Robust Performance of Sampled-data Adaptive Control of a Servo drive. From simulation to Experimental Results

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Abstract:

The paper considers robustness aspects of adaptive control in application to sampled-data systems with pole-placement controller subject to plant-model mismatch, in the sense of mistuning to model parameters with respect to their „true" values. The paper extends the results presented in author's previous work respecting comparison of sampled-data and discrete-time control systems to experimental results obtained from the control system of a servo drive. Firstly, the adaptive sampled-data controller is introduced and is applied by means of simulation, secondly, it is implemented in real-time control system to extend robust performance issues to real-world control systems. Finally, the regions of robust performance are shown on parameter surface, i.e. visualisation of parameters span for which there is no severe performance degradation in comparison to the best plant-model matching, as a function of sampling interval.

Keywords: *adaptive control, robustness, sampled-data systems, servo*

1. Introduction

Full knowledge of the plants allows one to design the control systems with parameters tuned using such laws as of Ziegler-Nichols for PID controllers. The proper tuning enables the control system to operate with high fidelity, good performance. Nevertheless, when the plant is not fully known, i.e. when there is a mismatch between „true" plant parameters and its model, or when plant parameters change in time, the tuning to fixed parameters becomes ambiguous, giving rise the need of adaptive control, see e.g. [3].

In the situation, when the identification procedure goes wrong, improper parameter estimates on the basis of certainty equivalence rule cause the controller to mistune, and the performance of the system deteriorates. One can address this situation from robust performance of adaptive control viewpoint, by checking, having chosen certain acceptable performance level, what parameter changes do not cause significant deterioration.

The paper addresses robustness issues for control of LTI continuous-time plant with unknown structure and parameters and sampled-data controller with simulation results shown for the mathematical model of the plant, as well as in the case of servo drive control.

Firstly, the plant description method is explained. Secondly, the control and identification issues in sampled-data systems are introduced. Furthermore, a case study for pole-placement control is given and, finally, the results concerning robustness issues in adaptive control of real-world control system are described.

For a discussion concerning new results in robustness of adaptive control see, e.g. [2].

2. Pole-placement Control

In order to illustrate the issues presented in the paper, the pole-placement control has been chosen, which aims at designing a controller to place the poles of a closed-loop system in prescribed locations for a discretised model (with discrete-time controller and discretised plant of known-structure) control system. Such a controller can be written in RST form, and control signal [3]

$$u_t = \left(1 - \hat{R}(q^{-1})\right) u_t - \hat{S}(q^{-1}) y_t + \hat{T}(q^{-1}) r_t \,, \quad (1)$$

where r_t is the reference signal to be tracked by y_t.

Having omitted estimate symbols, controller polynomials:

$$R(q^{-1}) = 1 + r_1 q^{-1} + \cdots + r_{nB+d-1} q^{-nB-d+1} \,, \quad (2)$$

$$S(q^{-1}) = s_0 + s_1 q^{-1} + \cdots + s_{nA-1} q^{-nA+1} \,. \quad (3)$$

On the basis of the latter polynomials and $A(q^{-1})$, $B(q^{-1})$ describing the model of the plant, known value of d and characteristic polynomial

$$A_M(q^{-1}) = 1 + a_{M,1} q^{-1} + \cdots + a_{M,nA_M} q^{-nA_M} \quad (4)$$

of the closed-loop system one can introduce a Diophantine equation

$$A(q^{-1})R(q^{-1}) + q^{-d} B(q^{-1}) S(q^{-1}) = A_M(q^{-1}) \,, \quad (5)$$

with

$$T(q^{-1}) = \frac{A_M(1)}{B(1)} \,. \quad (6)$$

By solving the Diophantine equation, one can obtain the controller polynomials, which, in turn, enable one to compute current value of the control signal. The discrete-time model of the closed-loop system

$$\frac{y_t}{r_t} = \frac{q^{-d} B(q^{-1}) T(q^{-1})}{A_M(q^{-1})} \,. \quad (7)$$

