of the paper is organized as follows: The modeling of the four-rotor rotorcraft based on Lagrange approach is presented in section II. The adaptive PD-sliding mode controller (A-PD-SMC) and the proposed adaptive fuzzy-PD-sliding mode controller (A-F-PD-SMC) development for quadrotor trajectory tracking are summarized in section III. Some simulation results are given and discussed in section IV. Finally, some conclusions are drawn in Section V.

## 2. Dynamic of the Quadrotor UAV

The quadrotor configuration consists of a rigid body equipped with four rotors which generate the propeller forces  $F_i$  ( $i = 1, 2, 3, 4$ ). In this type of helicopters the front and the rear motors (1 and 3) rotate clockwise while the other two motors (2 and 4) rotate counter-clockwise as shown in Fig. 1 [1-6]. The quadrotor rotorcraft does not have a swash plate

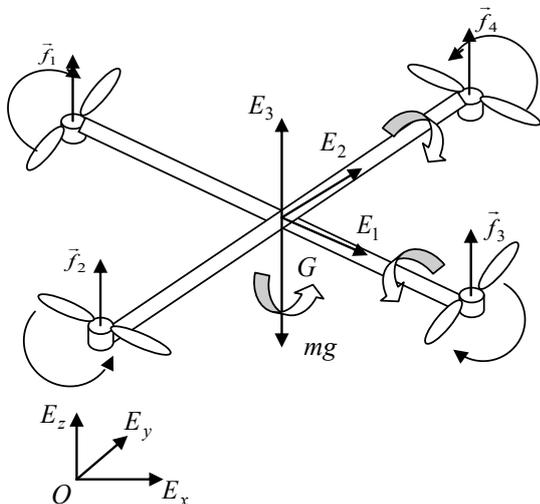


Fig. 1. Basic representation quadrotor unmanned aerial vehicle

as standard helicopters. The main thrust is the sum of the thrusts of each motor. Pitch movement is accomplished by increasing (reducing) the speed of the rear motor while reducing (increasing) the speed of the front motor. The roll movement is obtained similarly using the lateral motors. The yaw movement is achieved by increasing (decreasing) the speed of the front and rear motors while decreasing (increasing) the speed of the lateral motors [1-3]. This should be done while maintaining the total thrust constant.

The dynamic model of a quadrotor mini-aircraft derived using Euler-Lagrange formulation can be expressed as follows [1-5, 19]

$$\begin{cases} m\ddot{\xi} = u \begin{pmatrix} -s_\theta \\ c_\theta s_\phi \\ c_\theta c_\phi \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -mg \end{pmatrix} \\ \ddot{\eta} = \begin{pmatrix} \tau_\theta \\ \tau_\phi \\ \tau_\psi \end{pmatrix} = \mathfrak{S}^{-1}(\tau - C(\eta, \dot{\eta})\dot{\eta}) \end{cases} \quad (1)$$

where  $x$  and  $y$  are the coordinates in the horizontal plane, and  $z$  is the vertical position (see Fig. 1).  $\psi$  is the yaw angle around the  $z$  axis,  $\theta$  is the roll angle around the (new)  $x$  axis, and  $\phi$  is the pitch angle around the (new)  $y$  axis. The control inputs  $u_3$ ,  $\tau_\theta$ ,  $\tau_\phi$  and  $\tau_\psi$  are the total thrust or collective input (directed out the bottom of the aircraft) and the new angular moments (rolling moment, pitching moment and yawing moment).

Here, the quadrotor rotational dynamics can be expressed as follow [13, 19, 20]:

$$\begin{cases} \ddot{\theta} = u_4 \\ \ddot{\phi} = u_5 \\ \ddot{\psi} = u_6 \end{cases} \quad (2)$$

and the translational dynamics are given by:

$$\begin{cases} m\ddot{x} = -S_\theta u_3 \\ m\ddot{y} = S_\phi C_\theta u_3 \\ m\ddot{z} = C_\theta C_\phi u_3 - mg \end{cases} \quad (3)$$

## 3. Adaptive Fuzzy PD-Sliding Mode Control of the Quadrotor Rotorcraft

### 3.1. Sliding Mode Control

Variable structure control (VSC) with sliding mode (SMC) is one of the effective nonlinear robust control approaches because it provides system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode [8, 10, 11, 13, 21-23]. The first step of SMC design is to select a sliding surface that models the desired closed-loop performance in state-variable space. The control is then designed such that the nonlinear system-state trajectories are driven on a specified user-chosen

surface in state-space (phase plan). The system state trajectory in the period of time before reaching the sliding surface is called the reaching phase. Once the system trajectory reaches the sliding surface, it remains on it for all subsequent time and slides to the origin. The most important feature of sliding mode control is the insensitivity of the controlled system to uncertainties, but not during the reaching phase.

Without loss of generality, the possible choice of the nonlinear control law in the SMC can be defined as [8, 10, 11, 21-23]

$$u = u_{eq} + k \cdot \text{sgn}(s) \quad (4)$$

Where  $u_{eq}$  is called the equivalent control;  $k$  is a constant design parameter, representing the maximum controller output required to overcome parameter uncertainties and disturbances; sign is the signum function and  $s$  is called the switching function. In the conventional SMC design, a second-order system  $s$  is chosen in the state-space by the following scalar function [22]:

$$s = \left( \frac{\partial}{\partial t} + \lambda \right)^{r-1} \cdot e \quad (5)$$

Where  $e = x_d - x$  denotes the tracking position and  $x_d$  is the desired state;  $\lambda$  is a positive constant and  $r$  is the order of the sliding surface.

