Control Methods Design for a Model of Asymmetrical Quadrocopter

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Abstract:
The paper describes the results of quadrocopters motion properties for the control based on the inverse dynamics method and optimal control method with synthesis linear-quadratic regulator (LQR). Motion of quadrocopters is tested for composite trajectories. The new model of asymmetrical quadrocopters, taking into account the rotation and shift of one arm relative to the other, was developed. A few criteria for evaluation of the effectiveness of control methods of quadrocopters are presented in this paper. An analysis of the results allows selecting a method for solving the problem of quadrocopters control and making recommendations for the formation of trajectories.

Keywords: *linear-quadratic regulator, inverse dynamics, quadrocopter, dynamic mode*

1. Introduction

Recently the development of unmanned aerial vehicles (UAV) has been started. Quadrocopter is an example of such vehicle. Quadrocopter is a vehicle with four rotors, which are rigidly fixed to the body [1]. These features include the fact that they are maneuverable, can still be over a given point in space and carry additional equipment. However, there are several problems associated with using this type of construction. The main problem is calculation of the effective control of quadrocopters.

The first prototype of the aircraft with this configuration appeared in 1907 [2]. Vehicle was operated with a complex transmission, which make it difficult to control. The first full quadrocopter was developed in the 50s [2]. For a number of characteristics of these models gave way to aircraft and helicopters, so widespread use they have not received. The most popular quadrocopters obtained with using UAVs. Today quadrocopters are used in various fields of human activity.

Modern quadrocopters and most of the research on them based on simple construction, models, and, therefore, use simple control algorithm. In most cases, it reduces the effectiveness of control and is not always reasonable.

An analysis of the literature proved that mathematical models can be classified as follows:

1. Linear model. Used for simple maneuvers [3] or for the calculation of the control algorithm by complex methods of high computational cost (LQR, model predictive control) [4], [5], [6].

2. Non-linear symmetrical model. By symmetric model we mean a model, whose center of gravity coincides with the geometric center. By the geometric center of the construction we understand the point of intersection of center lines of the arm. These models allow implementing the regulator by on-line methods [1], [6–9].

3. Asymmetric model. Consider a model with such precision is necessary for the implementation of complex maneuvers that require high control precision. HoverBike is an example of this kind asymmetric construction [10].

The new asymmetric model of quadrocopters is presented in this paper, as having the biggest number of perspectives. However, the efficiency of this model will not be improved if it uses the control algorithm for a symmetric or linear model. Therefore, it is necessary to analyze the control methods for this model.

Quadrocopters control most commonly uses the following: PD/PID – regulators [3], [6], [7], [11], [12], LQR [4], [5], model predictive control [6], [9], backstepping control [7], [8], sliding mode control [7] and inverse control [5], [7], [13]. In this paper, the methods are chosen to control the synthesis of linear-quadratic regulator (LQR method), and the method of inverse dynamics.

The purpose of this paper is to develop a new mathematical model of quadrocopters and analyze the algorithms and principles of control for various kinds of trajectories, manoeuvers, and conditions. The mathematical model has to take into account the asymmetry of the design and the effects of external influences. The problem was solved by the example of motion along a predetermined path.

The paper consists of three main sections and conclusions. The first section describes the design of quadrocopters and obtained dynamic equations of motion of asymmetrical quadrocopters. The second section describes a synthesis of control algorithm for quadrocopters using LQR method and the method of inverse dynamics. The third section presents the results of motion simulation of asymmetric quadrocopter within two trajectories: a circle and an eight-shaped figure. These trajectories are described in the third section in details. To be concise, with respect to the trajectories, we will use the terms „circle" and „eight-shaped".

2. Development of a Mathematical Model

Most manufacturers simplify their tasks by developing symmetry with respect to frame design. This greatly simplifies the mathematical description of the motion of quadrocopters, but on the other hand, it is necessary to use additional equipment to comply with such symmetry. Manufactured devices differ significantly since the center of gravity with geometric center and the arm with the motors may be positioned at any angle relative to each other.

This paper presents a model of quadrocopters which has the center of gravity structure shifted, one of the arms is also shifted relative to the geometric center of quadrocopters and rotated at an angle α, generally not a right angle, relative to the other arm (Fig. 1). l_1 is the distance from the edge of the second platform to the intersection with the center of the first platform, $l_1 + l_2 = 2l$. l_s is the distance from the edge of the platform to the center of the motor. In Fig. 2, a dotted line shows a quadrocopter symmetric model.

The main elements of quadrocopters are (Fig. 2): the basic platform, two arms, four motors, unit with electrical system and accessories. The geometrical dimensions, weight and the center of gravity coordinates in the coordinate system associated with the quadrocopters geometric center are shown in Table 1.

Quadrocopter moves relative to the fixed inertial coordinate system (ICS) ($oXYZ$). Axis *0x, 0y* and *0z* form an orthogonal right-handed coordinate system. Axis *0z* is in the opposite direction to the vector of gravity (Fig. 3). Introduce two auxiliary coordinate systems (CS). The coordinate system $o_c X_c Y_c Z_c$ is related to the center of mass of quadrocopters (CSM), and the coordinate system $o_g X_g Y_g Z_g$ associated with the quadrocopters geometric center (CSG). The axis of the coordinate system are parallel to the axes of the inertial coordinate system. The quadrocopter related with the right movable orthogonal coordinate system $o_c X_p Y_p Z_p$ (MCS). MCS starts at the center of mass of quadrocopters. The axis $O_c x_p$ is connected with one of the arms of a quadrocopter, axis $O_c y_p$ lies in the plane of a quadrocopter, axis $O_c z_p$ is upwardly directed relative to a quadrocopter. The angular position of a quadrocopter is defined in MCS by Euler angles $\eta = (\phi, \theta, \psi)^T$: roll ϕ, pitch θ and yaw ψ.