3. Experimental Setup

The experimental setup comprises DC motor (voltage $12\,\text{V}$, power $77\,\text{W}$, torque $250\,\text{mNm}$, speed $3\,000\,\text{rpm}$, current $4.7\,\text{A}$), tachogenerator and inertia load (brass cylinder, weight $2\,\text{kg}$, diameter $66\,\text{mm}$, length $68\,\text{mm}$), as shown in the Figure 1. The DC motor drives the inertia load and tachogenerator that is connected directly to the DC motor, with voltage proportional to the angular velocity $y(t) = \dot{\theta}(t)$.

The command input to the plant comes from input-output card used by Matlab's Real-Time Workshop and Simulink in order to work in real-time. C-mex S-functions have been used to implement the controller actions and estimation scheme that has been downloaded to the FPGA board. The control voltage $e_a(t)$ is limited to $\pm 12\,\text{V}$, and is presented in the paper in dimensionless form as $|u(t)| \leq 1$.

The considered servo has nonlinear static characteristic that has been compensated by its inverse, leading to linear system equations (in the case of no saturation), as derived below.

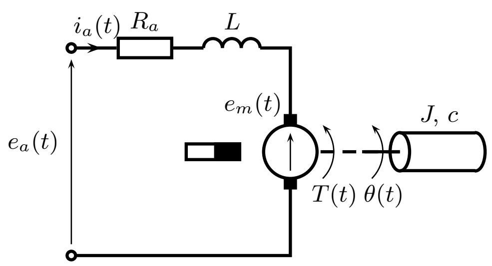

Fig. 1. Diagram of experimental setup

The armature loop equation (Fig. 1) is

$$e_a(t) = R_a i_a(t) + L\frac{di_a(t)}{dt} + e_m(t)\,. \tag{8}$$

Having assumed constant flux and electromagnetic force proportional to the angular velocity,

$$e_m(t) = k_e\dot{\theta}(t)\,, \tag{9}$$

and having performed the Laplace transform, the first equation can be put in a form

$$\begin{aligned} E_a(s) &= R_a I_a(s) + sLI_a(s) + sk_e\Theta(s) = \\ &= (R_a + sL)I_a(s) + k_e s\Theta(s)\,. \end{aligned} \tag{10}$$

The DC motor with constant flux has the electromechanical torque proportional to the armature current

$$T(t) = k_T i_a(t)\,. \tag{11}$$

The sought linear model can be derived assuming that static and kinetic frictions and saturation phenomena are neglected – using d'Alembert rule we get

$$T(t) = J\ddot{\theta}(t) + c\dot{\theta}(t)\,. \tag{12}$$

Combining the Laplace transform of the last two equations the following holds

$$k_T I_a(s) = (sJ + c)s\Theta(s)\,. \tag{13}$$

Using the equation including $E_a(s)$, $I_a(s)$ and $\Theta(s)$

$$\begin{aligned} k_T E_a(s) &= (sL + R_a)(sJ + c)s\Theta(s) + k_e k_T s\Theta(s) = \\ &= s\left((sL + R_a)(sJ + c) + k_e k_T\right)\Theta(s) \end{aligned} \tag{14}$$

one can write the following expression:

$$\begin{aligned} \frac{\Theta(s)}{E_a(s)} &= \frac{k_T}{s\left((sL + R_a)(sJ + c) + k_e k_T\right)} = \\ &\frac{k_T}{s\left(s^2 LJ + s(Lc + R_a J) + R_a c + k_e k_T\right)}\,. \end{aligned} \tag{15}$$

Since we are interested in angular, not position, control and armature inductance can be neglected, the final form of the „true" continuous-time model transfer function becomes

$$G(s) = \frac{k_T}{R_a J s + R_a c + k_e k_T} = \frac{k}{1 + sT} \tag{16}$$

for

$$k = \frac{k_T}{R_a c + k_e k_T}\,, \qquad T = \frac{R_a J}{R_a c + k_e k_T}\,. \tag{17}$$

In the real system, one can approximate the above parameters based on, e.g., step response of a plant, as $k = 169.2$, $T = 1.066\,\text{s}$.

