

# IDENTIFICATION OF INFLUENCE OF PART TOLERANCES OF 1PWR-SE PUMP ON ITS TOTAL EFFICIENCY TAKING INTO CONSIDERATION MULTI-VALUED LOGIC TREES

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*Adam Deptuła, Piotr Osiński, Marian A. Partyka*

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## Abstract:

*This paper presents a methodology for identifying the influence of the tolerances used in model pump (TYPE 1PWR – SE) construction on the total efficiency. The identification of the sensitive control dimensions (Value/Tolerance) of examined pumps has been made by means of multi-valued logic and inductive decision trees. The innovation of the prototype unit is based on oblique gears with involute teeth, modified in the lower and upper part of the profile. The modification in the lower part was made using the so-called tooth root undercutting technique. Through the use of multivalent logic trees, the designated rank of importance of both structural and operational parameters is identified, taking into account the effect of tolerances on construction. The area is increased by cutting the oblique teeth.*

**Keywords:** *multiple-valued logic function, optimization, gear pump after tooth root undercutting, degree of parameters importance*

## 1. Introduction

Hydraulic systems are often used because of their high power transmission capabilities and relatively high efficiency. One of the main elements of hydraulic systems are the energy generators in the liquid stream, also known as pumps. Industrial gear pumps with external gearing are the most widespread pumps used in the industry. Their share is estimated at around 50%. Such common use results from a simple and compact design, operational reliability, high resistance to contamination of the working medium, high efficiency, small dimensions (compared to other pumping units), and low production costs [3]. The basic element of each hydrostatic system is a positive displacement pump that performs the function of a liquid stream energy generator. The history of pump development has its origins in antiquity. In the mid-third century, a Greek constructor Ktesibios (285 BC–228 BC) invented a piston pump used to extinguish fires. The first industrial application of pumps dates back to the 15<sup>th</sup> century, when they were used to drain mines. One of the first documented description of a positive displacement pump took place in 1604, by German astronomer and mathematician Johannes Kepler (1571–1630), in one of the first pump patents. The original appli-

cation of the described solution at the time was to pump water out of mines [31, 29].

Currently, a variety of different construction methods are used. In the group of positive displacement pumps, the largest application was found by gear pumps with external gearing. Pumps of this type are characterized by relatively high working pressures of up to 32MPa. In addition, recently there have been solutions proposed with reduced pulsation obtained from the so-called zero lateral play [22]. The axles of the gears are mounted with a pre-determined clearance, and loaded with radial force acting on divided bearing bodies. Examples of this design include the Silence construction by Rexroth or the Caspar type Whisper pump constructed according to US Patent No. 5564225 or European Patent No. EP0692633. The production of gears with zero lateral play requires a highly accurate technological intervention. The gears are paired with each other and additionally selected for size based on the clearances in the plain bearings. Such Labor-intensive manufacturing technology generates significant costs [14, 30, 35]. Currently, pumps are among the most widespread working machines and are used in all fields of technology. Gear pumps are the most widespread among the positive displacement pumps used in hydraulic propulsion systems as energy generators. Their participation is estimated at more than half of all pumps manufactured. Such common use results from a simple and compact design, operational reliability, high efficiency factor and low production cost. The development of modern pumping units is currently associated with two trends: minimization of mass, vibrations, and efficiency of pulsation and the reduction of noise emission into the environment. The reduction of internal tolerances is connected with the minimization of energy losses, increasing the transferred power, and improved energy efficiency of the generator [12, 13]. The use of pumps in hydraulic systems is of particular importance, especially as a source of mechanical vibrations. An important source of vibrations are propulsion systems, for example, a combustion engine performing a periodic work cycle with variable characteristics. The working hydraulic system is also a significant source of mechanical vibrations, caused mainly by shock pressure changes, and the periodic nature of displacement pump operation. Vibrations generated in this way are characteristic at different frequencies, so there are different ways

in which they propagate through hydraulic and mechanical systems [11, 28]. This problem also applies to micropumps. The total efficiency of gear pumps which are produced currently is about 80–90% (for nominal pressures reaching 28 MPa). Such a large span is mainly connected with the adopted manufacturing tolerance. Due to the significant difficulty of establishing a precise relationship, the influence of the manufacturing tolerance of particular elements on the energy efficiency of the created pump, this paper tries to identify critical deviations using a method based on multiple-valued logic trees. The improvement of internal tightness is connected with the minimization of energy losses, increasing the transferred power, and energy efficiency of the generator.

The article presents only the results of part of the research concerning the identification of the impact of tolerances on the pump design model (1PWR-SE) on the overall efficiency. A performed identification of sensitive control dimensions of pumps surveyed by various heuristic methods suggests that it is reliable – then subsequently the multi-valued and inductive decision trees were used.

## 2. Construction and Design of the Volumetric Outline of Pumps for Total Efficiency with the Use of Logical Decision Trees – Assembling Technology Optimization

The increase in requirements related to accuracy results in higher and higher cost requirements for machine tools. The main attention is paid to increasing the efficiency while increasing the accuracy of manufacturing. In order to obtain sufficient accuracy at the level of micrometers, a whole range of errors should be constantly checked and compensated for, that is, kinematic errors, geometrical errors due to cutting forces, and accepted machining parameters. The errors mentioned can be significantly reduced, but it is not possible to eliminate them completely. Among the errors arising during the production process, the most important are:

- 1) thermal errors,
- 2) geometrical errors,
- 3) kinematic errors,
- 4) errors forced during processing,
- 5) drive system errors,
- 6) errors of control elements and regulators,
- 7) errors of measuring systems,
- 8) and others.

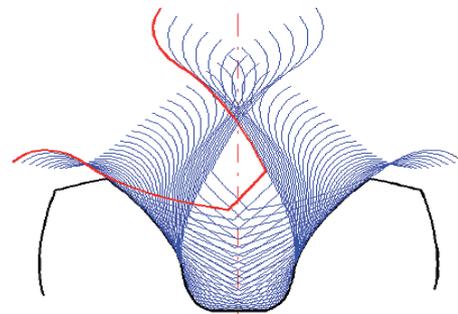
Another type of errors are those that appear during the machine's operation and result from the operation of individual drives and the inaccuracies of the interpolators. Control circuits operate in a feedback loop, and thus do not react to the error in real time. Therefore, one should take into account the occurrence of a certain time delay depending on the sampling frequency. The correct selection of the sampling frequency relative to the set machining speed has a significant effect on the kinematic error.

The working environment of machine tools is related to the occurrence of vibrations, which are

the result of both the machining process on which the process takes place as well as being the result of working machines in the immediate vicinity. The elimination of this type of interference is extremely difficult, and often involves the use of properly selected vibroisolators. The choice of criteria was determined by the main goal of the pumps being constructed, that is, the improvement of selected hydraulic and acoustic parameters. Not achieving the assumed effect was synonymous with a technologically unsuccessful product. The optimization process was carried out by taking into account the following five basic criteria:

**I. Technical feasibility.** Envelope methods are the basic methods of machining gears with involute tooth contours. Unfortunately, it is not possible to machine all tooth contours with enveloping methods. Thus, a formative method is an alternative. The tool in this machining method has a rebate shape. It is important in the context of achieving accuracy of involute outlines when cutting teeth with simple tools. For this reason, the envelope method is used as the basic method of machining gears. The envelope method uses the geometric process of involute formation (see, for example, Fig. 1) [13].

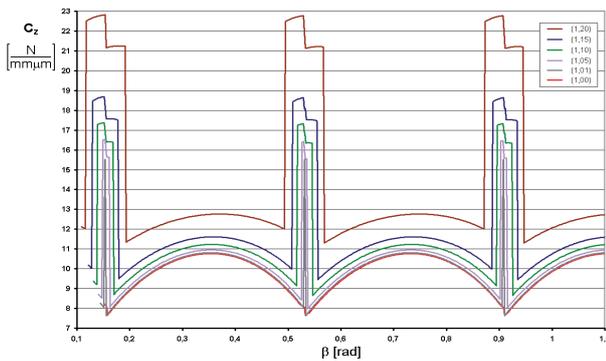
**II. Obtaining the minimum compression ratio.** Liquid sealing in the notches of cooperating gears occurs in pumps with external gearing when the number of buttresses is greater than one.



