Reliability and Efficiency of Differential Evolution Based Method of Determination of Jiles-Atherton Model Parameters for X30Cr13 Corrosion Resisting Martensitic Steel

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Abstract:
Paper presents new possibilities of Jiles-Atherton model of magnetic hysteresis parameters determination. The main problem connected with this model is the fact that its parameters have to be determined during the optimization process. However, due to the local minima on the target function, the gradient optimization methods are not effective, whereas evolutionary strategies, such as (μ+λ) strategy, are very time consuming. Results of calculation presented in the paper indicate that differential strategies create possibility of reliable and fast determination of Jiles-Atherton model parameters. Paper also presents guidelines for practical determination of model's parameters, which is very important from practical point of view.

Keywords: *corrosion resisting steel, Jiles-Atherton model, magnetic hysteresis*

1. Introduction

Jiles-Atherton model of magnetic hysteresis was introduced in 1986. This early model uses 6 parameters; however, it considered only isotropic materials. In 1996, Jiles-Atherton model was extended by Ramesh et al. As a result of this extension, the possibility of effective modeling of the magnetic hysteresis loops of anisotropic materials was created. On the other hand, extended model utilizes 9 parameters of magnetic material.

The parameters of Jiles-Atherton model of magnetic hysteresis are clearly connected with physical properties of magnetic materials. However, it is not defined how to calculate value of each Jiles-Atherton model parameter on the base of measurements of physical properties of given sample of magnetic material. This is the main drawback of Jiles-Atherton model, limiting its usability in engineering applications as well as limiting possibility of verification of correctness of this model.

To overcome this problem, the different methods of determination of Jiles-Atherton model parameters were presented. In general, all methods utilize optimization algorithms, targeting the minimization of target function defined as a sum of squares of differences between experimental data and the results of modeling. However, the target function connected with Jiles-Atherton model exhibit many local minima. For this reason, the results of gradient optimization proposed previously [1] are strongly dependent on given starting point. Moreover, methods of estimation of starting point in Jiles-Atherton model are connected only with isotropic magnetic materials, which are rarely used in technical applications. For this reason, the new methods of determination of parameters of Jiles-Atherton model, on the base of experimentally measured magnetic hysteresis loops, are intensively developed. This paper presents the results of systematic experimental research on differential optimization as a very promising method for determination of Jiles-Atherton model's parameters.

2. Jiles-Atherton Model of Magnetic Hysteresis

Jiles-Atherton model of magnetic hysteresis is based on the idea of a hysteretic magnetization M_{ah}. While, at the beginning, such magnetization was determined only for isotropic materials as a modified Langevin function, recently it is known [2] and verified [3] for anisotropic materials. In this case, anisotropic anhysteretic magnetization M_{ah}^{aniso} is given by the following set of equations:

$$M_{ah}^{aniso} = M_s \left[\frac{\int_0^{\pi} e^{\frac{E(1)+E(2)}{2}} \sin\theta \cdot \cos\theta \cdot d\theta}{\int_0^{\pi} e^{\frac{E(1)+E(2)}{2}} \sin\theta \cdot d\theta} \right], \quad (1)$$

$$E(1) = \frac{H_e}{a}\cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a}\sin^2(\psi - \theta), \quad (2)$$

$$E(2) = \frac{H_e}{a}\cos\theta - \frac{K_{an}}{M_s \cdot \mu_0 \cdot a}\sin^2(\psi + \theta). \quad (3)$$

Where a quantifies domain wall density, M_s is saturation magnetization of the material, K_{an} is average anisotropy energy density, ψ is an angle between direction of magnetizing field H and anisotropy easy axis. Moreover, effective magnetizing field $H_e = H + \alpha M$ is calculated for the total magnetization M and inter-domain coupling α.

It should be stressed that for isotropic materials (where average anisotropy energy density $K_{an} = 0$ J/m^3) equation (1) reduces to modified Langevin equation:

$$M_{ah}^{iso} = M_s \left[\coth\left(\frac{H_e}{a}\right) - \left(\frac{a}{H_e}\right) \right] \quad (4)$$

presented in [4].

