Mathematical Modeling of the Bicycle Robot with the Reaction Wheel

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Abstract:

The article presents the influence of the reaction wheels on the stability of the bicycle robot. The precise mathematical model is presented. The robot is treated as an inverted pendulum with rigid body physical nature. The mathematical analysis can be useful to develop control techniques, to make many computer simulations, to check how the system will behave in reality.

Keywords: *inertia wheel pendulum, reaction wheel, bicycle robot, dynamic model*

1. Introduction

This article presents the mathematical equations describing the two wheeled robot with additional stabilization unit. The reaction wheel (also called the inertia wheel) is the rotating mass accelerated by an electric motor. It can be used to stabilize robots in unstable equilibrium points. Using this system it is possible to keep the bicycle robot in the vertical position even if this machine has zero velocity in the horizontal direction.

Finding the literature concerning the dynamic model of such two wheeled robot is problematic. Usually it is focused on the bicycle differential equations. The detailed mathematical model was presented in [1]. It takes into account many physical effects typical for bicycles like: front fork forces, self-stabilization and gyroscopic effects. It describes results of rider lean and it takes into consideration the stability analysis. This also includes extensive research about various bicycle configurations like: controlling bicycle with a big or small wheels, front-wheel steering and rear-wheel steering. Some more detailed information about dynamics like: tyre modeling, aerodynamic forces, suspension and cornering models can be found in [2], [18]. On the other hand it is possible to find results of reaction wheel studies. The [4] presents the mathematical model of the monorail tram stabilized by an inertia wheel. It is focused on the stabilization unit but it does not take into account the model of two wheel machine. Other works like [12] [17] presents control law concepts for inertial wheel stabilization and [12], [13] includes global asymptotic stabilization approach.

Available works usually are focused on bicycle dynamics or IWP dynamics and never boots at the same time. The approach presented in this article is a combination of these two subjects. This article can be useful to develop control techniques to make many computer simulations, to check how the system will behave in the reality. This paper is associated with considerations from [14].

Currently, there are many publications related to bicycles and control theory. Given that this problem is very large, the work usually focuses on limited areas of knowledge. Modeling the bicycle has always caused trouble and creating a precise mathematical model is problematic. Works related to the mathematical modeling are [7], [5], [19], [8]. There are papers that take into account the work of the active damper suspension system [3]. Some studies refer to the description of the dynamics of the bike during acceleration [11]. A large proportion of researchers working on the bicycle theory are focused on designing controllers [9], [6]. Some works describes control methods based on gyroscope-balancing or steering-balancing [15], [10].

Publications describing the bicycle with the flywheel are not always associated with the stabilization based on the flywheel. They often refer to the kinetic energy recovery system (KERS).

2. Dynamic Model

The main goal of this article is to present the mathematical model useful to simulate the real machine which would be sufficient to solve the control law stabilizing it in the vertical pose. This research also takes into account movements in more difficult conditions like moving on the inclined plane and moving in an arc. The friction forces are also considered because of their great importance especially in rotating mass joint.

Additionally it is important to know Figure 1 presents the dynamic scheme of the robot. It has four degrees of freedom and three actuators. This makes this machine underactuated. Many symbols should be used to make precise model. Every symbol is described below.

- $\underline{x}$ – state vector,
- $\underline{x}_0$ – initial state vector,
- x_1 – angle of the robot from the vertical,
- x_2 – angular velocity of the robot,
- x_3 – rotation angle of the reaction wheel,
- x_4 – angular velocity of the reaction wheel,
- x_5 – horizontal movement (bicycle forward or backward motion),
- x_6 – horizontal velocity (bicycle forward or backward velocity),