**Fig. 1. Envelope method of teeth cutting – course of the boundary positions of the cutting edge [21]**

In this case, a certain volume of liquid is separated from both the suction and discharge chambers. Improper unloading leads to an increase in noise and the occurrence of dynamic forces in the meshing that generates additional sound-shaking vibrations.

**III. The occurrence of small changes in dynamic forces in meshing.** In the literature [13], the influence of modifications to the involute outline on the properties of a dynamic gear pump was demonstrated. The course of the change in stiffness in meshing was determined based on the method of calculation presented in the paper [13]. In the calculations, an even distribution of dynamic loading force ( $P_d$ ) along the width of the wheels was assumed. In the literature [13], according to the above assumptions, the stiffness of meshing was calculated, which in a further stage was used to determine the stiffness of six gears with different degrees of coverage  $e = 1.20, 1.15, 1.10, 1.05, 1.01$ , and  $1.00$ . The results of calculations are shown in Figure 2.



**Fig. 2. The course of meshing stiffness at different degrees of undercutting of the tooth's tooth for  $z = 10$ ,  $m_0 = 4$ ,  $\alpha_0 = 20^\circ$ ,  $\gamma = 1.17$ ,  $x = 0.5$**

The optimized involute poly contour should provide small changes in dynamic force in meshing as a result of the modifications and corrections used.

**IV. Obtaining the minimum pulsation rate of performance.** The measure of performance pulsation is the so-called efficiency  $\delta$  coefficient of performance, defined as the ratio of the difference between the maximum and minimum efficiency and the average efficiency. The value of performance pulsation for typical toothed units with external gearing is on average around 18%. The use of glacial contours, skewed teeth, and zero interdental clearance is an alternative to high-frequency conventional pumps. In addition, such constructions can largely be performed on machines used to manufacture conventional units.

**V. Ensure high energy efficiency.** An important direction for the development of gear pumps is to minimize energy losses and increase the transferred power, and thus the tendency of changes directed to an even greater increase in the energy efficiency of the generator. The research results presented in the literature indicate the possibility of increasing the efficiency to values well above 90%. Increasing the energy efficiency of prototype units is mainly due to high internal tightness.

### 2.1 Identification of the impact of construction technology used in the manufacturing of polyinvolute pumps

The purpose of the identification of the impact of the technology used in the construction of polyvalent pumps was an attempt to determine the sensitive control dimensions (values and tolerances) of the tested pumps for teeth made using chip technology [17] and grinding [18], [19], and [20]. The control dimensions concerned six details of the pump data: the raking gear, the driven gear, the set of bearings, the body, the plate, the cover, and the tightening force of the screws.

Tests carried out for 40 model units and prototype pumps have shown that it is not possible to analyze only the selected details, such as gear wheels, for the overall efficiency of the gear pump. Testing the total efficiency of two units with the same wheels mounted in different bodies gave different results. Therefore, a properly conducted analysis should be extended to control dimensions for all elements. For example, to be able to include in the assessment of the unit in

which the wheels were correctly made, but the technological criteria of other elements were not met.

The analysis first employed a heuristic method based on local searching of the largest deviations between data values [8, 15]. Then an inductive algorithm of decision trees based on the growth of entropy was applied [33, 34]. A neural network was implemented, learning the measurement relationships between the data, and finding the most important control points [9, 10]. An evolutionary (genetic) algorithm was used as the last method [16].

The first publication cycle concerns the application of the multi-valued logical tree method. In the work [5], multi-valued logical trees were used to determine the sensitivity of measurement points for (3PWR-SE) model pumps. The work [4] concerns type 2PWR-SE model pumps. This article presents the results of the use of multi-valued logic trees for the model pump number: 1PWR-SE from the point of view of the criterion of total efficiency. The Quine-McCluskey algorithm of minimizing the partial multivalent logic functions allows the use of tree structures as the tools of application and support for process design, optimization, and decision-making [2, 23]. If the design and/or operating parameters, taking on numeric values from a specific range, were designated by a set of logic variables, we can perform the discretization of such numerical ranges. The set of all number combinations is a tree with the number of levels equal to the number of design and/or operational parameters.

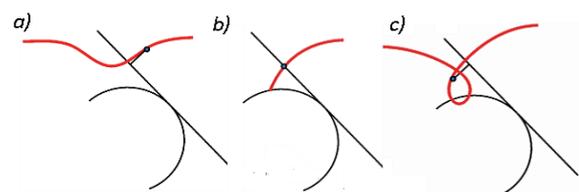
## 3. The Research Objective

The object of this research was the construction of both model and prototype pumps, the outline of which was optimized using multi-valued logical trees.

The process of optimization has been carried out with respect to the five basic criteria:

- I. Manufacturability,
- II. Obtaining a minimum compression ratio,
- III. The occurrence of small changes in the dynamic forces in the engagement,
- IV. Obtaining a minimum ratio of pulsation efficiency,
- V. Ensuring high energy efficiency.

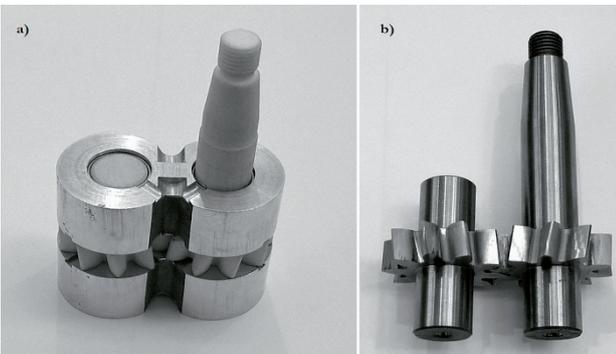
The gear profiles were selected in the optimization algorithm and are characterized by the presence of two involute ordinary and one involute extended profiles. The analysis includes the basic geometrical relationships gear (Fig. 3) [6].



**Fig. 3. Elements of design: a) involute summary, b) involute regular, c) involute extended**

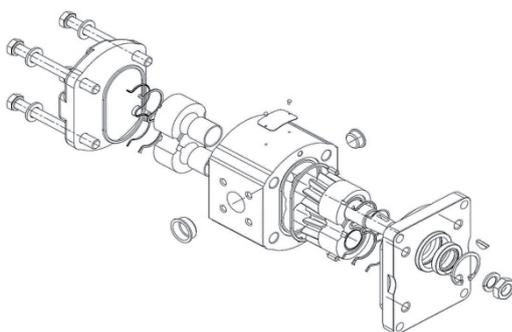
Gear pumps of the prototype series 1PWR – SE belonging to group I [12] were the research object. Wrocław University of Technology in cooperation

with the company HYDROTOR S.A. designed the units. The experimental pump has been designed with taking into consideration the technological capabilities of the company HYDROTOR S.A.. The innovation of the prototype unit is based on using oblique gears with an involute teeth modified in the lower and upper part of the profile. The modification in the lower part was made using the so-called tooth root undercutting. The outline has an elongated shape in the normal plane and its application causes the shortening of a part of the buttress and subsequent reduction of the sealed area [21]. The area has been increased by cutting the oblique teeth. The inclination of the tooth line also favors a decrease in the pressure pulsation [21]. An exemplary outline of teeth for gear wheels is shown in Figure 4a. Before the wheels were made, the kinematics of optimized three-way gearing on wheels printed using 3D printing technology was evaluated. The model wheels shown in Figure 4b were the equivalent of wheels with a unit capacity of  $q = 8 \text{ cm}^3/\text{rev}$ .



**Fig. 4. a) Three involute wheels made using 3D printing technology, b) Ground gears**

The application of a correction in the region of the apex was produced in order to improve the cooperation of gears at the moment of entering the subsequent pair of gears into cooperation. Finally, the gears got apolyevolvent outline. In order to determine the influence of the manufacturing technology on the level of emitted noise [21], it was decided that the gears should be made using classical grinding technology (Fig. 5). The pump prototype was fabricated entirely by the company HYDROTOR S.A.. The research types of gear pumps with serial numbers and performance are summarized in the table 1.



**Fig. 5. The exploded view of the prototype gear pump type 1PWR-SE**

The analyzed pump is characterized with a three-part construction. The consecutive parts are composed of:

- a plate with sealing of a roller, flanges fastening the pump to the coupling casing, holes for screws connecting the pump elements, dowel holes,
- casing, gears, slide bearing casings, discharge and suction ports, flow holes for screws connecting the casing elements, holes for screws of the flange connections and dowel holes.