In some specific materials, such as constructional steels, isotropic and anisotropic phases can be mixed. In such case, the total anhysteretic magnetization is calculated from following equation [5]:

$$M_{ah} = t M_{ah}^{aniso} + (1+t) M_{ah}^{iso}, \quad (5)$$

where $t \in <0,1>$ describes participation of anisotropic phase in the material.

Magnetic hysteresis is introduced to Jiles-Atherton model by differential equation:

$$\frac{dM}{dH} = \frac{\delta_M}{1+c} \frac{M_{ah} - M}{\delta \cdot k - \alpha (M_{ah} - M)} + \frac{c}{1+c} \frac{dM_{ah}}{dH}, \quad (6)$$

where $c \in <0,1>$ describes the reversibility of magnetization process, whereas parameter k quantifies the average energy required to break pining site. Moreover, in equation (6), the parameter δ causes hysteretic magnetization due to the fact that it is equal to 1 during the increase of H and -1 during its decrease. It should be highlighted that parameter δ_M guarantees that incremental susceptibility is physically justified (not smaller than 0) [6].

It should be stressed that for diversified values of the amplitude of magnetizing field, the original Jiles-Atherton model guarantees the good agreement between experimental hysteresis loop and results of the modeling for single hysteresis loop [7]. For other values of the amplitude of magnetizing field, another set of model's parameters have to be determined. It was proven previously that to overcome this problem model parameter k should change during the each magnetization process. Changes of parameter k are connected with changes of the average energy required to break pining site for different values of material's magnetization [4].

Due to the fact that the magnetic state of the material is determined by the value of magnetization M, the changes of parameter k might be described by the following equation [7]:

$$k = k_0 + \frac{e^{k_2 \cdot (1 - |M|/Ms)} - 1}{e^{k_2} - 1} \cdot (k_1 - k_0), \quad (7)$$

where parameters k_0, k_1 and k_2 describe shape of function determining k as follow: k_0 is maximum value of k, k_1 is its minimum value and k_2 is the shape parameter of $k(M)$ function.

Table 1 presents the set of parameters of Jiles-Atherton model of anisotropic magnetic materials.

It should be stressed that to solve equation (6) numerical Runge-Kutta algorithm based methods has to be applied [8]. Moreover, due to the lack of antiderrivative for equation (1), Gauss-Kronrod method should be applied for numerical integration [9].

Table 1. The set of Jiles-Atherton parameters of material

Parameter	Units	Description
M_s	A/m	Saturation magnetization of the material
a	A/m	Quantifies domain wall density
α	–	Interdomain coupling
k	A/m	Quantifies average energy required to break the pinning site
k_0	A/m	Maximum value of k (if k described by eq. 7)
k_1	A/m	Minimum value of k (if k described by eq. 7)
k_2	–	Shape parameter of $k(M)$ function (if k described by eq. 7)
c	–	Magnetization reversibility
t	–	Participation of anisotropic phase
K_{an}	J/m^3	Magnetic anisotropy energy density

3. Experimental Results

Experimental measurements of magnetic hysteresis loops were done for X30Cr13 corrosion resisting martensitic steel. This steel is widely used in construction of energetic structures. As a result, it is often subjected to non-destructive testing procedures, also based on magnetic properties. For proper analyse of such results of non-destructive testing, the Jiles-Atherton model is often applied. For this reason the determination of model's parameters is required.

For effective measurements of B-H magnetic hysteresis loops, the samples with closed magnetic circuit should be used. For this reason, the frame-shaped samples made of X30Cr13 corrosion resisting martensitic steel were utilized [10]. In addition, such frame-shaped sample may be also used for testing of the stress dependence of magnetic B-H characteristics, which is especially important from the point of view of non-destructive testing applications [11].

Frame shaped sample made of X30Cr13 corrosion resisting martensitic steel was wound by magnetizing and sensing winding. Quasistatic B-H characteristics were tested using digitally controlled hysteresis graph HBPL2.