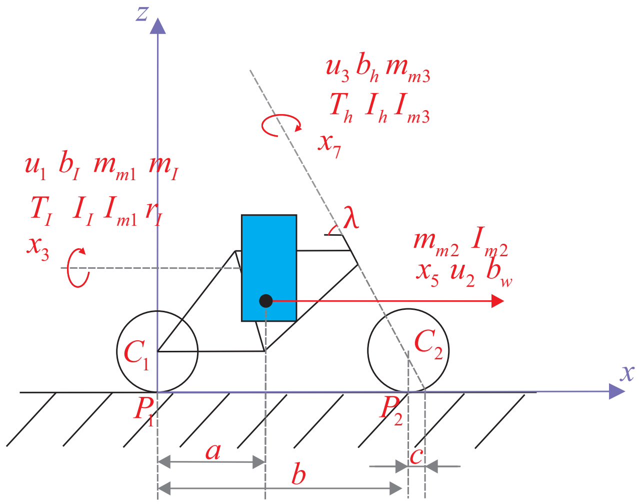

Fig. 1. The bicycle robot – main view

- x_7 – rotation of the steering wheel,
- x_8 – angular velocity of the steering wheel,
- $\underline{u}$ – control vector,
- u_1 – current of the first motor,
- u_2 – current of the second motor,
- u_3 – current of the third motor,
- m_f – frame weight,
- m_{cover} – cover weight,
- m_I – reaction wheel weight,
- m_{mr1} – weight of the rotor of the motor 1,
- m_{ms1} – weight of the stator of the motor 1,
- m_{m1} – weight of the first motor,
- m_{m2} – weight of the second motor,
- m_{m3} – weight of the third motor,
- m_r – weight of the robot,
- I_I – moment of inertia of the reaction wheel,
- I_{ms1} – moment of inertia of the stator of the motor 1,
- I_{mr1} – moment of inertia of the rotor of the motor 1,
- I_{mi} – moment of inertia of the i-th motor,
- I_h – moment of inertia of the handlebar,
- I_{Ig} – moment of inertia of the reaction wheel relative to the ground,
- I_{fg} – moment of inertia of the frame relative to the ground,
- I_{coverg} – moment of inertia of the cover relative to the ground,
- I_{mgi} – moment of inertia of the i-th motor relative to the ground,
- I_{hg} – moment of inertia of the handlebar relative to the ground,
- I_{rg} – moment of inertia of the robot relative to the ground,

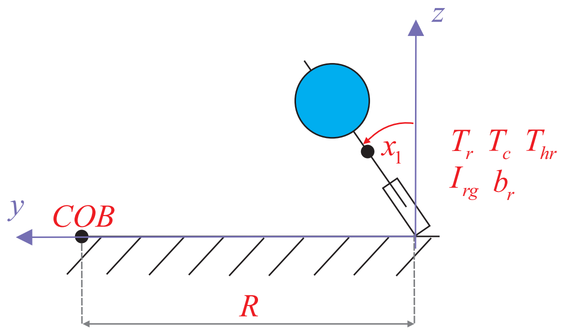

Fig. 2. The bicycle robot – side view

- h_I – distance from the ground to the center of mass of the inertia,
- h_{mi} – distance from the ground to the center of mass of the i-th motor,
- h_r – distance from the ground to the center of mass of the robot,
- r_I – radius of the reaction wheel,
- r_w – radius of the wheel,
- r_{mr1} – radius of the rotor of the motor 1,
- P_1, P_2 – contact points of the wheels with the ground,
- P_3 – intersection of the steer axis with the ground on the plane defined by front wheel,
- C_1 – center of the rear wheel,
- C_2 – center of the front wheel,
- g – gravity of the Earth,
- a – distance from a vertical line through the center of mass to P_1,
- b – wheel base,
- c – trail,
- λ – head angle,
- α – angle of inclination of the slope,
- R – radius of the bend,
- k_{mi} – i-th motor constant,
- b_r – coefficient of friction in the robot rotation,
- b_w – coefficient of friction in the robot horizontal movement,
- b_h – coefficient of friction in the rotation of the handlebar,
- b_I – coefficient of friction in the rotation of the reaction wheel,
- F_d – inclined plane force,
- F_w – resultant force,
- F_p – pressure force on the ground,
- F_g – gravitational force,
- F_{bw} – dissipation force,
- F_{m2} – motor 2 force,
- T_r – robot torque,
- T_I – inertial drive torque,