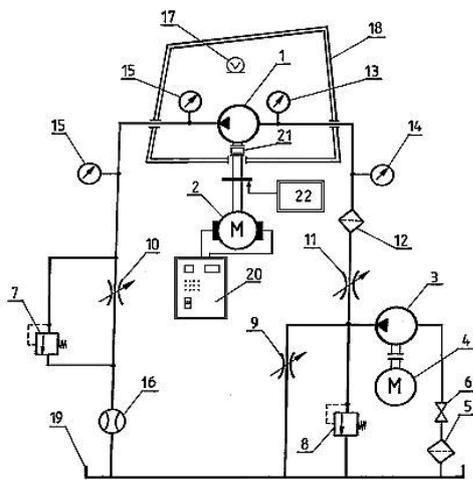
**Table 1. Tested gear pumps**

#	Pump model	Serial number
1.	1PWR - SE - 4/28 - 2 - 776	A 150 10001
2.	1PWR - SE - 4/28 - 2 - 776	A 150 10002
3.	1PWR - SE - 4/28 - 2 - 776	A 150 10003
4.	1PWR - SE - 4/28 - 2 - 776	A 150 10004
5.	1PWR - SE - 4/28 - 2 - 776	A 150 10005
6.	1PWR - SE - 4/28 - 2 - 776	A 150 10006
7.	1PWR - SE - 4/28 - 2 - 776	A 150 10007
8.	1PWR - SE - 4/28 - 2 - 776	A 150 10008
9.	1PWR - SE - 4/28 - 2 - 776	A 150 10009
10.	1PWR - SE - 4/28 - 2 - 776	A 150 10010

The optimization of the analyzed units of model pumps was composed of two parts: The first part concerned the optimization of the pump tooth outline with the use of multi-valued logic trees. The second part was based on the use of logic decision structures in the optimization of processing technology of elements having an influence on the total efficiency of the newly designed unit (the issue has been more deeply analyzed in this article). It resulted in the limitation of dimensions and size tolerance where it is necessary and decreasing the accuracy requirement in places of little importance. The optimization of the technology caused a decrease of the pumps production and an increase in their effectiveness. An advantage of shaved pumps is their higher total efficiency caused by low hydraulic and mechanical losses. Low values of roughness parameters of the tooth profile contribute to the improvement of the lubrication conditions of cooperating teeth. Besides the increase in the volumetric efficiency, the increase in the total efficiency as well as in the hydraulic and mechanical efficiency also took place. It is of particular importance in control distributors in difficult working conditions, designed for marine drilling equipment where there is high flow and small drop in pressure. It is also possible to mention here hydraulic systems used to change the propeller pitch of ships, as well as pump applications in the control and fuel systems of ship engines.



**Fig. 6. Test stand: DC drive motor, feed pump, AC motor [13]**



**Fig. 7. Schematic layout: 1-tested pump, 2-driving DC motor, 3-feed pump, 4-AC motor, 5-suction filter, 6-cut-off valve, 7,8-safety valves, 9,10,11-cut-off valve, 12-drain filter, 13,14-manovacuumeter, 15-pressure gauge, 16-flowmeter with microammeter, 17-measuring microphones, 18-sound chamber, 19-tank, 20-electronic rpm adjustment system, 21-torque sensor with recorder, 22-photocell with measuring counter**

#### 4. The Measurement Rig

The static characteristics of the pump with a multi-evolutional profile were determined using the rig shown in figs. 6-7. In the rig, the tested pump 1 is driven by 100 kW DC motor 2 working in tandem with a SCR control system 20. The Pxob-94a motor and the DSI-0360/MN-503 thyristor control system make it possible for the smooth changing of pump rotational speed from 0 to 2000 rpm [13]. Preliminary pressure pump 3 and tested pump 1 are protected by safety valves 7 and 8. Tested pump 1 is loaded through throttle valve 10. The pump's actual delivery  $Q_{rz}$  is measured by PT-M1 turbine flowmeter 16 with a PT15-100 flow sensor with a measuring range of 0-100 dm<sup>3</sup>/min. Instantaneous flow rates are record-

ed using a METEX microammeter type M-3650B. The pump output torque  $M$  is measured by Mt1000 torque sensor 21 with a measuring range of 0-000 Nm and a Beta 2002 recording system. The pump's number of revolutions  $n$  is controlled by the driving motor shaft by a measuring system consisting of a photocell and digital counter 22. For this purpose a disc with internal holes is mounted on the motor shaft. Temperature  $t$  of the liquid in the tank was measured by a set of thermistors (sensor PU 391/2 and meter PU 381/1).

Tests were carried out after the trial starting of the rig, that is, after the operation of the pump and the safety valve and the indications of all the measuring instruments had been checked. Measurement began with setting the prescribed shaft rotational speeds to  $n = 500, 800, 1000, 1500,$  and  $2000$  rpm. Pump loading was effected for  $p_t = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30,$  and  $32$  MPa [13]. The maximum forcing pressure was limited by the measuring range of the torque meter 21. The static characteristics were tested at a constant working fluid temperature of  $50^\circ\text{C}$ .

#### 5. Identification of the Impact of Tolerances on the Pump Design Model (TYPE 1PWR – SE)

The object of the analysis was the control measurements (value and tolerance) for the prototype gear pump series. The aim of the analysis was to identify the sensitive dimensions of the controls (value and tolerance) by testing the pumps. The dimensions of six pieces of data related to pumps: measurement of tightening PDs, active gear KZP, passive gear KZPn, a set of bearings KL and KLa, corps KR, plate PLt, cover PKr. The values and ranges of sensitive dimensions of the audits have not been given due to data protection and tolerance design company producing pumps tested. The degree of sensitivity of control dimensions (values and tolerances) implies differences in the values of the analyzed pumps efficiency. The value or tolerance change vulnerability during the measurement of control dimensions of parameters for particular pumps determines their importance rank. The determination of the most important places of particular parameters would make it possible to create a range of pumps characterized by the best efficiency parameters in the manufacturing process. The following 5 classification and optimization methods were used in the analysis:

- heuristic method – based on the local searching of the biggest deviations of given values,
- greedy algorithm. In order to determine a solution, a greedy, that is, the most promising at a given moment, choice of a partial solution is made in every step,
- application of the neural network. The Neuronix program and the multi-layered unidirectional network with a learning backpropagation algorithm were used in the analysis,
- application of the evolutionary algorithm.

A detailed calculation analysis of each of the methods will be presented in subsequent publications. Control measurements, both the most important ones and the less important ones, were determined in the iden-

tification of control measurements analysis. Especially important is the correct evaluation of tolerance during the construction of integrated decision-making methods [18]. The most important measurements are those specified by all applied research methods. Less important measurements are control measurements determined by at least one of the applied research methods. As a result of the calculations, 23 of the most important control measurements and 56 of the less important measurements were obtained. Figures 8-13 show the most important and less important control points for the parameters: measurement of tightening PDs, active gear KZP, passive gear KZPn, a set of bearings KL and KL<sub>a</sub>, corps KR, plate PLt, cover PKr.