4. Methods of Determination of Model's Parameters

Since its introduction in 1984, the most sensitive part of Jiles-Atherton model was the method of determination of its parameters. In general, all methods of determination of model's parameters [12] are based on different types of minimisation of target function G given as:

$$G = \sum_{i=1}^{n} (M_{calcJ-A}(H_i) - M_{meas}(H_i))^2, \quad (8)$$

where $M_{calcJ\text{-}A}$ were the results of the modeling of magnetization and M_{meas} were the results of the experi-

mental measurements of magnetization (calculated from the flux density B), all for the given value H_i of magnetizing field. Moreover, Jiles-Atherton model's parameters were determined simultaneously for given number of B-H hysteresis loops (e.g. three hysteresis loops) measured for different values of the amplitude of magnetizing field H.

4.1. Gradient Optimization

An algorithms from the family of gradient optimization methods (covering linear gradient methods [13], Newton-like algorithms [14] or congruent gradients methods [15]), are known to be time efficient local search methods. However, as strictly local optimizers, all these methods stuck in local minima, which leads to useless solutions of Jiles-Atherton model parameters. As a result, gradient optimization may be used only for final adjustment of Jiles-Atherton model parameters around the local minima of target function G determined in other way.

4.2. Evolutionary Strategies

Evolutionary strategies are a set of stochastic algorithms suitable for optimization of continuous functions with local minima [16, 17]. Such strategies are based on simplified model of evolution. In case of such algorithm, from the population of N vectors representing the possible solutions, the set of μ parents is selected. On the base of parents, λ descendants are generated with the use of mutation and crossing-over operators [16]. Mutation operator is connected with the change of randomly selected value of descendant vector accordingly to the normal distribution, where the centre of this distribution is the previous value. Crossing-over operator creates descendant vector on the base of two parental vectors when part of vector is taken from one parent, whereas second part is taken from the second parent. Next, in the case of (μ+λ) strategy, from the set of μ parents and λ descendants, μ best vectors are selected and returned to the population. In the case of (μ, λ) strategy, this selection is made only among λ descendants vectors.

Both (μ+λ) and (μ, λ) are global search methods but the (μ+λ) was selected for finding parameters of Jiles-Atherton model because it is not losing promising results when they are identified [18].

4.3. Covariance Matrix Adaptation Evolution Strategy (CMA-ES)

Covariance Matrix Adaptation Evolution Strategy [19] is a successor of evolutionary strategies but unlike them, it does not sore, nor perturb set of solutions called population. In CMA-ES, population is represented by multivariate normal distribution. That distribution is represented by its mean and its symmetric and positive definite covariance matrix.

The algorithm is iterative. At each step, the distribution is sampled, and resulting solutions are assessed by objective function. After that, distribution mean and covariance matrix are updated. The algorithm can be fast for difficult problems with costly objective function evaluation because the number of evaluated individuals can be much smaller than in population-based algorithms. The CMA-ES is known to be a robust local search strategy, and because of its robustness, it was successfully applied for many global optimization problems.

4.4. Differential Evolution

The differential evolution algorithm (DE) [20] is a relatively new member of Evolutionary Algorithms family. To apply DE, the problem to be solved is encoded by a vector of real numbers called individual. Each element of individual is a value of design variable. The algorithm is iterative – at each iteration *t (generation),* group of N individuals (population) from *t-1* is perturbed, i.e. modified by genetic operators (mutation and crossover), then is assessed by objective function. It is assumed that this function should be minimized, i.e. smaller value means better solution. The algorithm stops after specified number of generations. The number of generations and population size N are parameters of the algorithm. The thing that distinguishes DE from the rest of the family is differential mutation operator.

During mutation phase, a temporary population is created form current population. Each i-th new individual v_i (i in <1, N>) is created by adding weighted difference between selected individuals to the third individual. That schema can be implemented in many ways. First of all individuals can be selected in many ways from the population. What is more, the information from those selected individuals can be combined in many ways. Therefore there exist many versions of DE algorithm. Following variants of DE were tested from the point of view of its usability for Jiles-Atherton model parameters estimation:

DE algorithm 1:

In the canonical version of the algorithm (known also as "DE /rand/1/bin"), a mutant v_i is generated by adding difference between two randomly selected solutions to the third randomly selected solution, i.e.:

$$v_i = x_{r0} + F(x_{r1} - x_{r2}), \tag{9}$$

where $F \in (0,1)$ is scale factor and it is a parameter of the algorithm. The process of v_i generation is sketched in Fig. 1.