Having performed step-invariant transform

$$G(z) = (1 - z^{-1})\mathscr{Z}\left\{\mathscr{L}^{-1}\left\{\frac{G(s)}{s}\right\}\Big|_{t:=tT_S}\right\}\,, \tag{18}$$

and substituting operator z with q, we can obtain discrete-time CARMA transfer function where q is a shift operator in time domain. In the considered case,

$$G(s) = \frac{k}{1 + sT}\,. \tag{19}$$

the „true" discrete-time model can be derived according to the formulae:

$$G(z) = \frac{z^{-1}k\left(1 - \exp^{-\frac{T_S}{T}}\right)}{1 - z^{-1}\exp^{-\frac{T_S}{T}}}\,, \tag{20}$$

$$A(q^{-1}) = 1 - q^{-1}\exp^{-\frac{T_S}{T}}\,, \tag{21}$$

$$B(q^{-1}) = k\left(1 - \exp^{-\frac{T_S}{T}}\right)\,, \tag{22}$$

$$d = 1\,. \tag{23}$$

4. Structural Identification Experiment

Because the information in sampled-data systems is passed on in discrete-time instants only, the sampling frequency is crucial for the performance of control system. As a convention, x_t denotes a signal sample at time t (discrete-time value), and $x(t)$ is a signal value at time t. Let x_{t-1} denote a previous sample from instant t shifted by backwards T_S.

The considered sampled-data adaptive control system can work well only when the model is correct. Let us consider the AIC criterion [5], which for the introduced CARMA model with $nA = nB + 1 = n$ and $d = 1$ is described as

$$\mathrm{AIC}(n) = \log \sigma_n^2 + \frac{4n}{N} \,. \tag{24}$$

It has to be stressed that model structures no other than CARMA have been taken into consideration, based on the initial knowledge from linear continuous-time plant model.

The symbol N in (24) is a number of collected samples, σ_n^2 is the prediction error variance computed for parameter estimates $\underline{\hat{\theta}}_{N,n}$ of n-th order model, i.e.

$$\sigma_n^2 = \sum_{t=n+1}^{N} \epsilon_t^2 \,, \tag{25}$$

where:

$$\epsilon_t = y_t - \underline{\varphi}_{t,n}^T \underline{\hat{\theta}}_{N,n} \,, \tag{26}$$

$$\underline{\varphi}_{t,n} = [-y_{t-1}, \ldots, -y_{t-n}, u_{t-1}, \ldots, u_{t-n}]^T . \tag{27}$$

The closed-loop system has been excited by a pseudo-random signal with a unit amplitude, appropriate samples have been collected every $T_S = 0.1$ s, and the same initial values of estimates have been chosen in the experiment equal to 0.01 and $N = 1\,000$. The presented results are mean values from three identical experiments.

Table 1 illustrates values of AIC criterion for $n_{\max} = 5$. The numbers in bold corresponds to estimates of orders for models in CARMA structure that minimize AIC criterion.

Tab. 1. AIC values

n	1	2	3	4	5
σ_n^2	$\mathbf{0.86}$	0.94	0.96	0.98	1.06
AIC(n)	$-\mathbf{0.12}$	-0.03	-0.01	0.03	0.08

The „true” order of a discrete-time model of the plant can be described in the sense of a minimal value of AIC criterion by a first-order model, as obtained previously, on the basis of theoretical derivations.