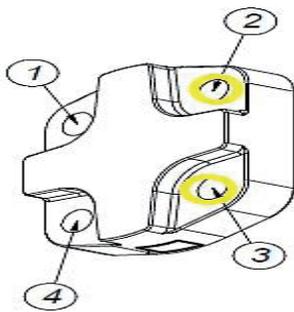


Fig. 8. The most important and less important control points for parameter – measurement of tightening PDs

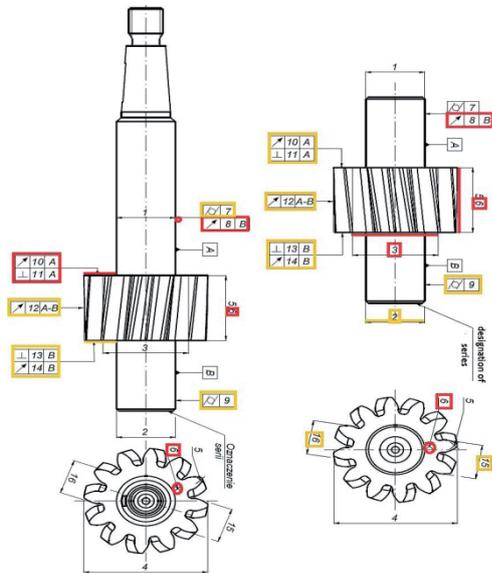


Fig. 9. The most important and less important control points for parameters: a) active gear KZP, b) passive gear KZPn

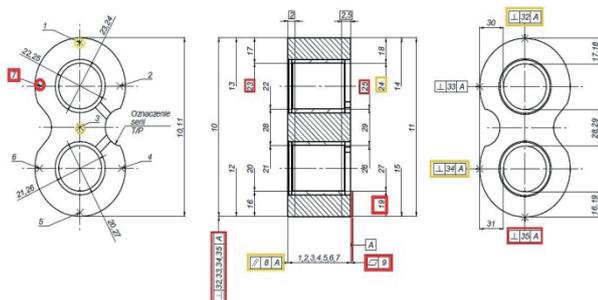


Fig. 10. The most important and less important control points for parameters: set of bearings KL

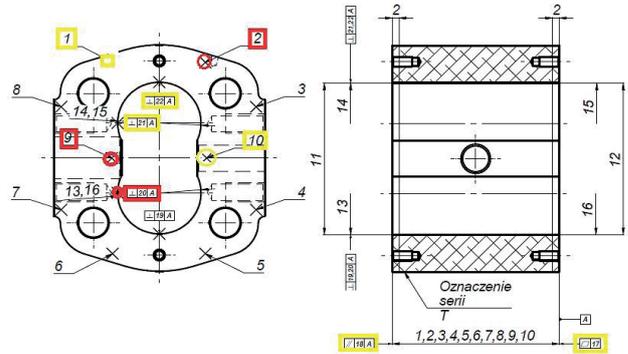


Fig. 11. The most important and less important control points for parameters: corps KR

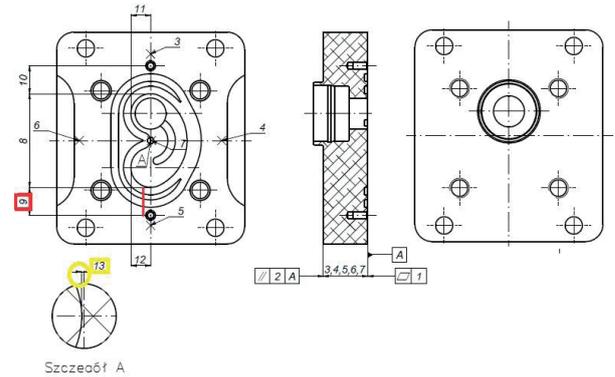


Fig. 12. The most important and less important control points for parameters: plate PLt

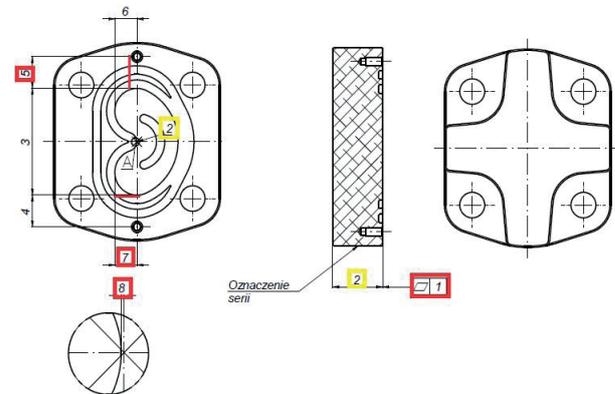


Fig. 13. The most important and less important control points for parameters: cover PKr

Using a genetic algorithm gave a greater effect, it was used to obtain about 200-300 results for pump designs. Then it would be the proper use of tools, which is reversed correlation matrix. Due to the complexity of computing the exact method, (formal) identification and classification can be used only for a small number of decision variables. Therefore, the sequential determination of the validity rank for the most important parameters can be performed by using multi-valued logic trees methods. It is applied to the algorithm of minimizing the individual logic functions. Additionally, for this purpose, the monotonicity of the value of the most important parameters according to the numbers of pumps is determined. In addition, Hedwig's algorithm is used for the optimal selection of variables. The Figures 14-18 show a comparison of the efficiency of the ten gear pumps 1PWR – SE for n = 500, 800, 1000, 1500, 2000 [rev / min].

## 6. Application of Multi-Valued Logic Trees

Using multivalent logic trees the designated rank of importance of structural and operational parameters, taking into account the effect of tolerances construction can be made. The study focused on two parameters selected: discharge pressure  $P_t$  and rotation speed  $n$ . Determination of the rank of the validity of parameters using multiple-valued trees logic requires the application of appropriate coding. Identification of influence of part tolerances of the 3PWR- SE pump on its total efficiency taking into consideration multi-valued logic trees was presented in [5].

### 6.1. The Quine–McCluskey Algorithm for the Minimization of Multiple-valued Logic Functions

The Quine-McCluskey algorithm makes it possible to find all prime implicants of a given logic function that is there is a shortened alternative normal form SAPN [7, 24, 25]. The terms of incomplete gluing and elementary absorption have the main role in the search for prime implicants and are used for the APN of a given logic function. The following transformation is called the consensus operation:

$$Aj_o(x_r) + \dots + Aj_{m_r-1}(x_r) = A \quad (1)$$

where:  $r = 1, \dots, n$  and  $A$  is a partial elementary product, the literals of which possess variables belonging to the set:  $\{x_1, \dots, x_{r-1}, \dots, x_{r+1}, \dots, x_n\}$ .

The following transformation is called the operation of reduction:

$$Aj_u(x_r) + A = A \quad (2)$$

where:  $0 \leq u \leq m_r - 1, 1 \leq r \leq n$ , and  $A$  is a partial elementary product, the literals of which possess variables belonging to the set  $\{x_1, \dots, x_{r-1}, x_{r+1}, \dots, x_n\}$ .

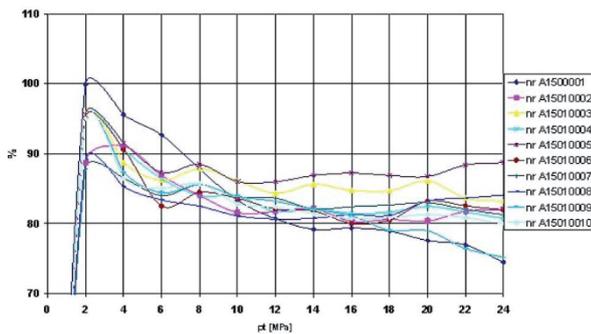


Fig. 14. Total efficiency  $\eta_c$  gear pumps 1PWR-SE for  $n=500$  [rev/min]

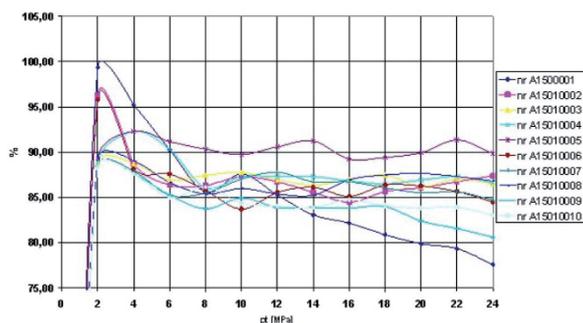


Fig. 15. Total efficiency  $\eta_c$  gear pumps 1PWR-SE for  $n=800$  [rev/min]

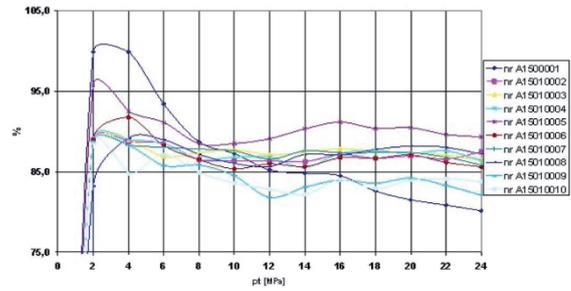


Fig. 16. Total efficiency  $\eta_c$  gear pumps 1PWR-SE for  $n=1000$  [rev/min]