To ensure that the system trajectories move toward and remain on the sliding surface  $s = 0$ , the following sliding mode condition must be guaranteed [21, 22]:

$$s\dot{s} \leq -\sigma \cdot |s| \Rightarrow s\dot{s} \leq -\sigma \cdot \text{sgn}(s) \Rightarrow \dot{s} \cdot \text{sgn}(s) \leq -\sigma \quad (6)$$

Where  $\sigma$  is a positive constant that ensures a finite time convergence to  $s = 0$ .

In practice, the main obstacle for application of sliding mode control is chattering phenomenon due to the using a sign function. This phenomenon can be reduced by introducing a boundary layer around the switching surface, such as [13, 21-22]:

$$u_s = k \cdot \text{sat}\left(\frac{s}{\zeta}\right) \quad (7)$$

Where the constant factor  $\zeta$  defines the thickness of the boundary layer

For simplification reason in the development of the equivalent control law in the sliding mode control and to make the design task very easy for practical engineers, Nandam and Sen [24, 25] have proposed an equivalent control action based on proportional and derivative law. The major advantage of this design method is its ability of nonlinear systems control with any dependency on system parameters. Y. Li and al. [26] extends this control strategy by incorporating an integration term and to form a generic controller structure given by:

$$u = k_1 \cdot e + k_2 \int e \cdot dt + k_3 \cdot \frac{de}{dt} \quad (8)$$

with the parameters  $k_i$  ( $i = 1, 2, 3$ ) are given by a generalized hard-switching law as:

$$k_1 = \begin{cases} k_{1a} & es < 0 \\ k_{1b} & es > 0 \end{cases}; k_2 = \begin{cases} k_{2a} & \dot{es} < \varepsilon \\ k_{2b} & \dot{es} > \varepsilon \end{cases}; k_3 = \begin{cases} k_{3a} & s < 0 \\ k_{3b} & s > 0 \end{cases}$$

In this work, only the proportional and derivative terms are used to design equivalent control law  $u_{eq}$  for SMC control of the quadrotor with modified hard-switching parameters and the controller structure can be defined as:

$$u = u_s + k_1 \cdot e + k_2 \frac{de}{dt} \quad (9)$$

where

$$k_1 = \begin{cases} k_{1a} & \text{if } es > \varepsilon \\ k_{1b} & \text{if } -\varepsilon \leq es \leq \varepsilon \\ k_{1c} & \text{if } es < -\varepsilon \end{cases} \quad (10)$$

$$k_2 = \begin{cases} k_{2a} & \text{if } \dot{es} > \varepsilon \\ k_{2b} & \text{if } -\varepsilon \leq \dot{es} \leq \varepsilon \\ k_{2c} & \text{if } \dot{es} < -\varepsilon \end{cases} \quad (11)$$

$\varepsilon$  is a small positive gain.

### 3.2. Adaptive PD-Sliding Mode Control of Quadrotor

This section describes the design procedure of the robust nonlinear control via the adaptive PD-SMC for quadrotor trajectory tracking and stabilization with external disturbances handling. The objective of this controller is to obtain the quadrotor control laws so as to achieve high-quality position and altitude performance. The overall control structure of the quadrotor aircraft using adaptive PD-sliding mode and adaptive fuzzy-PD-sliding mode is depicted in Fig. 2. The vertical input force  $u_3$  is used to stabilize the altitude of the quadrotor. The desired reference values of roll ( $\theta_d$ ) and pitch ( $\phi_d$ ) are formed on the rotational controller by the position subsystem. The rotation controller is used to stabilize the quadrotor with control inputs  $u_3, u_4, u_5$  and  $u_6$ . The design of the proposed sliding mode controller for the quadrotor rotorcraft trajectories control involves the following steps:

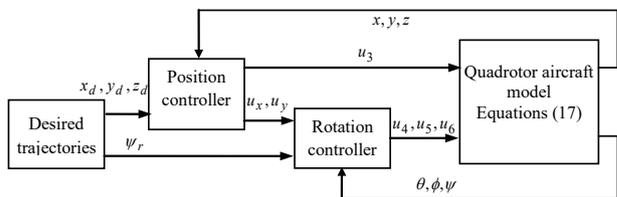


Fig. 2. Basic control structure of the quadrotor

**Altitude control.** The altitude can be controlled by the adaptive PD-sliding controller (see Fig. 3). Through the equation of the following movement

$$m\ddot{z} = C_\varphi C_\theta u_3 - m[g + a_3 \dot{z}] \quad (12)$$

The tracking error is defined is:

$$e_z = z_d - z \quad (13)$$

And the sliding surface can be given by the following linear function:

$$s_z = e_z + \lambda_z \dot{e}_z \quad (14)$$

The control of the vertical position can be obtained by using the following control input:

$$u_3 = \frac{m}{C_\phi C_\theta} [\ddot{z} + g + a_3 \dot{z}] \quad (15)$$

With  $a_3 = \frac{F_z}{m}$ , disturbances a long of position  $z$ .

And the expression of  $\ddot{z}$  can be given by

$$\ddot{z} = k_z \cdot \text{sat}(s_z) + u_z^{equ} \quad (16)$$

And the equivalent control  $u_z^{equ}$  for altitude control is defined by:

$$u_z^{equ} = k_{pz} \cdot e_z + k_{dz} \cdot \frac{de_z}{dt} \quad (17)$$

Where  $k_{pz}$  and  $k_{dz}$  are positive constants and are obtained using Eq. 10 and Eq. 11, as follow:

$$k_{pz} = \begin{cases} k_{pza} & \text{if } e_z s_z > \varepsilon \\ k_{pzb} & \text{if } -\varepsilon \leq e_z s_z \leq \varepsilon \\ k_{pzc} & \text{if } e_z s_z < -\varepsilon \end{cases} \quad (18)$$

And

$$k_{dz} = \begin{cases} k_{dza} & \text{if } \dot{e}_z s_z > \varepsilon \\ k_{dzb} & \text{if } -\varepsilon \leq \dot{e}_z s_z \leq \varepsilon \\ k_{dzc} & \text{if } \dot{e}_z s_z < -\varepsilon \end{cases} \quad (19)$$