5. Experimental Results

5.1. Discrete-time Model of the Plant (Servo Drive)

If the block compensating the static nonlinearity of the plant is attached in series between the controller and the servo drive, then model of the „new” plant (servo drive with inertia module), based on an open-loop identification experiment, can be approximated as first-order transfer function given for $T_S = 0.1$ s as

$$\frac{y_t}{u_t} = \frac{15.75q^{-1}}{1 - 0.907q^{-1}} \,, \tag{28}$$

from which:

$$A(q^{-1}) = 1 - 0.907q^{-1} \,, \quad B(q^{-1}) = 15.75 \tag{29}$$

and $d = 1$.

In the previous works [1] the performance of adaptive sampled-data control has been investigated together with evaluation of the impact of sampling frequency on the control performance. Since a dominating time constant of the plant is approx. 1 s, sampling with at least 10 samples per dominating time constant is a reasonable choice.

As it has been reported in [1], changing the sampling frequency to two times faster then 10 Hz improves the area of robust stability (as well as robust performance, in the sense of the span of estimates of k and T for which the sampled-data system has performance no worse than prescribed level) to a very minor percent. The conclusion has been that there is no point in sampling faster, as with Shannon-Kotielnikov frequency. This paper aims to check this for real-world system related to the previously performed simulations.

By combining parameter estimation algorithm with pole-placement controller one can obtain sampled-data-time adaptive control system as in Figure 2.

In order to illustrate the performance of the system the continuous-time plant has been chosen as firs-order inertia with gain and time constant as previously reported for the purpose of simulation, and in the case of experiment, the servo drive with inertia load has been used, together with static nonlinearity compensator. It has been assumed, that controller has the knowledge of the degrees of discrete-time model polynomials, i.e. $nA = 1, nB = 0$ and $d = 1$. For such as model, the controller has been chosen, with the closed-loop dynamics as $\frac{0.1054}{s+0.1054}$ (defining $A_M(q^{-1})$ for different sampling frequencies), $T_S = 0.1$ s, $\rho = 100, \underline{\hat{\theta}}_0 = 0.5\underline{\theta}, \lambda = 1$.

As it can be seen in Figures 3a and b, having obtained good estimates, the control signals generated by the adaptive control law assure tracking the reference signal with chosen dynamics in both systems. The profiles of control signals are alike due to similar adaptation behaviour, as well as comparable tracking properties. The control performance of real-world system can be estimated by means of simulation of a sampled-data control system with continuous-time plant model. Nevertheless, it has to be checked if the information obtained from simulation is trustworthy and how conservative this approximation is, what is the main aim of the paper.

6. Robustness of Adaptive Sampled-data Control Against Model Uncertainty

Below the results of simulations of robustness analysis of a fully discrete-time/sampled-data (simulated/experimental) systems are presented, with pole-placement controller, against parametric uncertainty of plant model. The simulations have been conducted for different sample times $T_S = 0.05$ s, $T_S =$

$0.1\,\mathrm{s}$, $T_S = 0.2\,\mathrm{s}$ and continuous-time plant

$$G(s) = \frac{k}{1+sT} \qquad (30)$$

or a servo drive attached to the real-time controller. In order to check the robust performance of adaptive control, it has been assumed that parameters of the above plant are accessible as $\hat{k} = \delta_k k$, $\hat{T} = \delta_T T$ only and available as the estimates $\hat{a_1}$, $\hat{b}_0$ of the discrete-time model, according to the formula:

$$G(z) = \frac{z^{-1}\hat{k}\left(1 - \exp^{-\frac{T_S}{\hat{T}}}\right)}{1 - z^{-1}\exp^{-\frac{T_S}{\hat{T}}}}, \qquad (31)$$

$$\hat{A}(q^{-1}) = 1 - q^{-1}\exp^{-\frac{T_S}{\hat{T}}}, \qquad (32)$$

$$\hat{B}(q^{-1}) = \hat{k}\left(1 - \exp^{-\frac{T_S}{\hat{T}}}\right), \qquad (33)$$

$$d = 1, \qquad (34)$$

what should model imperfect action of identification algorithm.