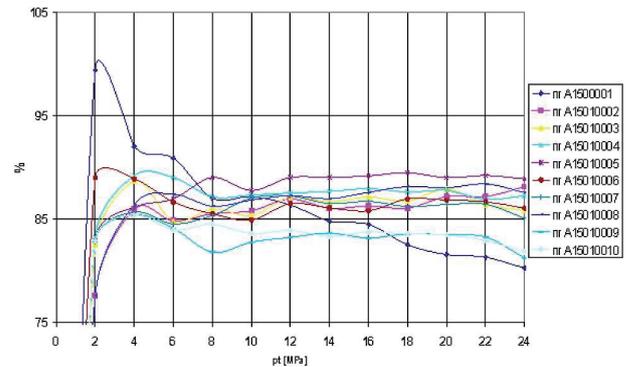


Fig. 17. Total efficiency  $\eta_c$  gear pumps 1PWR-SE for  $n=1500$  [rev/min]

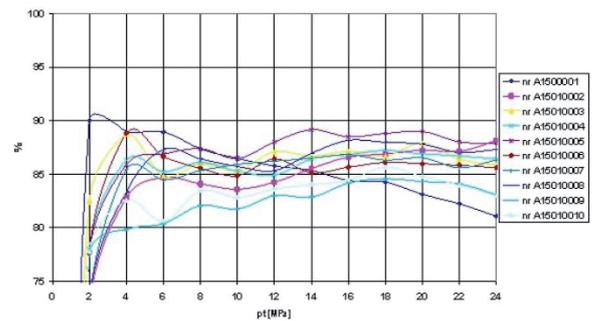


Fig. 18. Total efficiency  $\eta_c$  gear pumps 1PWR-SE for  $n=2000$  [rev/min]

In the case of multi-valued weighting factors, we obtain [1]:

$$w_o Aj_o(x_r) + \dots + w_{m_r-1} Aj_{m_r-1}(x_r) = \left( \min \{w_o, \dots, w_{m_r-1}\} \right) \cdot A + \sum_{S=I_o, \dots, I_{m_r-2}} w_s \cdot A \cdot j_s(x_r) \quad (3)$$

where:  $w_j$  is a polyvalent weighting factor.

For example using the formula:

$$Aj_o(x_r) + \dots + Aj_{m-1}(x_r) = A, \quad Aj_u(x_r) + A = A, \quad (4)$$

where:

$$A = A(x_1, \dots, x_{r-1}, x_{r+1}, \dots, x_n), \quad (5)$$

$$j_u(x_r) = \begin{cases} m-1 & , \quad u = x_r \\ 0 & , \quad u \neq x_r \end{cases} \quad (6)$$

successive stages of the multi-value logic function minimization: 020, 101, 200, 021, 111, 201, 210, 022, 121, 202, 211, 212, 221 can be presented in the following way (Table 2):

$$f(x_1, x_2, x_3) = j_0(x_1)(j_0(x_2)j_2(x_3) + j_1(x_2)j_0(x_3) + j_2(x_2)) + j_1(x_1) + j_2(x_1)(j_0(x_2)j_1(x_3) + j_1(x_2) + j_2(x_2)j_1(x_3))). \quad (7)$$

Table 2. NAPN and MAPN of a given logical function

	020	200	101	021	201	210	111	022	121	202	211	212	221
02-	*			*				*					
20-		*			*					*			
1-1			*				*		*				
21-						*			*		*	*	
-21				*					*		*	*	*
2-1					*						*	*	*

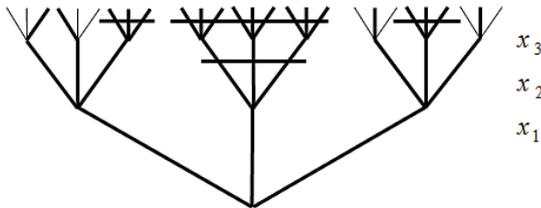


Fig. 19. A multi-valued decision tree for the parameters  $x_1, x_2, x_3$  with an appropriate layout of levels

Finally, we receive both NAPN and MAPN of a given logical function saved in the form of numbers of a m-position system [23]: {(02-), (20-), (1-1), (21-), (-21)} and {(02-), (20-), (1-1), (21-), (2-1)}.

For example, multiple-valued logical function  $f(x_1, x_2, x_3)$ , where  $x_1, x_2, x_3 = 0, 1, 2$ , written by means of numbers Canonical Alternative Normal Form): 100, 010, 002, 020, 101, 110, 021, 102, 210, 111, 201, 120, 022, 112, 211, 121, 212, 221, 122, there is one MZAPN (Minimal Complex Alternative Normal Form) after the application of the Quine–McCluskey algorithm based on the minimization of individual partial multi-valued logical functions having 13 literals (Fig. 19).

In the isomorphic interpretation of the Quine–McCluskey algorithm, three steps are taken for the graphic matrix formalization [23]:

- a) putting decision  $(m_1, \dots, m_n)$  – valued variables  $x_1, \dots, x_n$  in a certain order; creating the  $n!$  primary matrices, relative to all combinations of variables,

- b) prioritizing the numbers relative to  $(m_1, \dots, m_n)$  the valence in the increasing order from the left side of the matrix,
- c) combining numbers and removing them (minimization).

### 6.2. Determination of the Rank of the Validity of the Design Parameters and Operating Pumps Model (TYPE 1PWR-SE), taking into account the Effect of Tolerances Construction

Engineering practice requires correct evaluation of the mathematical model describing a given system with some variables. A proper mathematical model contains a group of functions joining different variables and describing connections between quantities in the system. Decision tables and logical functions [1, 5, 7] can be applied to the simulation of the machine or in data classification and scheduling of measurement points. Besides, there are dependence digraphs of the signal flow. The game structure describes a space of possible solutions in order to find the optimum of objective functions. Determination of the rank of the validity of parameters using multiple-valued trees logic requires the application of appropriate coding. The values of arithmetic discharge pressure  $Pt$  and the rotation speed  $n$ , taking into account the efficiency of the pumps model were coded logic of the respective periods of the tables 3-4. Tables 5-6 show the specific and general logical coding for ranges of changes  $Pt$  and  $n$ , in which at least 7 correspond to pump efficiency defined with a tolerance of 5%.

Tables 7-8 show the specific and general logical coding for ranges of changes  $Pt$  and  $n$ , in which at least 10 correspond to pump efficiency defined with a tolerance of 5%.

The identification of the impact of the manufacturing technology of the model units showed that the important dimensions affecting the efficiency of the pumps are generally repeated in all details, regardless of the analyzed group. The indicated geometrical errors depend mainly on the fastening devices used, deformation of the part in the mounting, the number of fasteners, machining parameters, forces occurring during machining of the element, repeatability of the machine tools. The following is a summary of the most frequently occurring critical dimensions for individual components of the pump.

Figure 24 presents the dimensions and critical deviations: 1) the beating of plugs and the lateral surface of the toothed rim 2) perpendicularity of the side surface of the tooth to the pivots (wheel rotation axis) for the detail of the gears speeding and running.

**Table 3. General and specific logical encoding for the full range of change of the pressure Pt and n= 500 and 800 [rev /min]**