From Eq. 15, Eq. 16 and Eq. 17 it follows that the output altitude tracking control can be given as:

$$u_3 = \frac{m}{C_\phi C_\theta} \left( \left( k_z \cdot \text{sat}(s_z) + k_{pz} \cdot e_z + k_{dz} \cdot \frac{de_z}{dt} \right) + g + a_3 \dot{z} \right) \quad (20)$$

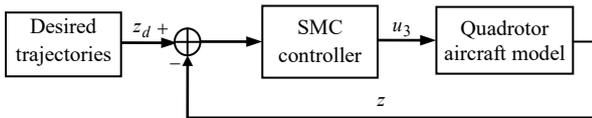


Fig. 3. Adaptive PD-Sliding mode control on  $z$  direction

**Linear  $x$  and  $y$  motion control.** From the model Eq. 3 we can see that the motion through the axes  $x$  and  $y$  depends on  $u_3$ . In fact  $u_3$  is the total thrust vector oriented to obtain the desired linear motion. If we considered  $u_x = -s_\theta$  and  $u_y = c_\theta s_\phi$  the orientations of  $u_3$  responsible for the motion through  $x$  and  $y$  axis respectively, we can then extract the roll and pitch angle necessary to compute the control  $u_x$  and  $u_y$  (Fig. 2).

Now, let denote the reference speeds in  $x$  and  $y$  directions by  $\dot{x}_d$  and  $\dot{y}_d$  respectively. Then, the tracking error is given by:

$$\begin{cases} \dot{e}_x = \dot{x}_d - \dot{x} \\ \dot{e}_y = \dot{y}_d - \dot{y} \end{cases} \quad (21)$$

Then, the sliding surfaces for this step can be given such as:

$$\begin{cases} s_x = e_x + \lambda_x \dot{e}_x \\ s_y = e_y + \lambda_y \dot{e}_y \end{cases} \quad (22)$$

The control laws can be obtained as follow:

$$\begin{cases} u_x = \frac{m}{u_3} \left( \left( k_x \cdot \text{sat}(s_x) + k_{px} \cdot e_x + k_{dx} \cdot \frac{de_x}{dt} \right) + a_1 \dot{x} \right) \\ u_y = \frac{m}{u_3} \left( \left( k_y \cdot \text{sat}(s_y) + k_{py} \cdot e_y + k_{dy} \cdot \frac{de_y}{dt} \right) + a_2 \dot{y} \right) \end{cases} \quad (23)$$

With  $a_1 = \frac{F_x}{m}$  and  $a_2 = \frac{F_y}{m}$ : disturbances a long positions  $x$  and  $y$ .

The reference roll and pitch angles can be computed by the following expressions:

$$\begin{cases} \theta_d = \arcsin \left( \frac{m}{u_3} \left( k_x \cdot \text{sat}(s_x) + k_{px} \cdot e_x + k_{dx} \cdot \frac{de_x}{dt} \right) \right) \\ \phi_d = \arcsin \left( \frac{m}{u_3 c_\theta} \left( k_y \cdot \text{sat}(s_y) + k_{py} \cdot e_y + k_{dy} \cdot \frac{de_y}{dt} \right) \right) \end{cases} \quad (24)$$

where  $k_x$  and  $k_y$  are positive design parameter. Parameters  $k_{px}$ ,  $k_{dx}$ ,  $k_{py}$  and  $k_{dy}$  are positive constants which can be obtained by the same manner used in Eq. 18 and Eq. 19.

**Attitude control ( $\psi$ ).** Attitude control is the heart of the control system because it keeps the 3D orientation. From the equations of dynamic model Eq. 18, the altitude subsystem containing vertical force input  $u_6$  is given by:

$$\ddot{\psi} = u_6 \quad (25)$$

The first step in adaptive PD-sliding control design is to consider the tracking error, such as:

$$e_\psi = \psi_d - \psi \quad (26)$$

We define also the sliding surface for the variable  $\psi$  as follows:

$$s_\psi = e_\psi + \lambda_\psi \dot{e}_\psi \quad (27)$$

The vertical force input  $u_6$  can be obtained by the same way as above. Then the equivalent control law  $u_6^{equ}$  is given by:

$$u_6^{equ} = k_{p\psi} \cdot e_\psi + k_{d\psi} \cdot \frac{de_\psi}{dt} \quad (28)$$

And the switching control law for the altitude controller is defined as:

$$u_6^{sw} = k_\psi \cdot \text{sat}(s_\psi) \quad (29)$$

**Roll and Pitch control ( $\theta$ ,  $\phi$ ).** The same steps are followed to extracted  $u_4$  and  $u_5$ . The control laws are derived using adaptive PD-sliding technique to control the quadrotor in  $(x, \theta)$  and  $(y, \phi)$  directions. The tracking errors for  $\theta$  and  $\phi$  are defined as:

$$\begin{cases} e_\theta = \theta_d - \theta \\ e_\phi = \phi_d - \phi \end{cases} \quad (30)$$

Then, the sliding surfaces for  $\theta$  and  $\phi$  are given by:

$$\begin{cases} s_\theta = e_\theta + \lambda_\theta \dot{e}_\theta \\ s_\phi = e_\phi + \lambda_\phi \dot{e}_\phi \end{cases} \quad (31)$$

The control laws are then:

$$\begin{cases} u_4 = \ddot{\theta} = k_\theta \cdot \text{sat}(s_\theta) + k_{p\theta} \cdot e_\theta + k_{d\theta} \cdot \frac{de_\theta}{dt} \\ u_5 = \ddot{\phi} = k_\phi \cdot \text{sat}(s_\phi) + k_{p\phi} \cdot e_\phi + k_{d\phi} \cdot \frac{de_\phi}{dt} \end{cases} \quad (32)$$