- T_c – torque acting on the robot from centrifugal force,
- T_p – torque acting on the robot from the gravity force,
- T_{br} – friction torque in the robot rotation,
- T_{hr} – torque acting on the robot from handlebar movements,
- T_h – handlebar torque,
- COM – Center Of Mass,
- COB – Center Of Bend,

Detailed symbol description is necessary to reach precise model of the robot.

The total weight of the structure is

$$m_r = m_f + m_{cover} + m_I + m_{m1} + m_{m2} + m_{m3}. \quad (1)$$

It contains every significant part that should be taken into account. One of the most important elements is electric motor 1. It accelerates the reaction wheel. The motor 1 is divided into two components – weight of the rotor and weight of the stator

$$m_{m1} = m_{mr1} + m_{ms1}. \quad (2)$$

The same applies to the moment of inertia of the motor

$$I_{m1} = I_{mr1} + I_{ms1}. \quad (3)$$

The motor 1 has the moment of inertia that can be calculated from the equation

$$I_{mr1} = \frac{m_{mr1} r_{mr1}^2}{2}. \quad (4)$$

It is important to know the moment of inertia of rotating mass in the inertial drive system which includes two important parts: the reaction wheel and the rotor of the motor 1. The moment of inertia of the reaction wheel is equal to

$$I_I = \frac{m_I r_I^2}{2}. \quad (5)$$

The first degree of freedom is the rotation about an axis lying on the ground. Every moment of inertia can be expressed about this axis using the Huygens-Steiner theorem:

$$I_{Ig} = I_I + m_I h_I^2, \quad (6)$$

$$I_{mg1} = I_{m1} + m_{m1} h_{m1}^2, \quad (7)$$

$$I_{mg2} = I_{m2} + m_{m2} h_{m2}^2, \quad (8)$$

$$I_{mg3} = I_{m3} + m_{m3} h_{m3}^2. \quad (9)$$

Having every distances h it is inevitable to make above calculations. This leads to the most important moment of inertia in dynamics of the robot, which is

$$I_{rg} = I_{fg} + I_{coverg} + I_{Ig} + I_{mg1} + I_{mg2} + I_{mg3}. \quad (10)$$

The distance from the ground to the center of mass COM is equal to

$$\begin{aligned} h_r = \frac{1}{m_r}(& m_f h_f + m_{cover} h_{cover} + m_I h_I \\ & m_{m1} h_{m1} + m_{m2} h_{m2} + m_{m3} h_{m3}). \end{aligned} \quad (11)$$

The basic equation with the gravity force acting the

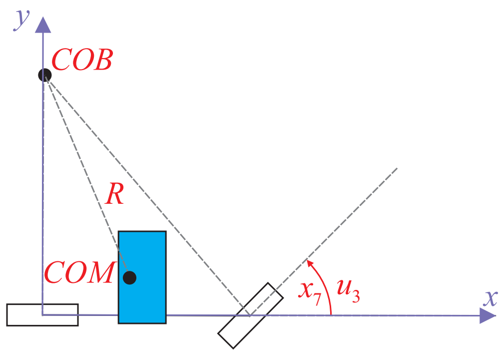

Fig. 3. The bicycle robot – top view

robot is

$$F_g = m_r g h_r. \quad (12)$$

The torque acting on the robot from the gravity is given by

$$T_p = \sin x_1 F_p. \quad (13)$$

The friction torque acting the robot rotation can be calculated by

$$T_{br} = -b_r x_2. \quad (14)$$

3. Reaction Wheel Torque

The inertial drive torque is created by accelerating reaction wheel and is equal to

$$T_I = (I_I + I_{mr1})\dot{x}_4. \quad (15)$$

It consists of two moments of intertia – both belong to rotating parts. The differential equation of electric motor 1 is

$$\dot{x}_4 = \frac{k_{m1} u_1}{(I_I + I_{mr1})} - \frac{b_I x_4}{(I_I + I_{mr1})}. \quad (16)$$