Pt		n		A15010001	A15010002	A15010003	A15010004	A15010005	A15010006	A15010007	A15010008	A15010009	A15010010		
c*	MPa	c*	min <sup>-1</sup>	%	%	%	%	%	%	%	%	%	%		
0	0	0	500	2	99.9	88.7	95.5	95.5	95.5	95.1	87.8	89.2	95.1	95.1	
				4	95.5	91.1	88.7	90.6	91.5	90.6	86.4	85.2	87.4	86.4	
	1			6	92.7	86.9	86.2	86.4	87.4	82.5	84	83.4	84.4	85.1	
				8	87.8	84	87.8	84	88.5	84.5	85.5	82.5	85.5	85.7	
	1			2	10	83.3	81.6	86.1	83.9	86	83.6	83.9	81.1	83.9	82.8
					12	80.7	81.7	84.3	81.7	86	82.1	83.6	80.6	83.2	82.3
				3	14	79.2	82.1	85.6	82.1	86.9	81.8	82.1	80.8	82	81.4
					16	79.4	80.3	84.8	81.4	87.2	80	82.4	81.3	81.1	80.7
2	4	18	78.9	80.6	84.7	81.6	86.9	80.5	82.7	81.2	79	80.8			
		20	77.6	80.4	86.1	82.5	86.7	83.2	83	83.2	79	81.3			
	5	22	76.9	81.7	83.7	81.7	88.4	82.5	82	83.7	76.4	80.8			
		24	74.5	82	83.2	80.7	88.7	82	81.3	84.1	75.2	80			
0	0	1	800	2	99.4	96.2	88.7	88.7	89.3	95.8	88.4	89.3	88.4	88.4	
				4	95.2	88.7	88.7	92.3	92.3	88.1	87.9	89	87.6	87.6	
	1			6	90.3	86.4	86.9	90.3	91.2	87.6	85.3	86.7	85.3	85	
				8	85.6	86.3	87.4	86	90.3	85.8	85.5	85.4	83.7	86.2	
	1			2	10	87.6	87.3	87.7	87.3	89.7	83.7	87	86	84.9	84.4
					12	85.3	86.7	86.9	87.3	90.6	85.5	87.8	85.4	83.9	84
				3	14	83.1	85.6	86.4	87.3	91.2	86.1	86.7	85.2	83.9	83.9
					16	82.2	84.5	87	86.8	89.2	85.1	86.7	87	83.8	84.7
2	4	18	80.9	85.7	87.5	86.5	89.4	86.4	86	87.5	84	84.1			
		20	79.9	86.1	86.2	86.9	89.9	86.3	85.5	87.6	82.4	83.8			
	5	22	79.3	86.7	87	87.3	91.3	85.7	85.6	87.3	81.6	83.9			
		24	77.6	87.4	86.5	86.7	89.9	84.5	84.7	86.8	80.6	83			

C\*- multivalent logic coding

**Table 4. General and specific logical encoding for the full range of change of the pressure Pt and n= 1000, 1500 and 2000 [rev /min]**

Pt			n		A15010001	A15010002	A15010003	A15010004	A15010005	A15010006	A15010007	A15010008	A15010009	A150100010	
c*	MPa	c*	min <sup>-1</sup>	%	%	%	%	%	%	%	%	%	%	%	
0	0	2	1000	2	99.9	88.94	88.7	89.2	95.8	88.9	88.7	83.2	88.7	88.5	
				4	99.9	88.72	88.9	88.9	92.5	91.8	88.3	89.2	88.3	84.8	
	1			6	93.4	88.49	86.9	88.5	91.1	88.3	88	88.9	85.8	87.1	
				8	88.7	86.49	87.4	86.5	88.5	86.5	87.8	87.2	85.8	84.9	
	1			2	10	87.2	86.35	87.7	86.8	88.5	85.4	87.6	86	84.5	83.6
					12	85.2	86.32	87.1	86.8	89.1	86	86.5	85.7	81.8	82.8
				3	14	84.8	86.24	87.4	85.5	90.3	85.6	87.6	87.2	83.1	82.2
					16	84.5	87.12	87.8	87.3	91.2	86.7	87.4	87.1	84	84
2	4	18	82.6	86.66	87.5	87.6	90.4	86.7	87.4	87.8	83.6	82.9			
		20	81.5	86.96	87.4	87.2	90.5	87.1	87.4	88.2	84.2	84			
	5	22	80.8	86.58	87	87.6	89.6	86.1	86.8	88	83.3	84.1			
		24	80.1	87.45	86.5	86.4	89.3	85.6	85.8	87.2	82.1	83.8			
0	0	3	1500	2	99.4	77.6	82.5	83.2	82.9	89	82.8	77.6	82.8	82.9	
				4	92	86	88.7	89.2	86.1	88.9	85.7	86.4	85.4	85.4	
	1			6	90.9	84.8	84.8	89	87	86.7	84.5	87.4	84.1	84	
				8	87	85.5	86	87.2	89	85.5	85.4	86.2	81.8	84.5	
	1			2	10	87.2	85.8	85.4	87.3	87.8	84.9	87	86.8	82.8	83.6
					12	86.4	86.9	87.1	87.5	89	86.5	87.2	87.2	83.2	83.9
				3	14	84.8	86.1	86.7	87.7	89	86.1	86.5	87	83.6	83.3
					16	84.5	86.3	87.1	87.9	89.2	85.8	86.7	87.6	83.2	83.8
2	4	18	82.5	86.1	86.7	87.6	89.5	87	86.2	88.1	83.6	83.6			
		20	81.6	87.2	87.9	87.8	89	86.8	86.4	88	83.5	83.5			
	5	22	81.3	87.2	86.4	86.9	89.2	86.7	86.4	88.4	83.3	82.9			
		24	80.3	88.1	85.8	87.2	88.9	86	85.1	87.6	81.2	81.8			
0	0	4	2000	2	90	72.7	82.5	78	77.6	77.6	72.9	73.2	77.6	77.7	
				4	88.9	82.8	88.7	86.3	85.9	88.8	85.4	83.3	79.8	82.6	
	1			6	88.9	84.7	84.8	85.2	86.9	86.7	84.5	87.3	80.4	80.6	
				8	87.4	84.1	86	86.1	87.4	85.5	85.3	86.4	82.1	83.5	
	1			2	10	86.5	83.6	85.4	85.3	86.5	84.9	85.9	85.7	81.7	82.8
					12	85.8	84.3	87.1	84.9	88	86.4	86.2	85.3	83	83.6
				3	14	85.3	85.5	86.7	86.4	89.2	85.1	86.5	87	82.8	84
					16	84.4	86.6	87.1	86.8	88.5	85.7	86.8	88.2	84.2	84.4
2	4	18	84.3	86.9	86.7	87.2	88.8	86.1	86.3	88	84.6	85.7			
		20	83.1	87.2	87.9	86.9	89	86	86.5	87.8	84.3	84.8			
	5	22	82.2	87.1	86.4	86.7	88	85.8	85.6	87	84.1	84			
		24	81.1	88.1	85.8	86.4	87.9	85.6	86.3	87.3	83.1	83			

C\*- multivalent logic coding

**Table 5. Specific logical coding for ranges of changes  $P_t$  and  $n$ , in which at least 7 correspond to pump efficiency defined with a tolerance of 5%**

$P_t$	$n$	$P_t$	$n$
0	0	0	0
1	0	1	0
2	0	2	0
2	1	2	1
3	1	1	2
4	1	3	1
1	2	2	2
2	2	1	3
3	2	4	1
4	2	3	2
5	2	2	3
1	3	1	4
2	3	4	2
3	3	3	3
4	3	2	4
5	3	5	2
1	4	4	3
2	4	3	4
3	4	5	3
4	4	4	4
5	4	5	4

**Table 6. General logical coding for ranges of changes  $P_t$  and  $n$ , in which at least 7 pumps meet prescribed performance with a tolerance of 5%**

$P_t$	$n$	$P_t$	$n$
0	0	0	0
1	0	1	0
1	1	1	1
2	1	0	2
0	2	2	1
1	2	1	2
2	2	0	3
0	3	2	2
1	3	1	3
2	3	0	4
0	4	2	3
1	4	1	4
2	4	2	4

**Table 7. Specific logical coding for ranges of changes  $P_t$  and  $n$ , in which at least 10 pumps meet prescribed performance with a tolerance of 5%**

$P_t$	$n$	$P_t$	$n$
1	2	1	2
3	4	3	4

**Table 8. General logical coding for ranges of changes  $P_t$  and  $n$ , in which at least 10 pumps meet prescribed performance with a tolerance of 5%**

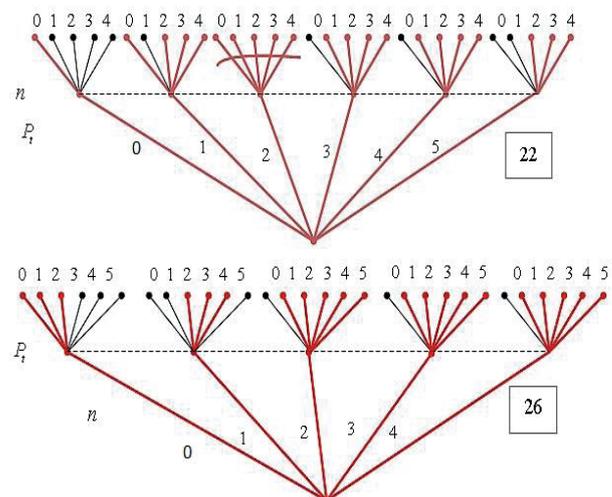
$P_t$	$n$	$P_t$	$n$
0	2	0	2
1	4	1	4

### 7. Conclusions

The complexity of logic tables or truth tables grows exponentially in relation to the number of variables. In case of a larger number of decision variables, there are practical geometric problems which need to be solved in order to extract the least and the most important data. What is more, the graphic matrix formalization can be a computer record of parametric game trees as an adjacency matrix.