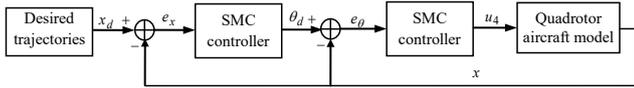


Fig. 4. Adaptive PD-sliding mode control on  $(x, \theta)$  direction

### 3.3. Adaptive Fuzzy PD-sliding Mode Controller

In this section, hybridization between the fuzzy logic and the sliding mode control is proposed in order to adapt the parameters of the equivalent component  $u_{eq}$  (Eq. 17) by two fuzzy logic controllers. In this approach, the fuzzy logic controllers are used to generate, in a soft way, the equivalent control law parameters given by (Eq. 18 and Eq. 19). So, the fuzzy logic controllers are designed to replace the inequalities which determine the parameters of the equivalent control action. The key idea of this controller is that instead of inequalities used to compute the parameters  $k_{pz}$  and  $k_{dz}$ . In this proposed adaptive fuzzy-PD-sliding mode control scheme, the sliding surface  $(s)$ , the error  $e$  and its time derivative  $\dot{e}$  form the inputs of the fuzzy implications of the switching rules. The first FLC, which is responsible for tuning  $k_{pz}$ , has two inputs  $(s$  and  $e)$  whereas the second FLC which generate  $k_{dz}$  has  $s$  and  $\dot{e}$  as inputs. The basic structure of the adaptive fuzzy PD-sliding mode controller is depicted in Fig. 5. The membership functions of the FLC1 inputs and output are shown in Fig. 6, whereas the membership functions of the FLC2 inputs and output are depicted in Fig. 7. The membership functions of the inputs are chosen triangular whereas the output membership functions are singleton. We note here that the triangular membership functions for the input variables are taken for their simplicity. The singleton parameters are taken equal to the  $k_{pz}$  and  $k_{dz}$  values (for example:  $B = 51$ ,  $M = 50$  and  $S = 49$  (see fig. 6)). After several tests, identical method is adopted for the membership output of the second regulator of  $k_{dz}$  tuning. Five fuzzy values are selected for linguistic expression of  $s$ ,  $e$  and  $\dot{e}$  inputs of the controllers: BN, MN, ZE, MP and BP. Big negative is represented as NB, medium negative as NM, zero as ZE, medium positive as PM and big positive as PB. For linguistic expression of outputs  $k_{pz}$  and  $k_{dz}$  of the controller, three fuzzy values are selected: B, M, and S. Big are represented as B, medium as M and small as s.

With the fuzzy implications,  $k_{pz}$  and  $k_{dz}$  have become nonlinear switched parameters, and the fuzzy inference mechanism can therefore replace the hard switching law generated by the 'if' conditioning of the adaptive PD-sliding mode controller. This new smooth nonlinear switching of the parameters of the sliding mode control works in such way that when  $(s$  and  $e)$  are in the positive direction  $k_{pz}$  has a positive value (red sub-tables) and when  $(s$  and  $e)$  are in the negative sense,  $k_{pz}$  has a negative value (blue sub-tables). And for the second parameter  $k_{dz}$ , the same logic is used to generate its values using  $s$  and  $\dot{e}$  as inputs variables. The resulting rule bases for the two output variables ( $k_{pz}$  and  $k_{dz}$ ) are presented in Table I. Likewise, the same design method has been applied for the other controller of translation  $(x$  and  $y)$  and rotational positions  $(\theta, \phi$  and  $\psi)$ . All these controllers have the same membership functions for the input variables  $(s, e$  and  $\dot{e})$  as shown in Fig. 6 and Fig. 7. However, their membership functions for the output variables can take different values as shown in Figs. 8-12.

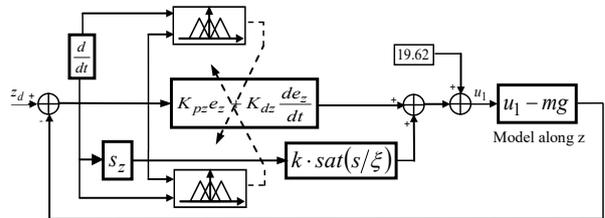


Fig. 5. Basic structure of the proposed adaptive Fuzzy-PD-SMC for quadrotor trajectories tracking (in z direction)

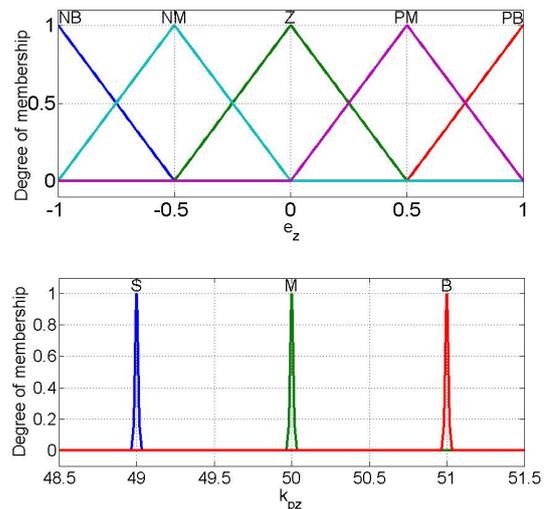


Fig. 6. Membership functions of the surface  $s$ , error  $e$  and variable  $k_{pz}$

### 4. Simulation Results

In order to verify the validity and the effectiveness of the proposed adaptive fuzzy-PD sliding mode controller, the designed controller has been tested for quadrotor trajectory tracking by numerical simulation using Matlab/Simulink software. The model parameters values of the quadrotor are given in table 2.