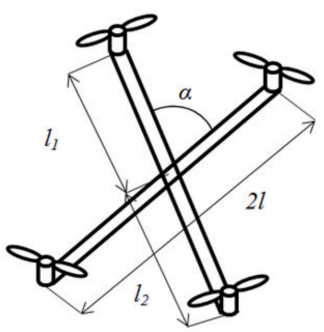

Fig. 1. Geometric model of quadrocopter

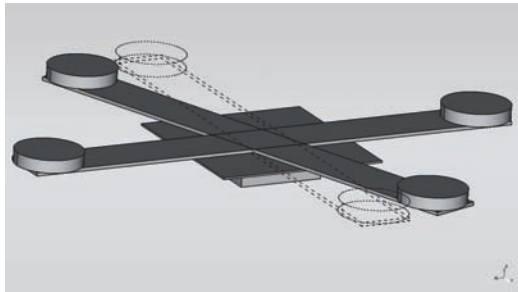

Fig. 2. Design quadrocopters

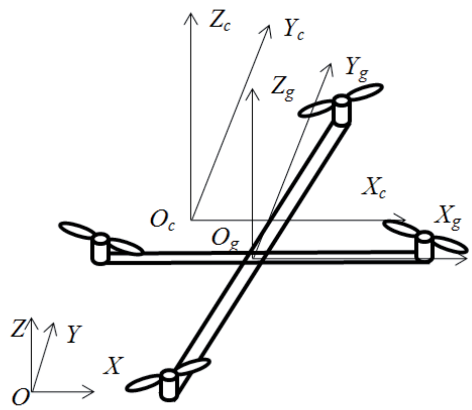

Fig. 3. Coordinate systems

The center of mass of a quadrocopter is defined by vector $X = (x, y, z)^T$ in ICS. The linear velocity vector of a quadrocopter is defined as $V_c = (v_{xc}, v_{yc}, v_{zc})^T$ and the angular velocity vector as $\Omega = (p, q, r)^T$ in CSM. Rotation matrix from CSM to the ICS has the form [14]:

$$Rot(\eta) = \begin{pmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\phi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{pmatrix},$$

where $c_\bullet = cos(\cdot), s_\bullet = sin(\cdot)$.

The connection between the linear speed in the ICS and the CSM has the form (1).

$$\dot{X} = Rot(\eta) \cdot V_c \qquad (1)$$

The transition matrix Λ for the angular velocity of the CSM to the MCS is described in [14]. The angular velocities connection in the form (2).

$$\Omega = \Lambda \cdot \eta = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & s_\phi c_\theta \\ 0 & -s_\phi & c_\phi c_\theta \end{bmatrix} \cdot \eta \qquad (2)$$

We use Köenig's theorem and Lagrange equation (3) for obtaining the dynamic equations of quadrocopters motion [15]. We form the kinetic energy of the system T. Vector coordinates of the center of mass X and the angular orientation of quadrocop-

Table 1. Description of the geometric dimensions, weight and center of gravity coordinates of structural elements

Structural component	Weight of the structural element [kg]	Length, width, thickness [m]	The coordinates of the gravity center [m]
The platform	M_1	a,b,h_1	$X_{c1}=(0 \quad 0 \quad 0);$
The unit with equipment	M_2	c,d,h_2	$X_{c2}=\left(0 \quad 0 \quad -\frac{h_1+h_2}{2}\right);$
The first arm	M_3	$2l,\Delta l,h_3$	$X_{c3}=\left(0 \quad 0 \quad \frac{h_1+h_3}{2}\right);$
The second arm	$M_4 \ (M_4=M_3)$	$2l,\Delta l,h_3$	$X_{c4}=\left(\frac{l_1-l_2}{2}cos(\alpha) \quad \frac{l_1-l_2}{2}sin(\alpha) \quad \frac{h_1+h_3}{2}\right);$
The motor 1	M_5	$2r,2r,h_5$	$X_{c5}=(l-l_s \quad 0 \quad h^*);$ $h^*=0.5(h_1+2h_3+h_5)$
The motor 2	$M_6 \ (M_6=M_5)$	$2r,2r,h_5$	$X_{c6}=(-(l-l_s) \quad 0 \quad h^*);$
The motor 3	$M_7 \ (M_7=M_5)$	$2r,2r,h_5$	$X_{c7}=(-(l_2-l_s)cos(\alpha) \quad -(l_2-l_s)sin(\alpha) \quad h^*);$
The motor 4	$M_8 \ (M_8=M_5)$	$2r,2r,h_5$	$X_{c8}=((l_1-l_s)cos(\alpha) \quad (l_1-l_s)sin(\alpha) \quad h^*);$

ters in CSM $\Theta=(\theta_1,\theta_2,\theta_3)^T$ were selected as generalized coordinates.

$$q=\begin{bmatrix}\Theta\\X\end{bmatrix};\ \frac{d}{dt}(\frac{\partial T}{\partial \dot{q}})-\frac{\partial T}{\partial q}=Q; \qquad (3)$$

$$T=\frac{1}{2}\dot{X}^T M\dot{X}+\frac{1}{2}\Omega^T I\Omega, \qquad (4)$$

where Q is generalized force, M is the quadrocopter mass, $M=\sum_{j=1}^{8}M_j$, I is the inertia tensor of a quadrocopter, Ω is the angular velocity vector in the CSM, $\Omega=\dot{\Theta}$.

Quadrocopters inertia tensor I can be written as:

$$I=I_g+M\begin{pmatrix} y_m^2+z_m^2 & -x_m y_m & -x_m z_m \\ -x_m y_m & x_m^2+z_m^2 & -y_m z_m \\ -x_m z_m & -y_m z_m & x_m^2+y_m^2 \end{pmatrix},$$

where I_g is the inertia tensor in CSG, X_m is the vector coordinates of the mass center,

$$X_m=(x_m,y_m,z_m)^T,\ X_m=\frac{1}{M}\sum_{k=1}^{8}M_k X_{c,k}$$

The inertia tensor I_g is described by the relation $I_g=\sum_{k=1}^{8}I_{g,k}$, where $I_{g,k}$ is the inertia tensor of the structure element k in the CSG. Table 2 shows the formulas for calculating the inertia tensor of quadrocopter's elements. For simplicity, arms, the platform and the equipment unit are treated as rectangular parallelepiped elements. The motors are treated in the calculation inertial tensor as cylinders.

In Table 2 x_i, y_i, z_i are the components X_{ci} of the center of gravity of the structural element $i=1,...,8$ (Table 1).

In Table 2 $\begin{pmatrix} A_m & 0 & 0 \\ 0 & A_m & 0 \\ 0 & 0 & C_m \end{pmatrix}$ is the inertia tensor of motors relative to the principal axes of inertia.