For arbitrary chosen values δ_k i δ_T close to unity, the true plant parameters k i T are accessible as approximate values, thus model is not fully matched to the plant, what enables one to evaluate the impact of δ_k and δ_T on the requirement of keeping high (close to nominal) control performance in both simulated and experimental environments.

In order to evaluate the span of δ_k and δ_T that assures robust performance of the control system, a performance index is introduced, computed for a chosen simulation horizon (t_1, t_2) in which behaviour of the closed-loop system is repeatable for periodic reference signal

$$J = \frac{T_S}{t_2 - t_1 + 1}\sum_{t=t_1}^{t_2}(r_{M,t} - y_t)^2. \qquad (35)$$

The index J is computed as a sum-of-the-squares of deviations of output signal from reference model response to the same reference that is used to find $A_M(q^{-1})$.

For a span of δ_k and δ_T one can obtain performance index surface as a function of (δ_k, δ_T). By cutting this plot at a certain level one stipulates an acceptable tracking performance threshold, and lower values of J correspond to robust performance of the control system. In order to obtain comparable data, the threshold level has been chosen as squared value of acceptable steady-state tracking error for a square reference signal (i.e., 10% of reference amplitude). In the case of experiment, the maximum speed was $170\,\frac{\mathrm{rad}}{\mathrm{s}}$, what gave threshold at 289.

The transfer function $\frac{0.1054}{s+0.1054}$ has been chosen as a continuous-time reference model, and $A_M(q^{-1})$ equals $1 - 0.9487q^{-1}$ for $T_S = 0.05\,\mathrm{s}$, $1 - 0.9q^{-1}$ for $T_S = 0.1\,\mathrm{s}$ and $1 - 0.81q^{-1}$ for $T_S = 0.2\,\mathrm{s}$. In order to compare responses of discrete-time reference model with pole-placement control system, the nominator of discrete-time reference model is to be modified as $q^{-d}B(q^{-1})T(q^{-1})$, i.e. to the values corresponding to the nominator of closed-loop transfer function for $\delta_k = \delta_T = 1$.

In the case of a fully discrete-time system (thick solid line, taken from [1]) the robust performance region gets smaller at lower sampling frequencies. Should the estimates of k and T (i.e. indirectly $\hat{a}_1$ and $\hat{b}_0$ in the adaptive case) fall in the ellipsoid-like area, the performance of the system degrades gracefully. It can be seen, that the control system is more robust against time constant mismatch that gain mismatch. True values of plant are chosen as in the previous section. Sampled-data system (thick dashed line) behaviour can be approximated by behaviour of discrete-time control system but this approximation is conservative for fast sampling.

Real-world sampled data system (controlling the servo drive, thin solid line), including unmodeled dynamics, neglected phenomena and true plant, enables one to obtain experimental data, to verify the acceptance of the results of prior studies. As it can be seen from Figure 4a-c, the robust performance area for such a system is larger as it is suggested by its approximations. They tend to be too conservative, especially in the case of fast sampling, and less conservative in the case of violated sampling theorem conditions. The experiment depicted in Figure 4a shows why adaptive control is usually successful when applied to real plants at fast sampling. The area in which estimates may fall is the largest in this case.

This what is surprising, is the fact that the area of robust performance for higher sampling frequencies is larger for sampled-data system than it is suggested by the corresponding area for discrete-time system. One of the reasons might be that discrete-time systems are only approximation of real-world control systems.

7. Summary

As it has been presented in the paper, it is important to conduct simulations of both discrete-time and sampled-data systems in order to obtain any information (or approximation) of the behaviour of sampled-data systems with digital controllers. The latter is less expensive than real-world experiment and easier to perform. On the basis of the results presented, one can say that for both adaptive and non-adaptive control systems there is a good correspondence of results in the time domain.

It has been also shown that adaptive control is robust against plant mismodeling in the sense of lack of good information about its parameters. The presented results show that robust performance area expands when sampling frequency increases. The controller can use larger number of samples, in which tracking should be conducted with prescribed quality. One can also easily draw conclusions concerning properties of the closed-loop system with digital controller on the basis of simulation tests.