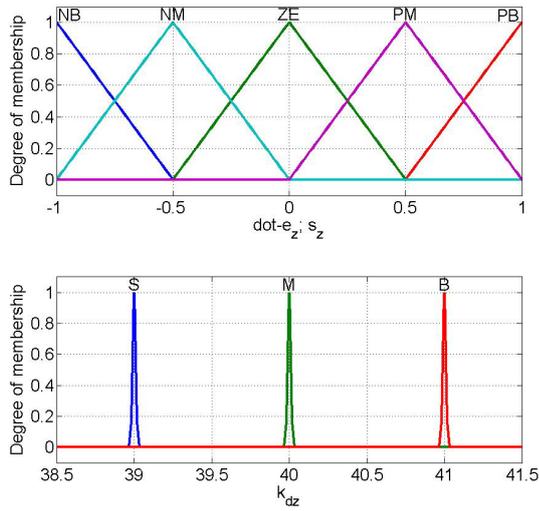


Fig. 7. Membership functions of  $s$ ,  $\dot{e}$  and variable  $k_{dz}$

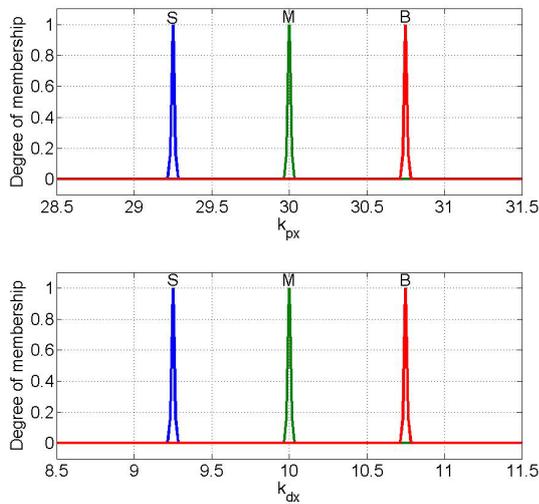


Fig. 8. Membership function of output variable  $k_{pz}/k_{dz}$

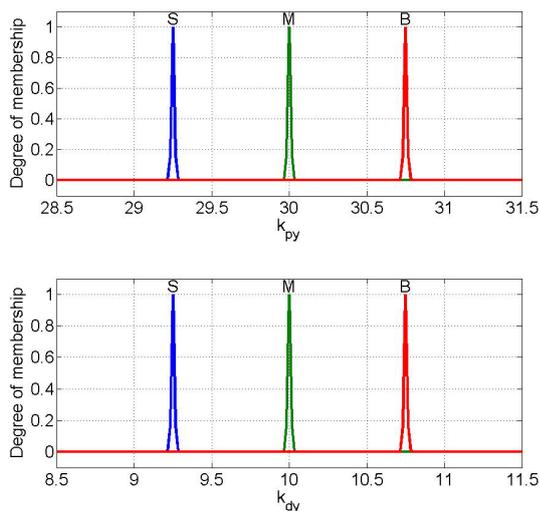


Fig. 9. Membership functions of output variable  $k_{pz}/k_{dz}$

A chosen trajectory, arc curves, is performed for the quadrotor to illustrate the operation of the proposed control scheme and its robustness against external disturbances. The performances of adaptive PD-sliding mode controller are compared with the

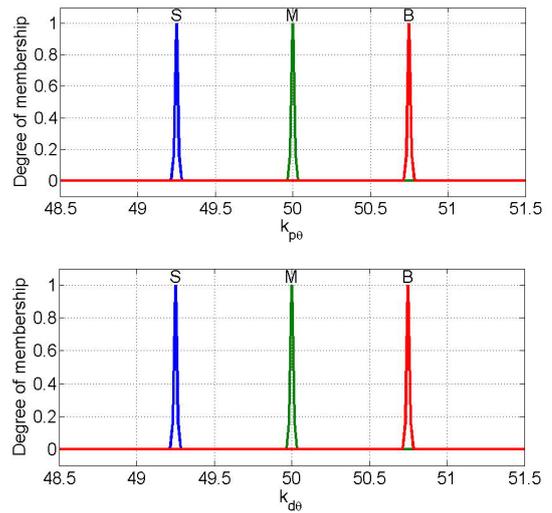


Fig. 10. Membership functions of output variable  $k_{\rho}/k_{d0}$

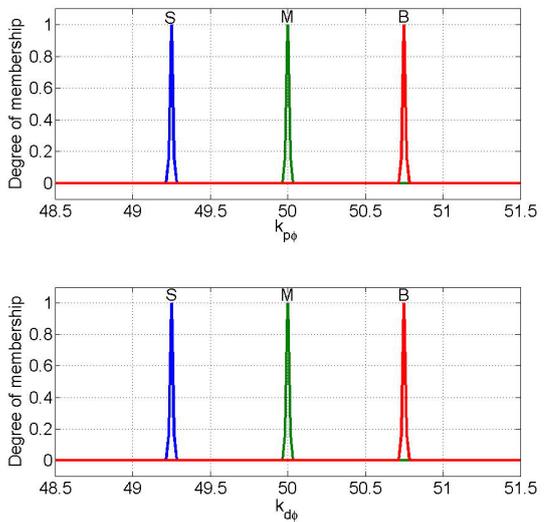


Fig. 11. Membership functions of output variable  $k_{p\phi}/k_{d\phi}$

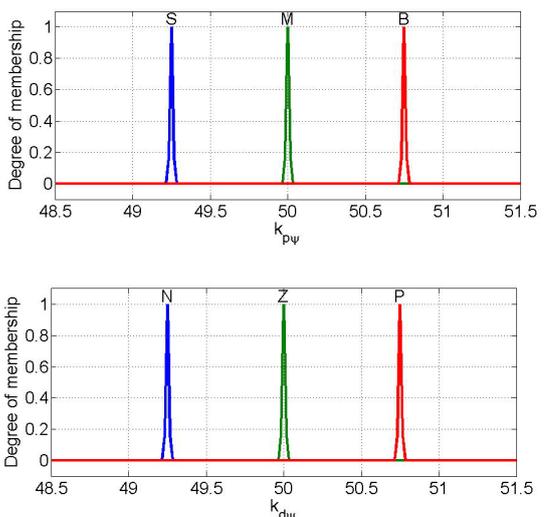


Fig.12. Membership function of output variable  $k_{p\psi}/k_{d\psi}$

performances of the adaptive fuzzy PD-sliding mode controller. To test the robustness of the controllers for arc curve tracking, the simulation has been done con-