The generalized force Q can be represented in the form $Q=(Q_M,Q_F)^T$, where Q_M is a generalized torque in the rotational motion, Q_F is a component of generalized force in translational motion. The main components of the generalized force can be written as (5) and (6).

$$Q_M=U+M_{gir}; \qquad (5)$$

Table 2. The inertia tensor of the structural elements

The structural elements	The inertia tensor
The platform	$I_1 = \begin{pmatrix} \frac{M_1}{12}(b^2+h_1^2) & 0 & 0 \\ 0 & \frac{M_1}{12}(a^2+h_1^2) & 0 \\ 0 & 0 & \frac{M_1}{12}(a^2+b^2) \end{pmatrix}$
The unit with equipment	$I_2 = \begin{pmatrix} \frac{M_2}{12}(d^2+h_2^2)+M_2 z_2^2 & 0 & 0 \\ 0 & \frac{M_2}{12}(c^2+h_2^2)+M_2 z_2^2 & 0 \\ 0 & 0 & \frac{M_2}{12}(c^2+d^2) \end{pmatrix}$
The first arm	$I_3 = \begin{pmatrix} \frac{M_3}{12}(\Delta l^2+h_3^2) & 0 & 0 \\ 0 & \frac{M_3}{12}(4l^2+h_3^2) & 0 \\ 0 & 0 & \frac{M_3}{12}(4l^2+\Delta l^2) \end{pmatrix}$
The second arm	$I_4 = \begin{pmatrix} I_{11}^{*} & \frac{M_3}{24}(4l^2-\Delta l^2)\sin 2\alpha - M_3 x_4 y_4 & -M_3 x_4 z_4 \\ \frac{M_3}{24}(4l^2-\Delta l^2)\sin 2\alpha - M_3 x_4 y_4 & I_{22}^{*} & -M_3 y_4 z_4 \\ -M_3 x_4 z_4 & -M_3 y_4 z_4 & I_{33}^{*} \end{pmatrix}$ $I_{11}^{*} = \frac{M_3}{12}(4l^2 \sin^2\alpha + \Delta l^2 \cos^2\alpha + h_3^2) + M_3(y_4^2+z_4^2);$ $I_{22}^{*} = \frac{M_3}{12}(4l^2 \cos^2\alpha + \Delta l^2 \sin^2\alpha + h_3^2) + M_3(x_4^2+z_4^2);$ $I_{33}^{*} = \frac{M_3}{12}(4l^2 + \Delta l^2) + M_3(x_4^2+y_4^2).$
The motor j	$I_j = \begin{pmatrix} A_m + M_5(y_j^2+z_j^2) & -M_5 x_j y_j & -M_5 x_j z_j \\ -M_5 x_j y_j & A_m + M_5(x_j^2+z_j^2) & -M_5 y_j z_j \\ -M_5 x_j z_j & -M_5 y_j z_j & C_m + M_5(x_j^2+y_j^2) \end{pmatrix}, \; j=5,..,8.$

$$Q_F = F_u + F_{mg} + F_{res}, \qquad (6)$$

where U is the vector of the rotational force caused by the operation of motors, $U=(U_1,U_2,U_3)^T$, M_{gir} is the gyroscopic moment, F_u is the traction of motors, F_{mg} is the force of gravity acting on the quadrocopter, F_{res} is resistance force, $F_{res} = -S \cdot sign(\dot{X})\dot{X}^2$, S is the aerodynamic force coefficient vector [2].

Project the generalized forces (6) on the base q and get the form (7).

$$Q_F = Rot(\eta)\begin{bmatrix} 0 \\ 0 \\ U_0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix} - \begin{bmatrix} S_x sign(\dot{x})\dot{x}^2 \\ S_y sign(\dot{y})\dot{y}^2 \\ S_z sign(\dot{z})\dot{z}^2 \end{bmatrix} \qquad (7)$$

where U_0 is lift force in the MCS, g is the acceleration of gravity.

After inserting (4), (5), (7) in (3) and adding supplement system kinematic relations (2) we finally obtain (8):

$$\begin{cases} I\dot{\Omega} + (\Omega \times I\Omega) + M_{gir} = U; \\ M\ddot{X} = Rot(\eta)\begin{bmatrix} 0 \\ 0 \\ U_0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix} - \begin{bmatrix} S_x sign(\dot{x})\dot{x}^2 \\ S_y sign(\dot{y})\dot{y}^2 \\ S_z sign(\dot{z})\dot{z}^2 \end{bmatrix}; \\ \Omega = \Lambda \cdot \dot{\eta} \end{cases} \qquad (8)$$

The system of equations (8) should be supplemented with the equations describing the forces and torques in quadrocopter motors. Areas of vectors of forces and moments in the CSG are shown in Fig. 4.

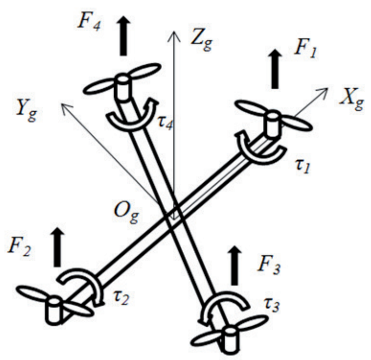

Fig. 4. The lifting force and torque motors

The lifting force and torque are directly proportional to the square of the rotation speed [4]. Formulas for traction and torque are of the form (9).

$$\begin{aligned}
U_0 &= \sum_{i=1}^{4} F_i = k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2;\\
U_1 &= \sum_{i=1}^{4} F_i \cdot (y_i - y_c) =\\
&= k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2 \cdot (y_i - y_c);\\
U_2 &= -\sum_{i=1}^{4} F_i \cdot (x_i - x_c) =\\
&= -k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2 \cdot (x_i - x_c);\\
U_3 &= -\tau_1 - \tau_2 + \tau_3 + \tau_4 = k_1(-sign(\omega_1)\omega_1^2 -\\
&- sign(\omega_2)\omega_2^2 + sign(\omega_3)\omega_3^2 + sign(\omega_4)\omega_4^2),
\end{aligned} \tag{9}$$

where k_1 and k_2 are constant coefficients, ω_i is the angular velocity of rotation of the motor i, $(x_i - x_c)$ and $(y_i - y_c)$ are distances from center of the motor i to the quadrocopter gravity center for axis Ox and Oy respectively.

