An Adequate Mathematical Model of Four-rotor Flying Robot in the Context of Control Simulations

Submitted: 23rd April 2013; accepted: 26th July 2013

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DOI: 10.14313/JAMRIS_2-2014/13

Abstract:
In this paper a model of the dynamics of four-rotor flying robot is described in details. Control design must be preceded by the modeling and subsequent analysis of the robot behavior in simulator. It is therefore necessary to develop the mathematical model as accurate as it is possible. The paper contains a detailed derivation of the mathematical model in the context of physics laws affecting the quadrocopter. The novelty of presented notation is an extention of Coriolis forces in linear acceleration and the gyroscopic effect on angular acceleration. In the validation phase, the mathematical model was verified with the use of proposed control algorithms. Simulation studies have demonstrated the adequacy of a MATLAB model to properly reflect the real quadrocopter dynamics. This would allow for its use in the simulator and afterwards to implement and verify of control laws on the real four-rotor flying robot.

Keywords: *unmanned aerial vehicle (UAV), quadrocopter, modeling, mathematical model*

1. Introduction

Understanding and control at the acceptable level of physical processes is possible due to proper modeling. By bringing together all the essential process features against the background of non-essential ones and the use of mathematics as well as computer, it is possible to build simulators to evaluate the more complex system properties such as dynamics. In the modeling phase of engineering work, in addition to the added value which is a new model and a possibility of acquiring a new knowledge through experimenting with it, modeling allows also for large savings on experiments costs. Studies on the model are repeatable and time-saving. They allow also for the introduction of certain extra conditions to be simulated. Finally, the results of simulation studies, which are normally available before the experiment, may be more easily established, described, and archived. To develop efficient and rapid control algorithms for unmanned flying vehicles having complex structure, good quality mathematical models, which reliably reflect the dynamics of these vehicles, are needed. In this article authors focused their attention on selected, four-rotor flying robot - called quadrocopter, which for the correct operation requires an appropriate control strategy. Naturally, proper control may be developed by experimental tests - on the real object, or by the use of simulator - which is a dedicated computer program. The first approach (physical modeling) is expensive and it is associated with tests carried out directly on the system hardware. The second way to its use requires an availability of the mathematical model. There are already many dynamics models focused on the four-rotor flying robot. They differ in the complexity and the level of the adopted simplifications. It is important that quadrocopter is a nonlinear dynamical object with multiple inputs and outputs. It is inherently unstable and its parameters are non-stationary in time. Of course, it would be beneficial to know all the necessary physical parameters of the quadrocopter's elements a priori [2], but without a wind tunnel experiment - it is impossible to directly evaluate quadrocopter's geometry, dynamics of its propellers, or other relevant aerodynamic parameters. The particular literature studies [4] let to draw the following conclusion: "To overcome the inflexibility of the complex models, the National Aeronautics and Space Administration (NASA) has developed a so-called Minimum-Complexity Helicopter Simulation Math Model (MCHSMM) [3], which is a math model depending only on the basic data sources with the intention of low cost real-time simulation possibilities (...). One of the additional benefits from a MCHSMM, is the potential for a more clear understanding of the helicopter and its dynamics in general." On the basis of MCHSMM, in further works, e.g. [4] there was a strive to develop methods for robust and optimal control of a robot based on this model. However, in the opinion of authors of this paper, too far-reaching simplifications narrow the spectrum of control methods used. While the robust control in its idea compensates modeling errors and guarantees stability [11], such a control is not optimal, which is not without the significance from the perspective of energy expenditure (as robot batteries allow up to tens of minutes of flight). On the other hand most often optimal control is associated with considerable amount of computing [8], which makes it difficult to perform the relevant real-time calculations in the case of robot's on-board control unit. Thus it would be preferable to use the adaptive control methods [8], among which efficient methods of low computational complexity are available. Therefore, authors postulated a compromise, i.e. to create a mathematical model of the certain simplified robot structure (solid body), but taking into account all relevant physical phenomena, which the robot is subjected to, and which are often not included in other commonly used models, such as introduction of relationship describing the linear acceleration due to Coriolis and angular acceleration due to the gyroscopic effect. In this way, this article is a supplement to previous approaches [10], [6], contributing to more faithfully reflect the dynamics of the robot.