sidering an external disturbance as wind influence along all directions ( $F_z = 2\text{N}$  at 15s,  $F_x = 1.5\text{N}$  at 25s and  $F_y = 2\text{N}$  applied at 40s). In addition, the desired  $\psi$  has been fixed at  $\pi/3$ .

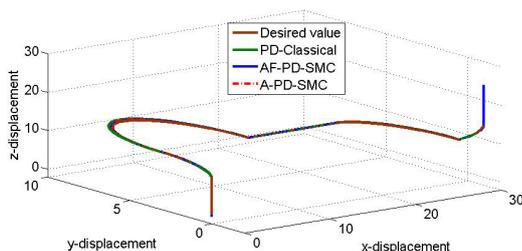
**Tab. 1.** Fuzzy rules of  $k_{pz}, k_{dz}$

$s \backslash e$	BN	MN	ZE	MP	BP
BN	B	B	S	S	S
MN	B	B	S	S	S
ZE	S	S	M	B	B
MP	S	S	B	B	B
BP	S	S	B	B	B

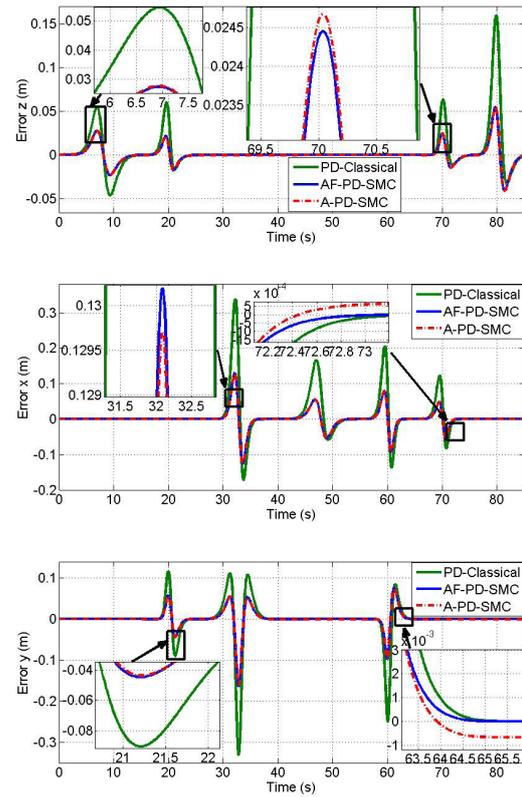
**Tab. 2.** Quadrotor model parameters

Parameter	Value	Units
$l$	0.23	m
$I_{xx}$	$2224931 \times 10^{-7}$	$\text{Kg} \cdot \text{m}^2$
$I_{yy}$	$222611 \times 10^{-7}$	$\text{Kg} \cdot \text{m}^2$
$I_{zz}$	$325130 \times 10^{-7}$	$\text{Kg} \cdot \text{m}^2$
$I_{xx}$	$10^{-5}$	$\text{N} \cdot \text{s}^2$
$K_M$	$9 \times 10^{-6}$	$\text{N} \cdot \text{s}^2 \cdot \text{m}$

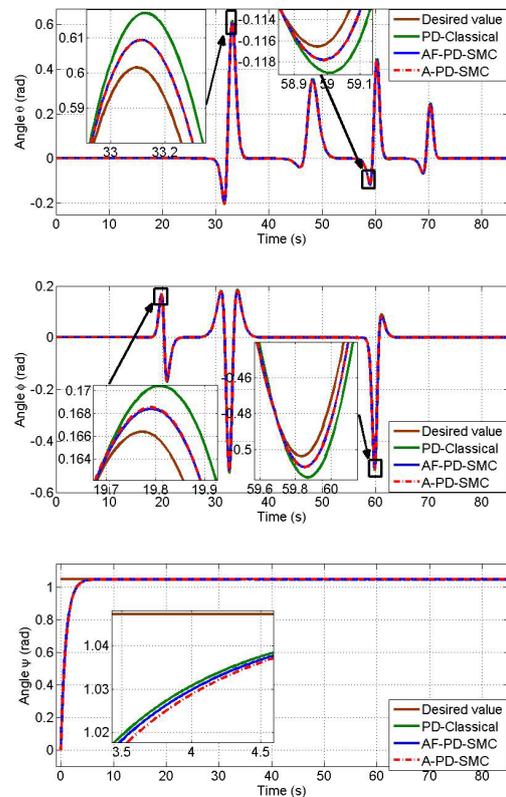
Fig. 13 shows the flying trajectory of the quadrotor stabilizing for arc curves tracking in case of adaptive fuzzy PD-sliding mode and adaptive fuzzy PD-Sliding mode control application. In this figure, the quadrotor flying response of the proposed controllers is clearly observed to present good performance for trajectory tracking and more robustness (minimal steady-state error against load force application). The errors on translation displacement positions in ( $x, y$  and  $z$ ) directions and rotational positions ( $\theta$  and  $\phi$ ) of the quadrotor are shown in Fig. 14 and Fig. 15 respectively, for the two types applied controllers (APD-SMC and AF-PD-SMC). It should be noted that, with the two proposed control scheme, the displacement errors converge to zero. However, we can note here that the proposed adaptive fuzzy-PD-Sliding mode controller presents good transient performances and it's more robust than the adaptive PD-Sliding mode controller for trajectory tracking (i.e. minimal error tracking). From Fig. 14, we observe that rotational positions ( $\theta$  and  $\phi$ )