The gyroscopic torque depends on the quadrocopters rotational speed and motors kinetic torque:

$$M_{gir} = \Omega \times K_m = \Omega \times \begin{bmatrix} 0 & 0 & C_m \cdot \sum_{i=1}^{4} \omega_i \end{bmatrix}^T \tag{10}$$

The dynamic equations of motion of quadrocopters (8) equations of traction, torque (9) and gyroscopic torque (10) create the system of quadrocopter equation. For further convenience, the equations of motion around the center of gravity of quadrocopters shift to the base η. The inertia tensor has the following form $J = \Lambda^T I \Lambda$. After simplification we obtain dynamic equations of quadrocopters motion in the final form (11).

$$\begin{cases}
J\Lambda\ddot{\eta} + J\dot{\Lambda}\dot{\eta} + \Lambda\dot{\eta} \times J\Lambda\dot{\eta} + \Lambda\dot{\eta} \times \begin{bmatrix} 0 \\ 0 \\ C_m \cdot \sum_{i=1}^{4} \omega_i \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix};\\
M\ddot{X} = Rot(\eta)\begin{bmatrix} 0 \\ 0 \\ U_0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ Mg \end{bmatrix} - \begin{bmatrix} S_x sign(\dot{x})\dot{x}^2 \\ S_y sign(\dot{y})\dot{y}^2 \\ S_z sign(\dot{z})\dot{z}^2 \end{bmatrix};\\
U_0 = k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2;\\
U_1 = k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2 \cdot (y_i - y_c);\\
U_2 = -k_2 \sum_{i=1}^{4} sign(\omega_i)\omega_i^2 \cdot (x_i - x_c);\\
U_3 = k_1(-sign(\omega_1)\omega_1^2 - sign(\omega_2)\omega_2^2 +\\
+ sign(\omega_3)\omega_3^2 + sign(\omega_4)\omega_4^2);
\end{cases} \tag{11}$$

In (11) the second equation describes the motion of the center of gravity of quadrocopters, and the first equation describes the motion around the center of gravity in the MCS. The main differences between symmetrical and asymmetrical models are the form of the equations (9). For asymmetrical model the equations (9) become more complicated and require additional analysis.

3. Control Development

The synthesis of the control algorithm was carried out by methods LQR and inverse dynamics. The following criteria were used for the comparison of selected methods:

1. The value of functional use in LQR method. We consider the part of functionality associated with the state vector, which is responsible for the achievement of the control objectives, and part of the functional related to the control, which is proportional to the energy costs as separate.
2. The standard deviation of the gravity center of predetermined trajectory.
3. The maximum deviation from the given position in absolute value.

3.1. Inverse Dynamics

The method of the inverse dynamics is used to find the forces acting on an object of known trajectory. The method of inverse dynamics is unstable. In practice, various modifications of the method were used to guarantee the stability of the closed system [15, 16].

Assume that the desired trajectory is defined as analytical functions of the vector position of the gravity center X_c^t and the yaw angle ψ^{tr}. This method of defining the desired trajectory is the most informative for the controlled quadrocopter operator.

The equation form for control of quadrocopters acceleration is as follows:

$$\ddot{X}^{*} = \ddot{X}^{tr} - C_1(X - X^{tr}) - C_2(\dot{X} - \dot{X}^{tr}), \tag{12}$$

where C_1, C_2 are matrixes of known feedback coefficients.

From the dynamic equations of gravity center motion it is possible to determine the thrust U_0 and the values of roll $\tilde{\phi}$ and pitch $\tilde{\theta}$ angles for the realization of different maneuvers.

$$\begin{aligned} U_0 &= \frac{M\ddot{z}^{*} + Mg + F_{res,z}}{c_\theta c_\phi}; \\ \tilde{\theta} &= arctg\left(\frac{(M\ddot{x}^{*} + F_{res,x})c_\psi + (M\ddot{y}^{*} + F_{res,y})s_\psi}{M\ddot{z} + Mg + F_{res}(\dot{z})}\right); \\ \tilde{\phi} &= arctg\left(\frac{(M\ddot{x}^{*} + F_{res,x})s_\psi - (M\ddot{y}^{*} + F_{res,y})c_\psi}{M\ddot{z} + Mg + F_{res}(\dot{z})} c_{\tilde{\theta}}\right); \end{aligned} \tag{13}$$

Similarly, the controls of angular acceleration of the angular position are:

$$\begin{aligned} \ddot{\phi}^{*} &= \ddot{\phi}^{tr} - C_3(\phi - \tilde{\phi}) - C_4(\dot{\phi} - \dot{\phi}^{tr}); \\ \ddot{\theta}^{*} &= \ddot{\theta}^{tr} - C_3(\theta - \tilde{\theta}) - C_4(\dot{\theta} - \dot{\theta}^{tr}); \\ \ddot{\psi}^{*} &= \ddot{\psi}^{tr} - C_3(\psi - \psi^{tr}) - C_4(\dot{\psi} - \dot{\psi}^{tr}); \end{aligned} \tag{14}$$

where C_3, C_4 are known feedback coefficients.

From the dynamic equations of motion around the gravity center we can determine the control torques.

$$U = J\Lambda\ddot{\eta}^{*} + J\dot{\Lambda}\dot{\eta}^{tr} + \Lambda\dot{\eta}^{tr} \times J\Lambda\dot{\eta}^{tr}, \tag{15}$$

where $\ddot{\eta}^{*} = (\ddot{\phi}^{*}, \ddot{\theta}^{*}, \ddot{\psi}^{*})^T, \dot{\eta}^{tr} = (\dot{\phi}^{tr}, \dot{\theta}^{tr}, \dot{\psi}^{tr})^T$.

To determine the angular velocities of motors we can use a system of equations (9). This system is linear relative to $sign(\omega_i)\omega_i^2$, coefficient matrix is constant for the configuration and does not degenerate. It means that the system of equations (9) provides a unique solution.

3.2. LQR Method

The LQR method is described in detail in [17]. Apply an algorithm to solve this problem. System of equations (11) was linearized and used in this method. The system of equations (11) can be written as (16). So the linearized system has the form (17).

$$\dot{Y} = F(Y, W) \tag{16}$$

$$\begin{aligned} \dot{Y} &= F(Y_0, W_0) + \frac{\partial F(Y_0, W_0)}{\partial Y}(Y - Y_0) + \\ &+ \frac{\partial F(Y_0, W_0)}{\partial W}(W - W_0) = A(Y_0, W_0)Y + \\ &+ B(Y_0, W_0)W + D(Y_0, W_0) \end{aligned} \tag{17}$$

where Y_0, W_0 are the state vector and control vector at some point.

Suppose, the criterion of control quality [17] has the functional form (18).

$$\begin{aligned} \Phi(Y, W) &= \frac{1}{2}\int_0^T \left[(Y^{tr} - Y)^T Q (Y^{tr} - Y) + W^T P W\right] dt = \\ &= \Phi(Y) + \Phi(W) \end{aligned} \tag{18}$$

where Y^{tr} is the vector of the desired trajectory of movement, Q and P are constant positive definite symmetric matrix.

The control is determined by formulas (19–21).

$$W = -P^{-1}B^T(RY + w) \tag{19}$$

$$\dot{R} = -RA - A^T R + RBP^{-1}B^T R - Q, \; R(T) = 0 \tag{20}$$

$$\begin{aligned} \dot{w} &= -(A^T - RBP^{-1}B^T)w + QY^{tr} - RD \\ w(T) &= -(B^T B)^{-1} BPW(T) \end{aligned} \tag{21}$$

The optimal control (19) is determined for a given trajectory with regard to minimizing the functional (18). The particularity of this method is that the equations (20) and (21) are integrated in the reverse time and require high computational cost.