4. Centrifugal Force

When the bicycle robot circles around some point (COB) the centrifugal force appears. First of all the radius of the bend can be calculated by

$$R = \sqrt{(b \tan(\pi/2 - x_7) - h_r \sin x_1)^2 + a^2}. \quad (17)$$

Figure 2 and Figure 3 show two views of robot when it tilts.

The centrifugal force creates torque acting on the robot

$$T_c = \frac{m_r x_6^2 h_r}{R}. \quad (18)$$

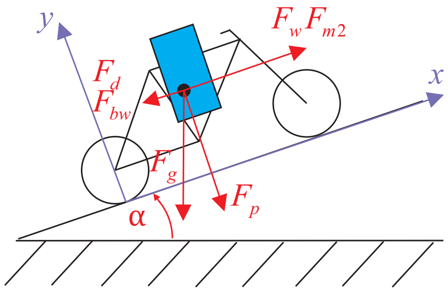

Fig. 4. The bicycle robot – top view

5. Climbing

Well prepared mathematical model should describe the forces in the inclined plane. This situation is presented in Figure 4. The pressure force is expressed by

$$F_p = \cos\alpha F_g. \tag{19}$$

The inclined plane force is

$$F_d = -\sin\alpha F_g, \tag{20}$$

the motor 2 force is

$$F_{m2} = \frac{k_{m2}u_2}{r_w}. \tag{21}$$

and the dissipation force is

$$F_{bw} = -b_w x_6. \tag{22}$$

The resultant force is equal to

$$F_w = F_{m2} + F_d + F_{bw}. \tag{23}$$

The equation 23 tells if robot goes up or down the inclined plane. The acceleration of the robot is expressed by

$$\dot{x}_6 = \frac{F_w}{m_r}. \tag{24}$$

It also includes the 2 mathematical model of the electric motor.

6. Front Fork Torque

The torque acting on the robot from handlebar movements is

$$T_{hr} = \frac{-m_r ac \sin\lambda x_7}{b}. \tag{25}$$

The motor 3 torque can be expressed by

$$T_{m3} = \frac{k_{m3}u_3}{I_h} - \frac{b_h x_8}{I_h}. \tag{26}$$

The torque acting the front fork is equal to

$$T_s = -\frac{am_r x_6^2(\sin\lambda)^2 cx_7}{b^2} - \frac{am_r g \sin\lambda x_1}{b} + \\ + \frac{am_r gc \sin\lambda \cos\lambda x_7}{b}. \tag{27}$$

As it can be seen it highly depends on the trail c. This parameter usually ranges from 0.03 to 0.08 m. This torque is the "bicycle phenomenon" which provides to the self-stabilization. The handlebar torque includes additionally the third motor torque

$$T_h = T_s + T_{m3}. \tag{28}$$

The differential equation of the handlebars is given by

$$\dot{x}_8 = \frac{T_h}{I_h}. \tag{29}$$

7. Full Rotating Torque

The full rotating torque is given by

$$T_r = T_p + T_I + T_c + T_{br} + T_{hr}. \tag{30}$$

It contains every torque which acts the first degree of freedom of the bicycle robot. Now is possible to arrange the differential equation

$$\dot{x}_2 = \frac{T_r}{I_{rg}}. \tag{31}$$

The rest derivatives of state variables are equal to

$$\dot{x}_1 = x_2, \tag{32}$$

$$\dot{x}_3 = x_4, \tag{33}$$

$$\dot{x}_5 = x_6, \tag{34}$$

$$\dot{x}_7 = x_8. \tag{35}$$

8. Results

It is important to check if the mathematical model is correct. The best way is to make computer simulation.