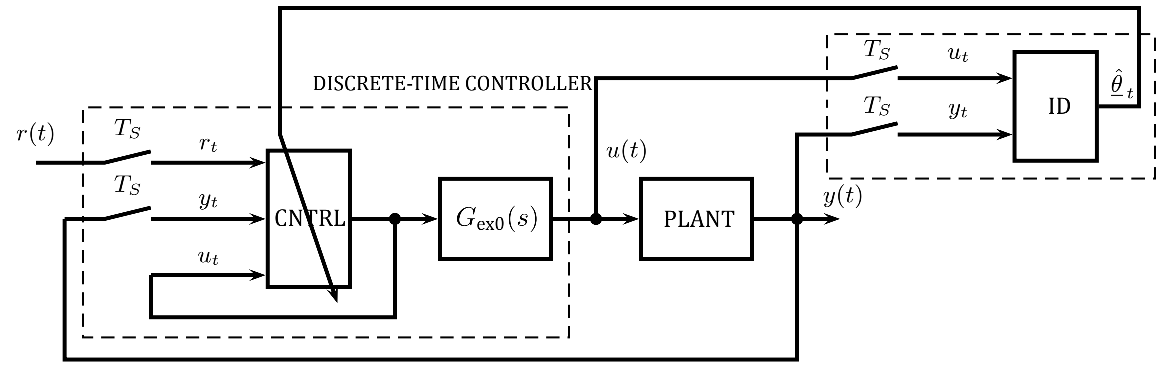

Fig. 2. Sampled-data adaptive control system

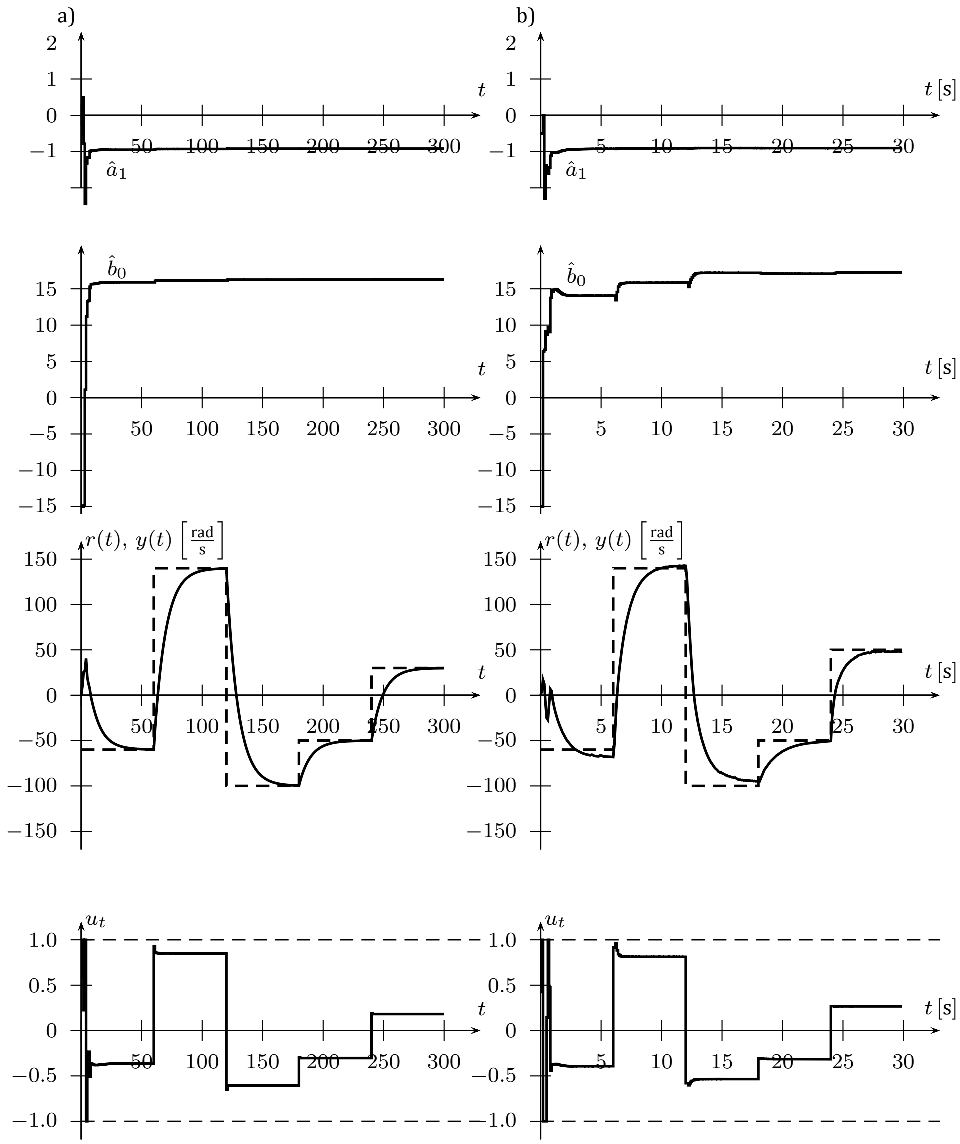

Fig. 3. Results of: a) simulation, b) experiment for the same reference profile