Considering the fact that the original system is not linear, to solve the original problem with this method it is necessary to know the matrix of the system at all points of the trajectory. For this, we need to know the trajectory of the object, which is set by the operator, and the planned control, which is unknown. We used an iterative approach to solve this problem. As a first approximation selected control obtained by the inverse dynamics. The functional (18) is the criterion for the process convergence.

4. Simulation

Based on the obtained mathematical models and control algorithms, a mathematical complex has been developed by using MATLAB R2014b. Quadrocopter AR.Drone 1.0 was taken as a basis [18]. The angular rotation speed of motors is limited to the equation $150 < \omega_i < 500, i = 1,..,4$. The model parameters are: a = 0.2 [m], b = 0.2 [m], c = 0.1 [m], d = 0.1 [m], l = 0.3 [m], l_1 = 0.33 [m], l_s = 0.03 [m], Δl = 0.1 [m], r = 0.05 [m], h_1 = 0.02 [m], h_2 = 0.05 [m], h_3 = 0.02 [m], h_5 = 0.02 [m], α = 75°, M_1 = 0.2 [kg], M_2 = 0.1 [kg], M_3 = 0.1 [kg], M_5 = 0.05 [kg], C_1 = 16, C_2 = 16, C_3 = 225, C_4 = 40, A_m = 0.005 [m^{2}kg], C_m = 0.001 [m^{2}kg], k_1 = 0.7426·10^{-6} [m^{2}kg], k_2 = 0.1485·10^{-6} [m^{2}kg], g = 9.8 [m/s^2], S_x = 0.0024 [kg/m], S_y = 0.0072 [kg/m], S_z = 0.0072 [kg/m].

For the comparison of the efficiency of the control algorithms with the new model, two trajectories were selected. Both trajectories consisted of three stages. In the first stage, a quadrocopter hovered motionless at a given point in space during 0.1 s. (22). In the second stage, the quadrocopter rose straight up and picked up speed for a maneuver (23). In the third stage, the quadrocopter was doing the maneuver. For the first trajectory, the quadrocopter flew around the ring in a vertical plane with a radius of 1 m and the angular speed $\theta = 2\pi/5$ rad/s (24). For the second trajectory, the quadrocopter flew along "eight-

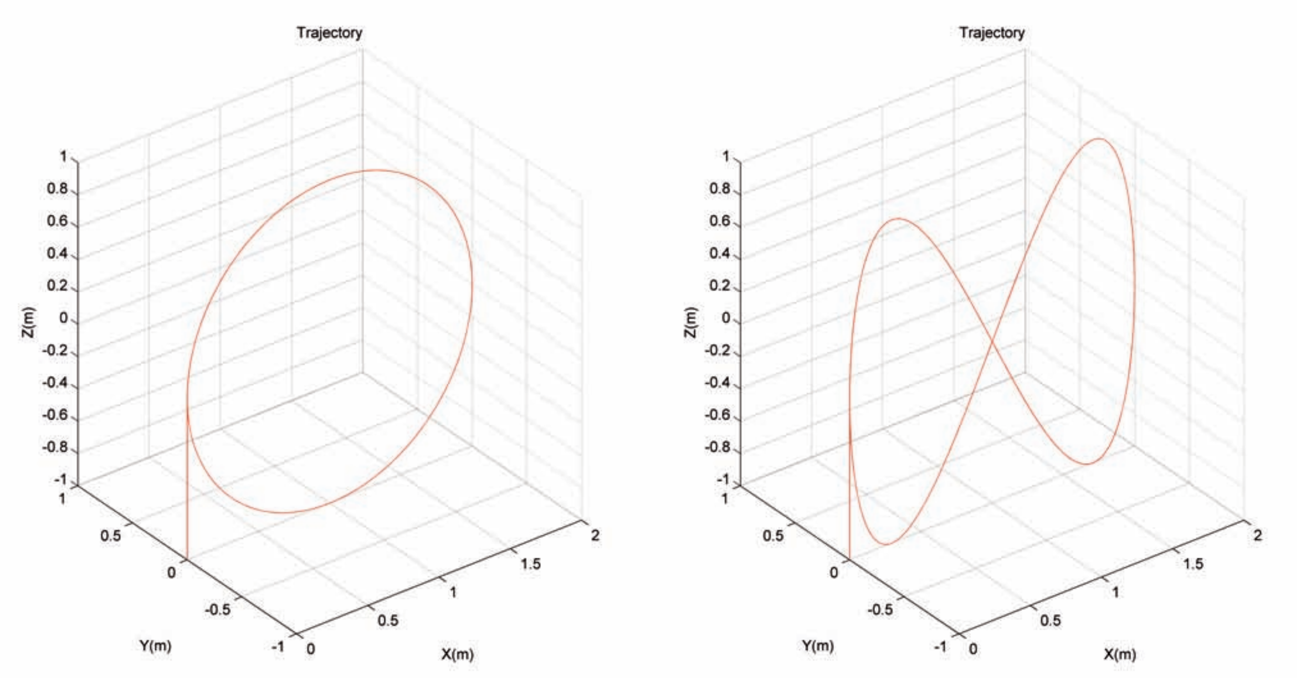

Fig. 5. Motion trajectories of quadrocopters

shaped" with a loop radius of 1 m and a time period of 5 s. (25). Trajectories are shown in Fig. 5. and described in detail in (22–25).

$$\begin{cases} 0 \le t < 0.1; x_I(t)=0; y_I(t)=0; \\ z_I(t)=-1; \psi_I(t)=0; \end{cases} \tag{22}$$

$$\begin{cases} 0.1 \le t < t_s; t_s = \dfrac{5}{\pi}+0.1; x_I(t)=0; y_I(t)=0; \\ z_I(t)=\dfrac{\pi}{5}(t-0.1)-1; \psi_I(t)=0; \end{cases} \tag{23}$$

$$\begin{cases} t_s \le t; x^o_{III}(t)=1-cos(\dfrac{2\pi}{5}(t-t_s)); y^o_{III}(t)=0; \\ z^o_{III}(t)=sin(\dfrac{2\pi}{5}(t-t_s)); \psi^o_{III}(t)=0; \end{cases} \tag{24}$$

$$\begin{cases} t_s \le t; x^\infty_{III}(t)=1-cos(\dfrac{2\pi}{5}(t-t_s)); y^\infty_{III}(t)=0; \\ z^\infty_{III}(t)=sin(\dfrac{4\pi}{5}(t-t_s)); \psi^\infty_{III}(t)=0; \end{cases} \tag{25}$$

The results of the simulation are shown in Figs. 6–9. Table 4 shows the numerical value of the evaluation criteria.