The system has the following parameters: $m_f = 1.0k$, $m_{cover} = 0.1kg$, $m_I = 2.8kg$, $m_{mr1} = 0.63kg$, $m_{ms1} = 0.53kg$, $m_{m2} = 0.1kg$, $m_{m3} = 0.1kg$, $h_f = 0.14m$, $h_{cover} = 0.14m$, $h_{m1} = 0.14m$, $h_I = 0.14m$, $h_{m2} = 0.04m$, $h_{m3} = 0.24m$, $I_{mr1} = 0.0008kg \cdot m^2$, $I_{ms1} = 0.0002kg \cdot m^2$, $I_{m2} = 0.0001kg \cdot m^2$, $I_{m3} = 0.0001kg \cdot m^2$, $I_h = 0.0001kg \cdot m^2$, $I_{coverg} = 0.0010kg \cdot m^2$, $I_{fg} = 0.0013kg \cdot m^2$, $r_I = 0.08m$, $r_{mr1} = 0.0366m$, $r_w = 0.03m$, $g = 9.80665\frac{m}{s^2}$, $a = 0.4m$, $b = 1.0m$, $c = 0.03m$, $\lambda = \pi/2rad$, $k_{m1} = 0.421\frac{Nm}{A}$, $k_{m2} = 0.421\frac{Nm}{A}$, $k_{m3} = 0.121\frac{Nm}{A}$, $b_r = 0.0013N \cdot m \cdot s$, $b_w = 0.0022N \cdot m \cdot s]$, $b_h = 0.0002N \cdot m \cdot s$. These parameters were measured with real parts which will probably form a new robot based on presented model. The first simulation is shown in the Figure 5. The initial state is equal to

$$\underline{x} = \begin{bmatrix} 0.2 & 0 & 0 & 0 & 0 & 0 & 3.0 & 0 \end{bmatrix}^T. \tag{36}$$

It means that the whole robot starts from inclined position and velocity which is not equal to zero. After 10 seconds the handlebar angle changes from 0 to $0.1rad$ and it creates the centrifugal force. This force changes the position of the equilibrium point and now

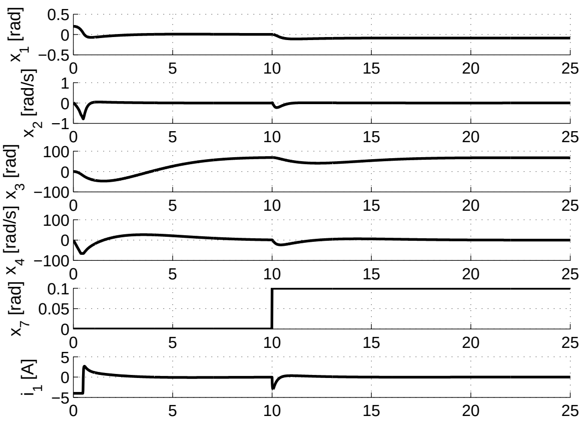

Fig. 5. The result of the centrifugal force – handlebar angle changes from 0 ***to*** $0.1rad$

the robot goes not to inclination $x_1 = 0rad$ but $x_1 = -0.1rad]$. Increasing velocity and handlebar angle can cause loss of stability. The Figure 6 presents analogous symulation but in this time the handlebar angle changes from 0 to $0.2rad$. As it can be seen it creates