**Fig. 13.** Response of quadrotor for arc curves tracking trajectory with A-PD-SMC and AF-PD-SMC

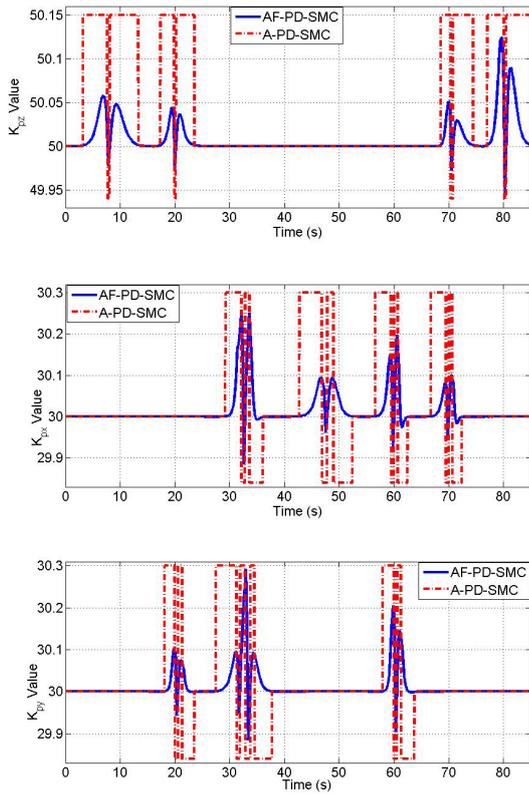


**Fig. 14.** Tracking errors according to ( $z, x, y$ ) directions in case of the AF-PD-SMC, A-PD-SMC and SMC application

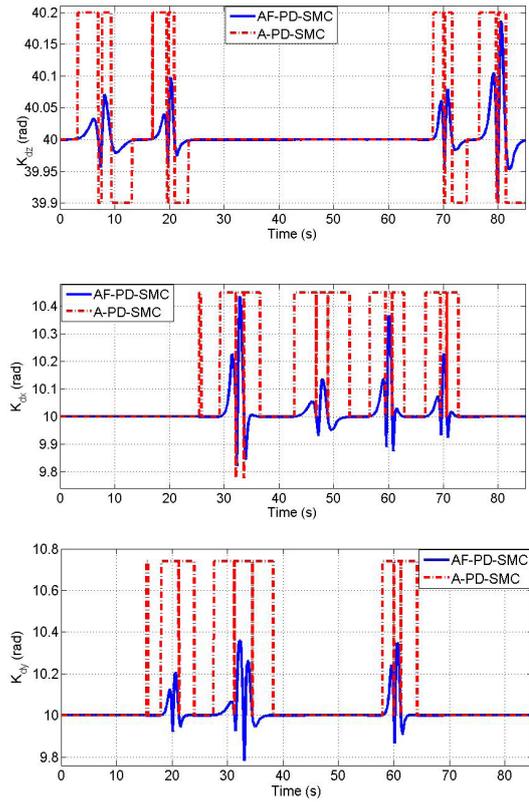


**Fig. 15.** The Pitch, Roll and Yaw angles ( $\theta, \phi, \psi$ ) of the quadrotor for the AF-PD-SMC, A-PD-SMC and SMC

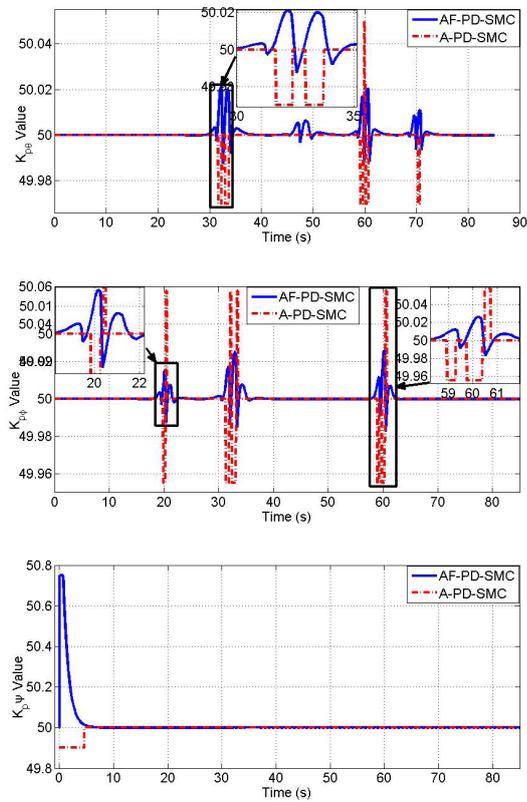
are converging to desired values (i.e. zero) after some transient affectation during transient variation on the directions which prove the stabilization of the quadro-