In Figs. 6–9 for the state vector dotted line indicates the desired trajectory. It should be noted that the deviation from the predetermined trajectory in the plane YZ equals less than 1.5 mm for absolute value in all cases.

According to the simulation results, we can conclude that both methods are able to solve the problem of control successfully. The control algorithm obtained by both methods is within the predetermined limits. The most difficult phase to control is the transition from the second to the third stage. This

Table 4. Properties of quadrocopters motion

	Trajectory "circle"		Trajectory "eight-shaped"	
Criteria	LQR	ID	LQR	ID
Functional $\Phi(Y,W)$	2.65	12.00	4.50	17.15
Part $\Phi(Y)$ of the functional	0.27	4.57	0.68	6.10
Part $\Phi(W)$ of the functional	2.39	7.43	3.82	11.05
Standard deviation [m]	0.06	0.05	0.10	0.14
Maximum deviation of the position [m]	0.15	0.17	0.38	0.65

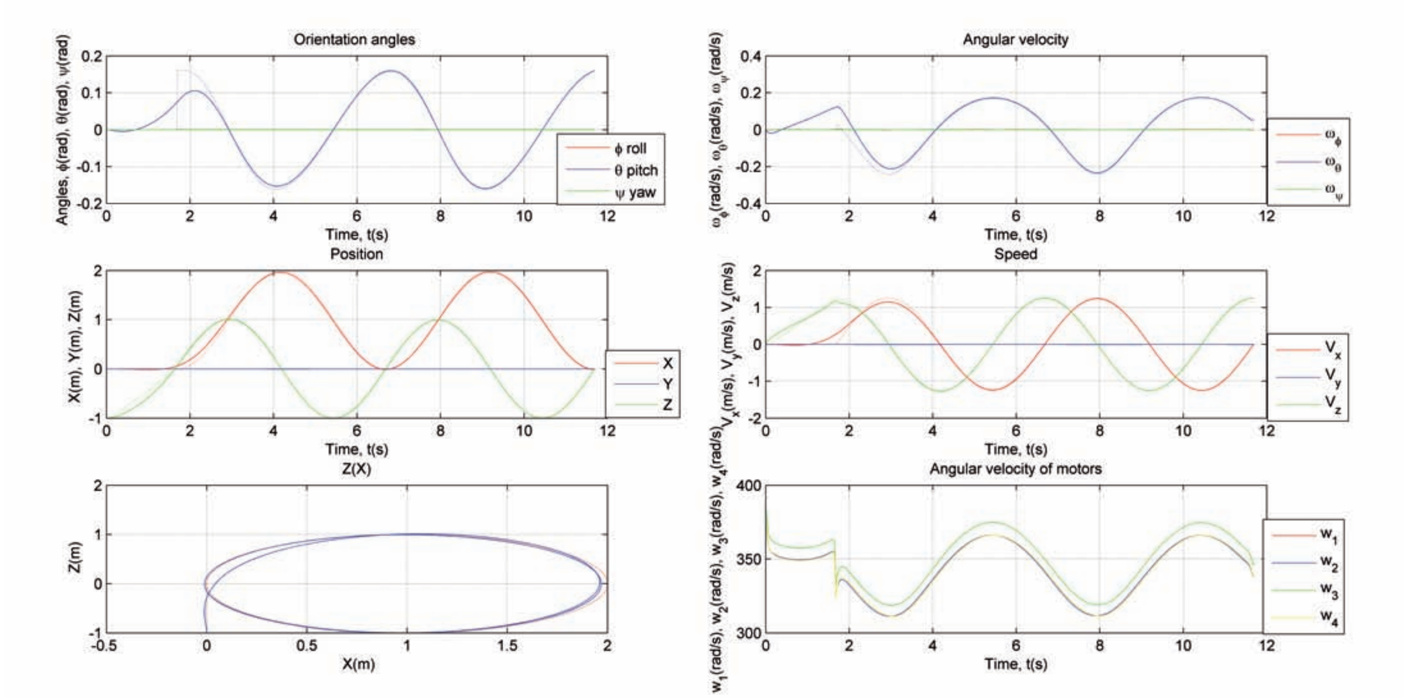

Fig. 6. The state vector and the control vector to the trajectory "circle" by the LQR method