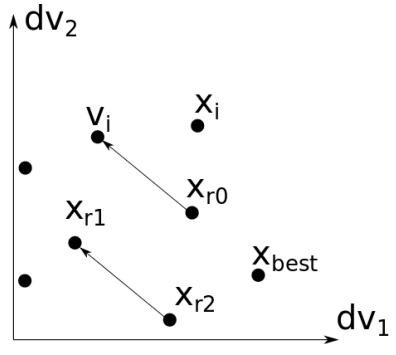

Fig. 1. Sketch of differential mutation procedure for DE's canonical schema (variant 1)

Bullets represent individuals in a space of two design variables (d_{v1}, d_{v2}), v_i is the resulting mutant that will compete with x_i to take i-th place in resulting population, x_{r0}, x_{r1}, x_{r2} are randomly drawn solution, x_{best} is the best solution in current generation. For sketch simplicity, it was assumed that F from equation (9) is equal to 1.

DE algorithm 2:

In variant called "DE/local-to-best/1/bin", the mutant v_i is a result of the sum of i-th solution x_i, difference of two randomly selected solutions and difference of the best solution in current population and x_i, i.e.:

$$v_i = x_i + F \cdot (x_{best} - x_i) + F \cdot (x_{r1} - x_{r2}) . \quad (10)$$

DE algorithm 3:

In the third variant called "DE/best/1/bin with jitter", new individual is calculated from:

$$v_i = x_{best} + jitter + F \cdot (x_{r1} - x_{r2}) , \quad (11)$$

where *jitter* is defined as 0.0001**rand*+F, where *rand* is real number uniformly drawn from <0, 1>.

DE algorithm 4:

In the fourth variant called "DE/rand/1/bin with per-vector-dither", a new parameter value is taken from values of three randomly drawn individuals:

$$v_i = x_{r0} + dither \cdot (x_{r1} - x_{r2}) , \quad (12)$$

where *dither* is defined as *F+rand*(1-F)*.

DE algorithm 5:

The fifth variant called "DE/rand/1/bin with per-generation-dither" is like fourth variant but *dither* is calculated only once per generation.

DE algorithm 6:

In the sixth variant called "DE/current-to--p-best/1" instead of the best solution an individual randomly selected from the set of 10**p***problem size* the best solutions is used (*pbest)*, where *p=0.2* is an algorithm parameter and *problem size* is the number of parameters to optimize:

$$v_i = x_i + F \cdot (x_{pbest} - x_i) + F \cdot (x_{r1} - x_{r2}) . \quad (13)$$

After mutation, a crossover is performed. For generation of each i-th trial individual u_i, a pair of individuals (x_i, v_i) is used, i.e. x_i – i-th individual form old population and v_i is i-th individual from temporary population. To perform crossover, first an index j of design variable is randomly drawn. Starting from that index a sequence of values from v_i is copied into u_i until uniformly drawn real number (from <0, 1>) is lesser than crossover probability (CR), which is a parameter of the algorithm. The rest of the u_i vector is copied from x_i. Therefore setting *CR=1* means that $u_i=v_i$.

After crossover, trial population is assessed by quality function. After that selection is performed. The i-th place in new population will be occupied by u_i, if its quality is not worse than quality of x_i, otherwise x_i will be used.

The model and search algorithm were implemented in R language [21]. In the presented research differential evolution algorithms implemented by Ardia et al. [22] were used. We accepted parameters of algorithm proposed by the implementation, i.e. stop after 200 generations, N=10*problem size, F=0.8, CF=0.5. The implementation uses DE-2 as default optimization method.

5. Results of Determination of Model's Parameters

5.1. Evolutionary Strategies

Evolutionary strategies, especially (μ+λ) evolutionary strategy were successfully applied previously for determination of Jiles-Atherton model parameters [23]. Such methods of optimization are able to escape from local minima. However, application of evolutionary strategy is time consuming. For presented research 70 iterations of (μ+λ) evolutionary strategy were performed, where population N was equal 900, number of parents μ was equal to 3 and number of offspring λ was equal to 12. In such case, 252 900 evaluations of target function were required. After (μ+λ) optimization, process value of target function G equal 0.401 was reached. The time run of calculation was 32 hours at the single node of Halo2 computer cluster (AMD Opteron 6272).