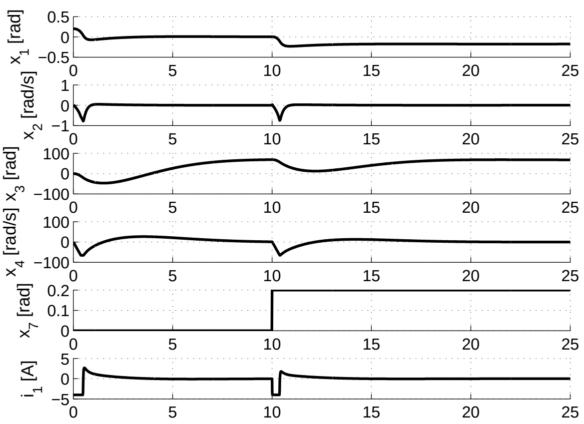

Fig. 6. The result of the centrifugal force – handlebar angle changes from 0 ***to*** $0.2rad$

bigger centrifugal force and the new equilibrium point is further from $x_1 = 0$. Also the bigger control signal is needed this time to maintain the stability.

The next simulation shows the behavior of the robot on the inclined ground. The results are in Figure 7.

The initial state is the same as in the previous simulation. As it can be seen the regulation time is shorter if the slope is more inclined. The control is less rapid and jerks of the state are smoother. The Figure 8 presents how the system reacts to bigger inclination. This time the stabilization process is much faster and it costs much less energy.

9. Conclusions

The dynamic model proposed here takes into account many detailed parameters. The description is

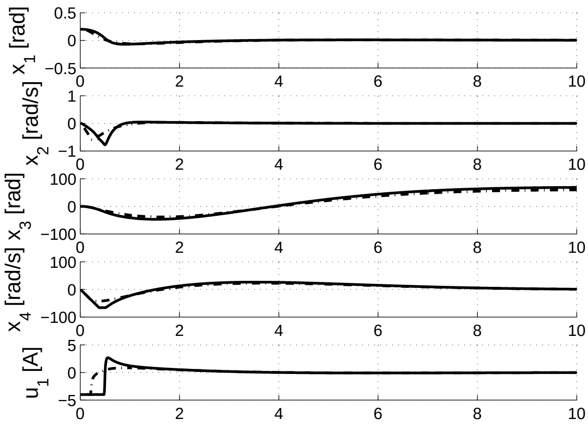

Fig. 7. Comparison of the behavior of the robot on the flat ground (solid line, $\alpha = 0rad$***) and inclined ground (dashed line,*** $\alpha = 0.79rad$***)***

based on the principles of rigid body physics. The final mathematical equations describe a complex system of inverted pendulum with reaction wheel.

The interesting part is connected with the description of torque acting on the handlebar. It is focused on the trail parameter and it explains why it is so important in the bicycle construction.

The simulation results show functioning of the mathematical model. The most important characteristics are shown. The simulations could be very helpful to construct the real machine.

The model does not contain information about the gyroscopic effect due to rotating reaction wheel. In nominal terms it can be negligible, because the angular velocity of the wheel should tends to zero value. However, wanting a more detailed description of the physical behaviour this effect should be taken into account. Another ignored factor is the reaction force from the ground when the robot travels on the rough surface. Such consideration could be highly interesting wanting to create a high level robot system.

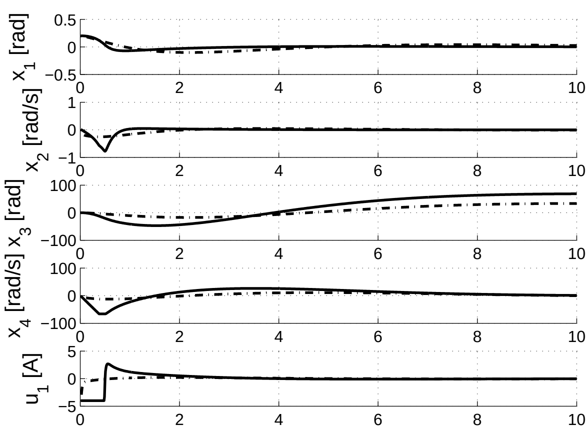

Fig. 8. Comparison of the behavior of the robot on the flat ground (solid line, $\alpha = 0[rad]$***) and inclined ground (dashed line,*** $\alpha = 1.48[rad]$***)***

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