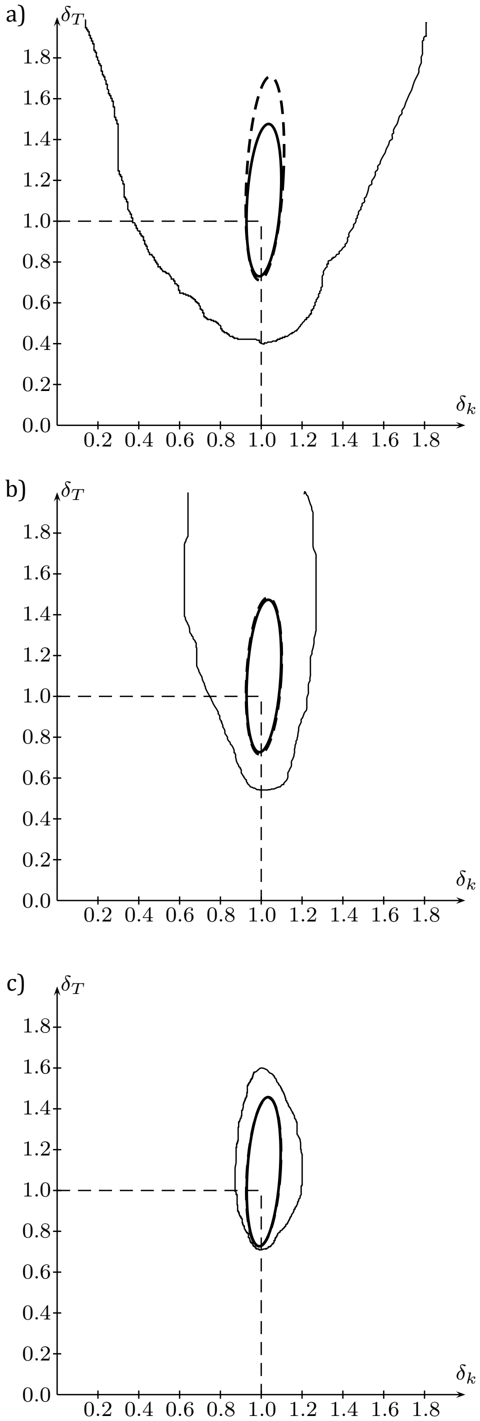

Fig. 4. Performance surfaces for a) $T_S = 0.05\,\mathrm{s}$*, b)* $T_S = 0.1\,\mathrm{s}$*, c)* $T_S = 0.2\,\mathrm{s}$

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