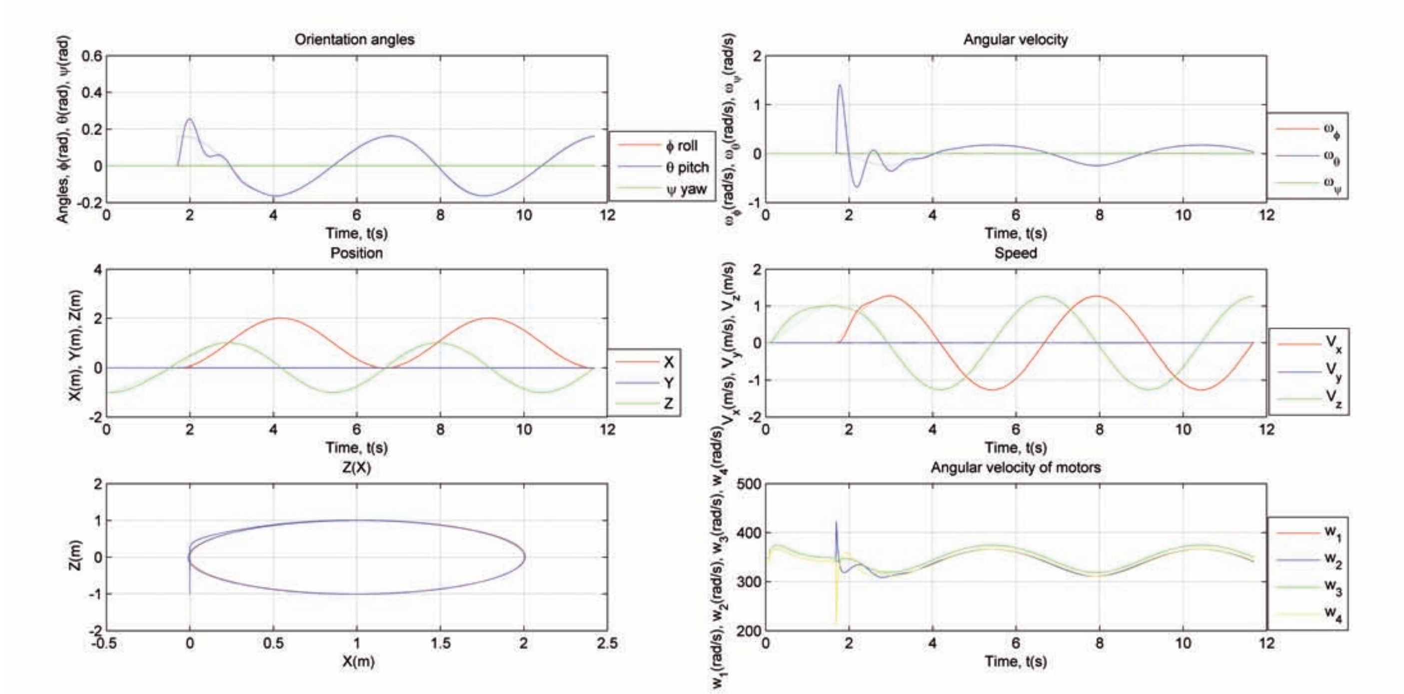

Fig. 7 .The state vector and the control vector to the trajectory "circle" by the inverse dynamics

is caused by discontinuity of the desired state vector, namely discontinuity of the angular position. This is particularly well illustrated by the trajectory "circle" (Figs. 6–7). LQR method was implemented smoothly around that time, as can be seen from the Fig. 6 and Fig. 8. The method of inverse dynamics (ID) could not do it smoothly. To continue the flight along the trajectory, it is necessary to create high moments, it is shown by the peaks in control in Fig. 7 and Fig. 9.

An analysis of the imposed criteria shows that the mean deviation and the maximum deviation of the gravity center from predetermined trajectory are approximately the same. However, the energy cost is higher in inverse dynamics.

The advantages of the inverse dynamics method mainly consist of their simplicity, computational speed in the calculation and the ability of application in on-line tasks.

5. Conclusions

The control problem of asymmetric quadrocopters was illustrated an example of complex trajectories "circle" and "eight-shaped". The new mathematical model which takes into account the asymmetry of

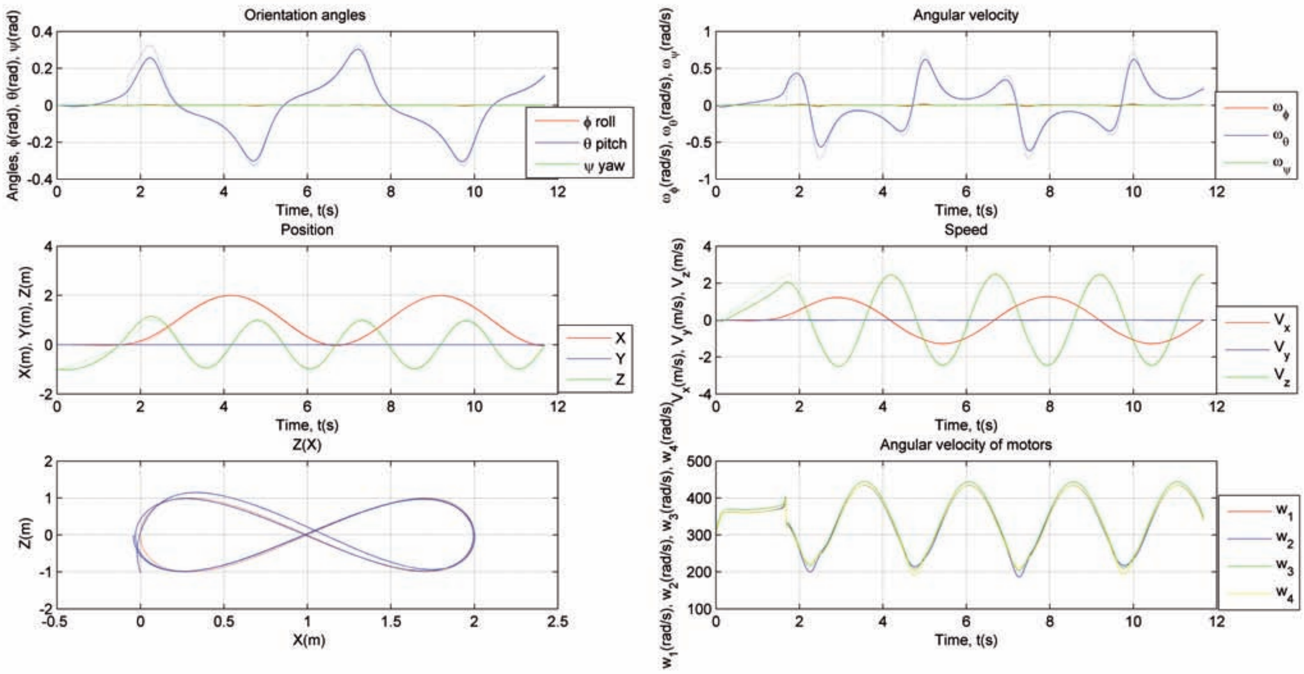

Fig. 8. The state vector and the control vector to the trajectory "eight-shaped" by the LQR method