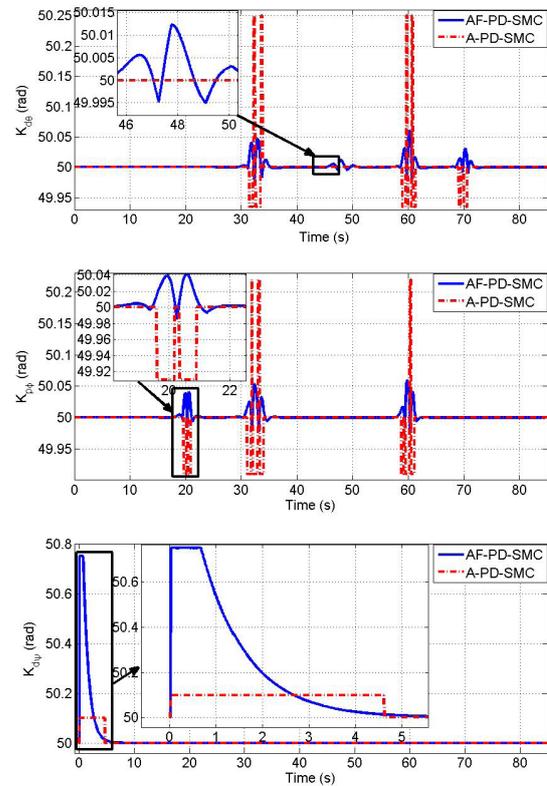
**Fig. 16a.** Variation in proportional gains values of the equivalent control laws (translational subsystem)



**Fig. 17a.** Variation of the derivative gains values of the equivalent control laws (translational subsystem)

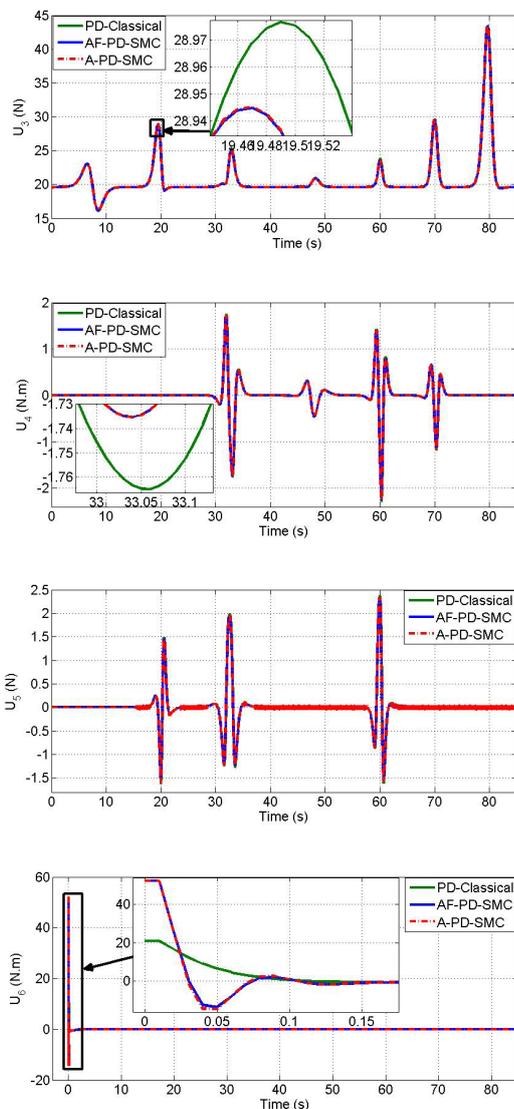


**Fig. 16b.** Variation in proportional gains values of the equivalent control laws (rotational subsystem)



**Fig. 17b.** Variation of the derivative gains values of the equivalent control laws (rotational subsystem)

tor along all directions. Moreover, Fig. 16 depicts the proportional gains of the different equivalent control laws  $k_{px}, k_{py}, k_{pz}, k_{p\phi}, k_{p\psi}$  whereas Fig. 16 shows the derivative gains  $k_{dx}, k_{dy}, k_{dz}, k_{d\phi}, k_{d\psi}$  for the two types applied controllers (A-PD-SMC and AF-PD-SMC). From Fig. 16 and Fig. 17, it can be observed that the adaptive fuzzy-PD-SMC has lower value gains in comparison with the adaptive PD-SMC. In addition, we can notice that these gains vary smoothly in nonlinear form for the AF-PD-SMC whereas they vary as a signum function for the A-PD-SMC. This performance allowed the proposed controller to reduce the chattering phenomenon with good transient tracking errors. In addition, it has been exploited to mitigate the variation caused by external disturbances. The control input signals ( $u_3, u_4, u_5$  and  $u_6$ ) are illustrated in Fig. 18. We notice that the inputs  $u_4$  and  $u_5$  converge to zero in steady-state after the quadrotor stabilization to its desired orientation. Furthermore, we notice here that the control inputs ( $u_3, u_4, u_5$  and  $u_6$ ) have minimal values and smooth signal in AF-PD-SMC compared to those in case of A-PD-SMC (see zoomed responses in Fig. 18).



**Fig. 18.** Control effort signals for the two types of controllers AF-PD-SMC and A-PD-SMC

## 5. Conclusion

In this paper, we have demonstrated the application of a novel hybrid adaptive fuzzy based PD sliding mode controller for stabilization and trajectories tracking of a quadrotor rotorcraft. The main control objective of the proposed controllers is to allow the quadrotor to track desired trajectories under external disturbance variation. First, dynamical model of the quadrotor has been developed and presented using Euler-Lagrange formulation. Then, an adaptive PD-Sliding mode controller was studied and developed for quadrotor control and stabilization. After that, fuzzy logic controllers have been introduced for designing a robust and adaptive PD-sliding mode control for trajectory tracking of the quadrotor. The combination between fuzzy logic and adaptive PD-sliding mode control has been successfully implemented and simulation. This combination allowed us to improve the control performance of the quadrotor and provide more robustness regarding to the external disturbance variation. The simulation results shown clearly that the proposed AFPDSM controller provides high precision trajectory tracking and good stabilization performance can be achieved enabling the rotorcraft to stabilize on desired values. Note also that the values of the amplitudes and periods of the AF-PD-SMC controller's coefficients vary according to the amplitudes and periods of the applied signals, whereas the A-PD-SMC controller's coefficients vary in period and the amplitudes remain fixed. These variations ensure a good follow-up of the trajectories.

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