In the case of 3-value coding {0, 1, 2} decision discharge pressure  $P_t$  {2, 4, 6, 8}, {10, 12, 14, 16}, {18, 20, 22, 24}, for 5- value coding for valuable rotational speed  $n$  {500, 800, 1000, 1500, 2000} obtained a higher rank of importance for  $n$  compared to  $P_t$ . The difference is approximately 20-25%, according to the contractual scale accuracy calculated for the number of branches in the relevant decision trees. Then a minimum of 7 pumps meet certain performance criteria with a tolerance of 5%. If the increase decisiveness discharge pressure  $P_t$  to 6-value for encoding {0, 1, 2, 3, 4, 5} respectively, for the values {2, 4}, {6, 8}, {10, 12}, {14, 16}, {18, 20}, {22,24}, at the same speed ranges  $n$ , it receives a higher rank of importance for  $P_t$  compared to  $n$ .

Figures 20-23 show the corresponding multi-valued logic trees for the tables 5, 6, 7 and 8.



**Fig. 20. Multi-valued logic trees for the Table 5**

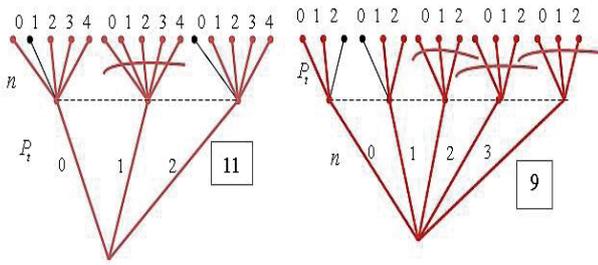


Fig. 21. Multi-valued logic trees for the Table 6

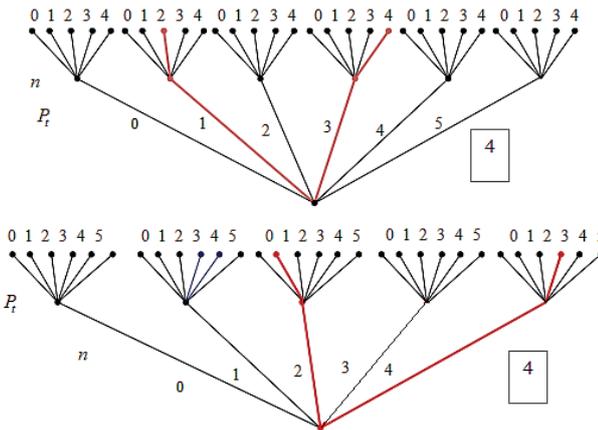


Fig. 22. Multi-valued logic trees for the table 7

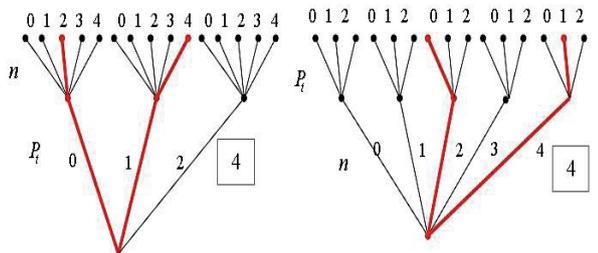


Fig. 23. Multi-valued logic trees for the table 8



Fig. 24. The dimensions and critical deviations: 1) the beating of plugs and the lateral surface of the toothed rim 2) perpendicularity of the side surface of the tooth to the pivots (wheel rotation axis) for the detail of the gears speeding and running

Figure 25 shows the perpendicularity of the sump to the pump head for detail: pump corpus.

The difference is approximately 25%, according to the contractual scale accuracy calculated for the number of branches in the relevant decision trees. Then a minimum of 7 pumps meet certain performance criteria with a tolerance of 5%. It is also possible for

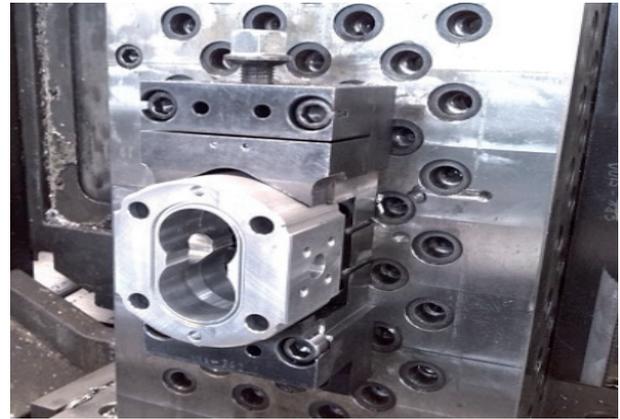


Fig. 25. The perpendicularity of the sump to the pump head for detail: pump corpus

the above various decision-making valence trading discharge pressure  $P_t$  and identical decision valence rotational speed  $n$ , assuming that the 10 pumps meet certain performance criteria with a tolerance of 5%. Then we receive always the same range of validity for  $n$  and  $P_t$ .

The minimization of geometrical and kinematic errors takes place through the appropriate selection of the geometrical-motor structure of the machine tool. Geometric inaccuracies include surface misalignment errors and shape errors. However, kinematic errors are related to the relative movements of individual machine tool components. In particular, they become important in the case of complex traffic. Properly designed and operated machine tools are characterized by a significant repeatability of kinematic errors, so it is relatively simple to compensate for such errors.

The temperature instability of the machine components is related to their nature of work and the impact of the environment. Often this type of error can be critical and decisive for the accuracy of the workpiece. It is generally assumed that the thermal error is directly proportional to the change in temperature and the coefficient of thermal expansion of the material being heated.

**AUTHORS**

**Adam Deptuła\*** – Opole University of Technology, Faculty of Production Engineering and Logistic, 75 Ozimska Street, 45-370 Opole, Poland, e-mail: a.deptula@po.opole.pl, www.a.deptula.po.opole.pl

**Marian A. Partyka** – Opole University of Technology, Faculty of Production Engineering and Logistic, 75 Ozimska Street, 45-370 Opole, Poland, e-mail: m.partyka@po.opole.pl

**Piotr Osiński** – Wrocław University of Technology, Faculty of Mechanical Engineering, 5 Łukasiewicza Street, 50-370 Wrocław, Poland, e-mail: piotr.osinski@pwr.edu.pl