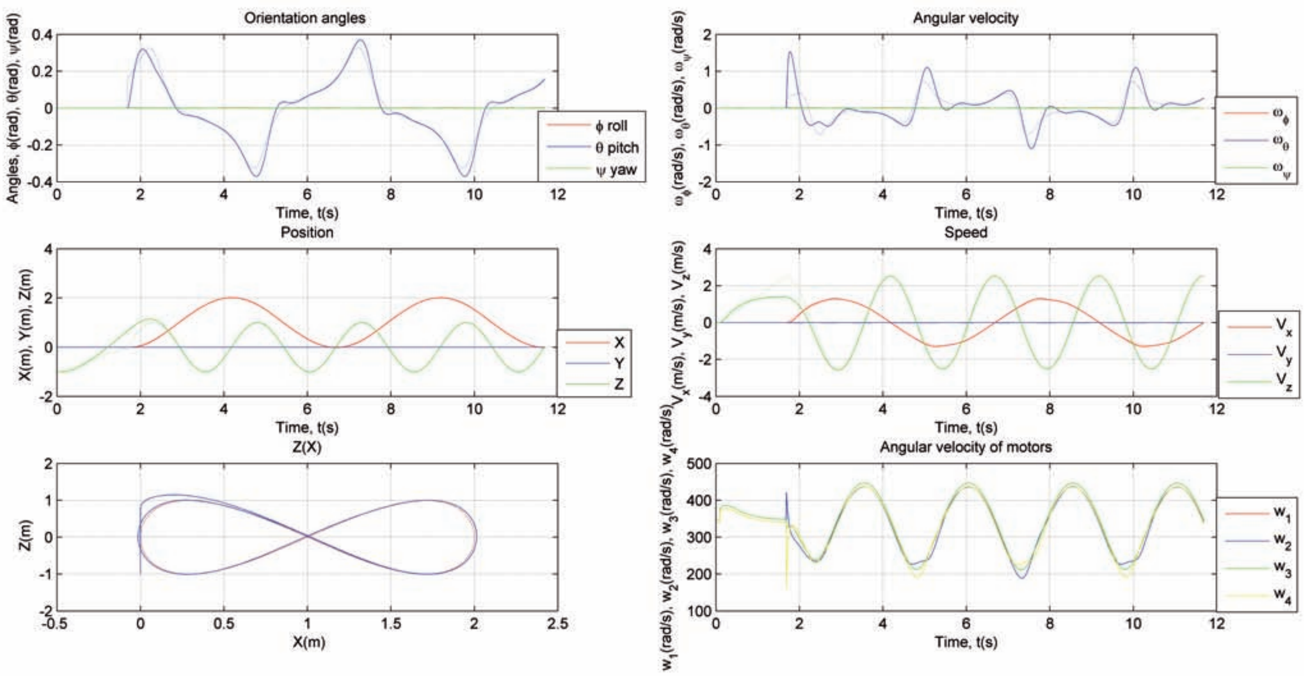

Fig. 9. The state vector and the control vector to the trajectory "eight-shaped" by the inverse dynamics

the quadrocopter design has been developed. In order to study the characteristics, of the new model, control algorithms by LQR method and inverse dynamics for the motion along a predetermined trajectory have been synthesized. We suppose that the asymmetric construction properties improve maneuverability. Therefore, in the control algorithms it is necessary to considered construction characteristics. This paper presents the solution algorithm for nonlinear model. The criteria entered for the efficiency evaluation of the synthesized control algorithms allow us to consider the choice of solution methods for conditions variety. LQR method involves large computational costs and requires a prior knowledge of the motion trajectory, but it allows decreasing the energy costs and obtaining a smooth motion of trajectory. The method of inverse dynamics can be used in on-line mode, it does not require high computational costs, but at the moments of trajectory discontinuity the system may lose stability especially in aggressive manoeuvers.

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REFERENCES

[1] Erdinc Altug, James P. Ostrowski, Robert Mahony, "Control of a Quadrotor Helicopter Using Visual Feedback". In: *Proceedings of the 2002 IEEE, International Conference on Robotics & Automation*, Washington, DC, May 2002, 72–77.

[2] J. Gordon Leishman, *Principles of Helicopter Aerodynamics*, 2nd edition, Cambridge University Press, 2006, 25–33.

[3] H. Bolandi, M. Rezaei, R. Mohsenipour, H. Nemati, S. Smailzadeh, "Attitude Control of a Quadrotor with Optimized PID Controller", *Intelligent Control and Automation*, vol. 4, no. 3, 2013, 335–342. DOI: 10.4236/ica.2013.43039.

[4] H. Voos, "Nonlinear State-Dependent Riccati Equation Control of a Quadrotor UAV". In: *Conference: Computer Aided Control System Design, 2006 IEEE International Conference on Control Applications, 2006 IEEE International Symposium on Intelligent Control*, Munich, Germany, 2006, 2547–2552. DOI: 10.1109/CACSD-CCA-ISIC.2006.4777039.

[5] Wei Dong, Guo-Ying Gu, Xiangyang Zhu, Han Ding, "Solving the Boundary Value Problem of an Under-Actuated Quadrotor with Subspace Stabilization Approach", *Journal of Intelligent and Robotic Systems*, vol. 80, no. 2, November 2015, 299–311. DOI: 10.1007/s10846-014-0161-3.

[6] Weihua Zhaoa, Tiauw Hiong Go, "Quadcopter formation flight control combining MPC and robust feedback linearization", *Journal of the Franklin Institute*, vol. 351, no. 3, March 2014, 1335–1355. DOI: 10.1016/ j.jfranklin.2013.10.021.

[7] İ. Can Dikmen, Aydemir Arısoy, Hakan Temeltaş, "Attitude Control of a Quadrotor", *Recent Advances in Space Technologies*, Istanbul, 2009, 722–727. DOI: 10.1109/ RAST.2009.5158286.

[8] Erdinç Altu˘g, James P. Ostrowski, Camillo J. Taylor, "Control of a Quadrotor Helicopter Using Dual CameraVisual Feedback", *International Journal of Robotics Research* vol. 24, no. 5, May 2005, 329–341. DOI: 10.1177/0278364905053804.

[9] A. Bemporad, C. Rocchi, "Decentralized Hybrid Model Predictive Control of a Formation of Unmanned Aerial Vehicles", *Decision and Control and European Control Conference (CDC-ECC)*, Orlando, FL, 2011, 7488–7493. DOI: 10.1109/ CDC.2011. 6160521.

[10] http://www.hover-bike.com/ - The official website of The Hoverbike.

[11] Abdelhamid Tayebi, Stephen McGilvray, "Attitude Stabilization of a VTOL Quadrotor Aircraft", *IEEE Transactions on Control Systems Technology*, vol. 14, no. 3, May 2006, 562–571. DOI: 10.1109/ TCST.2006. 872519

[12] Hanoch Efraim, Amir Shapiro, Gera Weiss, "Quadrotor with a Dihedral Angle: on the Effects of Tilting the Rotors Inwards", *Journal of Intelligent & Robotic Systems*, November 2015, vol. 80, no. 2, 313–324. DOI: 10.1007/s10846-015-0176-4.

[13] K. M. Zemalache, L. Beji, H. Maaref, "Control of a drone: study and analysis of the robustness", *Journal of Automation, Mobile Robotics & Intelligent Systems*, vol. 2, no. 1, 2008, 33–42.

[14] Thomas S. Alderete, "Simulator aero model implementation", NASA Ames Research Center, Moffett Field, California.

[15] Mark W. Spong, Seth Hutchinson, M. Vidyasagar, *Robot Dynamics and Control*, 2nd edition, 2004.

[16] Krzystof Piotr Jankowski, "Inverse dynamics control in robotics applications", Canada by Trafford Publishing, Ltd., Victoria, British Columbia, 2004. DOI:10.13140/RG.2.1. 1015. 3683.

[17] M. Athans, P.L. Falb, *Sterowanie optymalne*, Warszawa, 1966 (in Polish).

[18] Gurianov A.E. "Control Simulation of quadrocopters", *Electronic Science and Technology Journal Engineering Journal*, Moscow, BMSTU, 2014, 522-534, ISSN 2307–0595.