5.2. Covariance Matrix Adaptation Evolution Strategy (CMA-ES)

CMA-ES is known to be a robust local search strategy. Practical tests indicated that it is not suitable for minimization of G function determining quality of parameters of Jiles-Atherton model. It is results did not lead do acceptable solutions and they were strongly dependent on initial conditions (staring point). For these reasons, we resigned from further research on application of CMA-ES for minimization of G function.

5.3. Differential Evolution

To test the efficiency of the differential evolution for determination of the Jiles-Atherton model, the set of 25 optimization processes were carried out for each differential evolution algorithm. Moreover, algorithms were tested for both constant value of model's parameter k as well as for k(M) given by the equation (7). Table 2 presents the results of the tests of efficiency of optimization process for constant value of k, whereas table 3 presents similar results for k given by equation (7).

The number of function evaluations for each algorithm in table 2 was set to 10 000, whereas for algorithms shown in table 3 it was set to 14 000. The result of tuning algorithm's crossover ratio parameter (CR) is presented in table 4 for constant k, and in table 5 for k(M). We used only DE-2 for that tests because this algorithm was chosen to be default in used optimization package, which means that it was recognized to be good choice for various optimization tasks. For our task, according to tables 2 and 3, DE-2 and DE-3 are comparable and they are noticeably better than the rest of investigated algorithms.

Table 2. Result of test of the efficiency of differential evolution algorithm for 25 runs of optimization (constant value of k parameter, Mean G – average value of result target function, Best G – best value of target function, σ(G) – standard deviation of value G in 25 runs)

Algorithm	Mean *G*	Best *G*	*σ(G)*
DE-1	0.566	0.503	0.043
DE-2	0.507	**0.469**	0.026
DE-3	0.495	**0.460**	0.031
DE-4	0.606	0.509	0.054
DE-5	0.599	0.536	0.043
DE-6	0.622	0.503	0.062

Table 3. Result of tests of the efficiency of differential evolution algorithm for 25 runs of optimization (parameter k given by equation 7)

Algorithm	Mean *G*	Best *G*	*σ(G)*
DE-1	0.584	0.446	0.074
DE-2	0.476	**0.405**	0.042
DE-3	0.473	**0.408**	0.047
DE-4	0.591	0.430	0.092
DE-5	0.571	0.429	0.099
DE-6	0.605	0.431	0.127

Table 4. Result of tests of the efficiency of DE-2 with different crossover ratio (CR). 25 independent runs of optimization were performed, constant value of k

Crossover (CR)	Mean *G*	Best *G*	*σ(G)*
0.5	0.507	0.469	0.026
0.7	0.461	0.450	0.004
1	**0.458**	0.458	0.000

Table 5. Result of tests of the efficiency of DE-2 with different crossover ratio (CR). 25 independent runs of optimization were performed, parameter k was given by equation 7

Crossover (CR)	Mean *G*	Best *G*	*σ(G)*
0.5	0.476	0.405	0,042
0.7	0.409	0.373	0.020
1	**0.383**	**0.346**	0.013

According to tables 4 and 5, increasing CR from default 0.5 gives better average results and smaller standard deviation of the results. Therefore, after setting CR=1 it is highly probable to achieve solution that is close to the best possible in a single run of the algorithm.

The algorithm was run on single processor on Intel(R) Xeon(R) 3.50GHz CPU. The time of one run was about 26 minutes for problems in which *k* was a function of M, and 16 minutes for constant *k*.

6. Conclusion

From the practitioner's point of view, it is desired to achieve good result by the use of of-the-shelf optimization algorithm, i.e. without parameter tuning. For our problem, the optimizer's default algorithm DE-2 proved to be one of the best performing algorithms. Unfortunately, further tests revealed that default setting of crossover ratio (CR) gave substantially worse results than results achieved after tuning of the CR value. Therefore, to achieve the best possible results, still there is a need for parameter tuning. Fortunately, within the progress of developing modern optimization methods, the number of parameters that needs tuning decreases.

Presented results indicate that differential optimization algorithms can be used to determine Jiles-Atherton model parameters, and they are about 20 times faster than previously used evolutionary strategies.

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