\*Corresponding author

## REFERENCES

- [1] A. Deptuła, "Application of game graphs to describe the inverse problem in the designing of mechatronic vibrating systems". In: Zawiaślak S., Rysiński J. (eds.) *Graph-Based Modelling in Engineering. Mechanisms and Machine Science*, vol. 42, Springer, Cham, 2017, 189–199. DOI: 10.1007/978-3-319-39020-8\_14.
- [2] A. Deptuła, "Application of multi-valued weighting logical functions in the analysis of a degree of importance of construction parameters on the example of hydraulic valves", *Int. Journal of Applied Mechanics and Engineering*, vol. 19, no. 3, 2014, 539–548, DOI: 10.2478/ijame-2014-0036.
- [3] P. Osiński, A. Deptuła, M.A. Partyka, "Discrete optimization of a gear pump after tooth root undercutting by means of multi-valued logic trees", *Archives of Civil and Mechanical Engineering*, vol. 13, no. 4, 2013, 422–431, DOI: 10.1016/j.acme.2013.05.001.
- [4] A. Deptuła, P. Osiński, M.A. Partyka, "Identification of influence of part tolerances of 2PWR-SE pump on its total efficiency taking into consideration multi-valued logic trees", In: Rusiński E., Pietrusiak D. (eds.) *Proceedings of the 14th International Scientific Conference: Computer Aided Engineering. CAE 2018*, Lecture Notes in Mechanical Engineering, Springer, Cham, 2019, 128–135, DOI: 10.1007/978-3-030-04975-1\_16.
- [5] A. Deptuła, P. Osiński, M. A. Partyka, "Identification of Influence of Part Tolerances of 3PWR-SE Pump On Its Total Efficiency Taking Into Consideration Multi-Valued Logic Trees", *Polish Maritime Research*, vol. 24, no. 1, 2017, 47–59, DOI: 10.1515/pomr-2017-0006.
- [6] A. Deptuła, P. Osiński, "The Optimization of Three-Involute Tooth Outline with Taking into Consideration Multi-valued Logic Trees". In: Rusiński E., Pietrusiak D. (eds.) *Proceedings of the 13th International Scientific Conference. RE-SRB 2016*, Lecture Notes in Mechanical Engineering, Springer, Cham, 2017, 99–107, DOI: 10.1007/978-3-319-50938-9\_11.
- [7] A. Deptuła, M.A. Partyka, "Separate logical analysis of design guidelines in the machine systems modelling", *International Journal of Applied Mechanics and Engineering*, vol. 17, no. 3, 2012, 743–751.
- [8] B. Filipowicz, "Modele stochastyczne w badaniach operacyjnych: analiza i synteza systemów obsługi i sieci kolejkowych", *WNT*, Warszawa, 1996.
- [9] D.H. Greene, D.E. Knuth, "Mathematics for the Analysis of Algorithms", *Birkhäuser Boston*, 1982.
- [10] J.A. Hertz, A.S. Krogh, R.G. Palmer, S. Jankowski, "Wstęp do teorii obliczeń neuronowych", *WNT*, Warszawa, 1993.
- [11] W. Kollek, Z. Kudźma, M. Stosiak, J. Mackiewicz, "Possibilities of diagnosing cavitation in hydraulic systems", *Archives of Civil and Mechanical Engineering*, vol. 7, no. 1, 2007, 61–73, DOI: 10.1016/S1644-9665(12)60005-3.
- [12] W. Kollek, P. Osiński, M. Stosiak, A. Wilczyński, P. Cichoń, "Problems relating to high-pressure gear micropumps", *Archives of Civil and Mechanical Engineering*, vol. 14, no. 1, 2014, DOI: 10.1016/j.acme.2013.03.005.
- [13] W. Kollek, P. Osiński, "Modelling and design of gear pumps", Wrocław University of Technology Publishing House, Wrocław, 2009.
- [14] S. Kudźma, Z. Kudźma, "Refined model of passive branch damper of pressure fluctuations", *Journal of Theoretical and Applied Mechanics*, vol. 53, no. 3, 2015, 557–567, DOI: 10.15632/jtam-pl.53.3.557.
- [15] A. M. Law, W. D Kelton, "Simulation Modeling and Analysis", *McGraw-Hill*, Boston 2000.
- [16] M. Mahajan, P.R. Subramanya, V. Vinay, "A Combinatorial Algorithm for Pfaffians". In: Asano T., Imai H., Lee D.T., Nakano S., Tokuyama T. (eds.) *Computing and Combinatorics. COCOON 1999*. Lecture Notes in Computer Science, vol. 1627, Springer, Berlin, Heidelberg, 1999, 134–143, DOI: 10.1007/3-540-48686-0\_13.
- [17] P. Osiński, W. Kollek, M.A. Partyka, A. Deptuła, "Identyfikacja wpływu technologii wykonania konstrukcji pomp modelowych o nowym zarysie (typ 2PW-SEW) na sprawność całkowitą z uwzględnieniem logicznych struktur decyzyjnych", *Raporty Wydziału Mechanicznego Politechniki Wrocławskiej*, Ser. SPR no. 114, 2015.
- [18] P. Osiński, W. Kollek, M.A. Partyka, A. Deptuła, "Identyfikacja wpływu technologii wykonania konstrukcji pomp modelowych o nowym zarysie (typ 1PWR-SE) na sprawność całkowitą z uwzględnieniem logicznych struktur decyzyjnych", *Raporty Wydziału Mechanicznego Politechniki Wrocławskiej*, 2015, Ser. SPR no. 44, 2015.
- [19] P. Osiński, W. Kollek, M.A. Partyka, A. Deptuła, "Identyfikacja wpływu technologii wykonania konstrukcji pomp modelowych o nowym zarysie (typ 2PWR-SE) na sprawność całkowitą z uwzględnieniem logicznych struktur decyzyjnych", *Raporty Wydziału Mechanicznego Politechniki Wrocławskiej*, Ser. SPR nr 45, 2015.
- [20] P. Osiński, W. Kollek, M.A. Partyka, A. Deptuła, "Identyfikacja wpływu technologii wykonania konstrukcji pomp modelowych o nowym zarysie (typ 3PWR-SE) na sprawność całkowitą z uwzględnieniem logicznych struktur decyzyjnych", *Raporty Wydziału Mechanicznego Politechniki Wrocławskiej*, Ser. SPR nr 46, 2015.
- [21] P. Osiński, „Pompy zębate o obniżonym poziomie emisji hałasu”, *Oficyna Wydawnicza Politechniki Wrocławskiej*, Wrocław, 2017.
- [22] P. Osiński, "Wysokociśnieniowe i niskopulsacyjne pompy zębate o zazębieniu zewnętrznym", *Oficyna Wydawnicza Politechniki Wrocławskiej*, Wrocław, 2013.
- [23] M.A. Partyka, "Algorytm Quine'a-Mc Cluskeya minimalizacji indywidualnych cząstkowych wielowartościowych funkcji logicznych", *Studia i Monografie nr 109*, Politechnika Opolska. Oficyna Wydawnicza, Opole, 1999.
- [24] M.A. Partyka, "Some remarks on the Quine – Mc Cluskey minimization algorithm of multiple-valued partial functions for design structures", *7th Inter. Cong. Log. Method. Phil. Sc.*, Salzburg, Austria, 1983.
- [25] M.A. Partyka, "The Quine- Mc Cluskey minimization algorithm of individual multiple-valued partial functions for digital control systems", *3rd Inter. Confer. Syst. Engin.*, Wright State University, Dayton, USA, 1984.

- [26] Y.R. Shiue, R.S. Guh, "The optimization of attribute selection in decision tree-based production control systems", *The International Journal of Advanced Manufacturing Technology*, vol. 28, no. 7–8, 2006, 737–746, DOI: 10.1007/s00170-004-2430-y.
- [27] P.P. Shenoy, "Game Trees For Decision Analysis", *Theory and Decision*, vol. 44, no. 2, 1998, 149–171, DOI: 10.1023/A:1004982328196.
- [28] M. Stosiak, "Ways of reducing the impact of mechanical vibrations on hydraulic valves", *Archives of Civil and Mechanical Engineering*, vol. 15, no. 2, 2015, 392–400, DOI: 10.1016/j.acme.2014.06.003.
- [29] S. Stryczek, "Napęd hydrostatyczny", vol. I and II, *WNT*, Warsaw, 1984.
- [30] P. Śliwiński, "Flow of liquid in flat gaps of the satellite motor working mechanism", *Polish Maritime Research*, vol. 21, no. 2, 2014, 50–57, DOI: 10.2478/pomr-2014-0019.
- [31] J. Wojnowski (ed.), "Wielka Encyklopedia PWN", *Wydawnictwo Naukowe PWN*, Warszawa 2005.
- [32] W.V. Quine, "The Problem of Simplifying Truth-Functions", *American Mathematical Monthly*, vol. 59, no. 8, 1952, 521–531, DOI: 10.2307/2308219.
- [33] J.R. Quinlan, R.L. Rivest, "Inferring decision trees using the minimum description length principle", *Information and Computation*, vol. 80, no. 3, 1989, 227–248, DOI: 10.1016/0890-5401(89)90010-2.
- [34] J.R. Quinlan, "Induction of Decision Trees", *Machine learning*, vol. 1, no. 1, 1986, 81–106, DOI: 10.1007/BF00116251.
- [35] Z. Zarzycki, S. Kudźma, Z. Kudźma, M. Stosiak, "Simulation of transient flows in a hydraulic system with a long liquid line", *Journal of Theoretical and Applied Mechanics*, vol. 45, no. 4, 2007, 853–871.