2. Quadrocopter – Physical Object

Quadrocopters differ in technical details, but have one common feature: the design of a system is based on four rotors powered by DC motors and supplied by battery (Fig. 1). The layout of the quadrocopter is discussed in details in [5]. The use of such a structure reduces the instrumentation costs by eliminating a variable rotor blades compared to the classic solution used in helicopters. In this way an increased stability and a relatively large capacity of the flying robot can be achieved.

The robot's avionics consists of the following on-board sensors: inertial measurement unit (IMU) - used for the stabilization of the robot's position and orientation, an altimeter for the height stabilization, a thermometer and a power consumption meter [5]. The IMU contains three gyroscopes and triaxial accelerometer. This sensor communicates with on board processor via the SPI bus. Gyroscopes are used to determine the orientation of the robot in space (relative to the base). The accelerometers are used to reset the gyro drift.

In quadrocopters, currently powertrain consisting of a speed controller, brushless motor and propeller, are mostly used. Their task is to generate a lift force and appropriate power steering. These are essential features from the perspective of robot motion control. The propeller used in each unit is responsible for the generation of the lift force and it is rotating in one direction, which is due to a constant pitch of the blade. For this reason, the controller is used only to modulate its speed, and it is not responsible for the direction of the propeller rotation.

Fig. 1. Quadrocopter Hornet

3. Mathematical Model – Reference Frames

During mathematical modeling (for the general quadrocopter's geometric model form Figure 4), the physical parameters are described in three different reference systems (or with respect to these systems), i.e. the Earth system (EF), the robot system (local- BF) and auxiliary system (SF). The relationship between

Fig. 2. Quadrocopter's geometric model

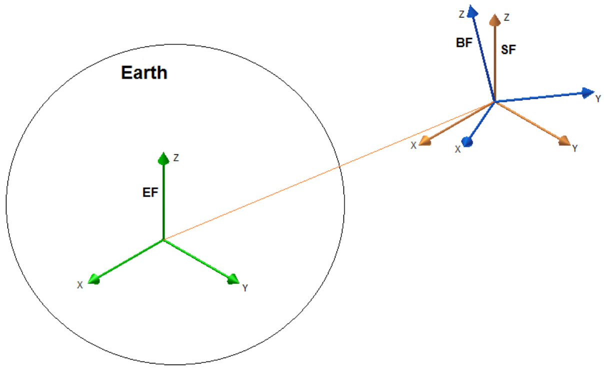

Fig. 3. Reference systems

EF, BF and SF are shown in Figure 3. System SF is a virtual system whose aim is to adjust the orientation of the global EF system to the local BF system. In the notation of the mathematical model all physical quantities except the Euler angles are expressed in the local coordinate system BF. Euler angles are the angles between the BF and SF (or EF) frame.

4. Mathematical Model, Equations

4.1. Euler angles

Euler angles define the orientation of the robot's frame (BF system) relative to the global system - the Earth EF [9]. During the following calculations we will apply the standard rotation matrices (transformations): matrix of rotation in the axis X about angle ϕ:

$$C_X(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & sin\phi \\ 0 & -sin\phi & cos\phi \end{bmatrix}, \tag{1}$$

matrix of rotation in the axis Y about angle θ:

$$C_Y(\theta) = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \tag{2}$$

and matrix of rotation in the axis Z about angle ψ:

$$C_Z(\psi) = \begin{bmatrix} cos\psi & sin\psi & 0 \\ -sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (3)$$

While determining the transformation matrix, that allows to translate the system SF to BF, authors follow the principle of rotation sequence "3-2-1" [7]. This sequence assumes that during rotations the following steps are performed:

- Rotation *Yaw*; about angle ψ in the axis Z of the local coordinate system (BF)
- Rotation *Pitch*; about angle θ in the axis Y of the new local coordinate system (BF')
- Rotation *Roll*; about angle ϕ in the axis X of the new local coordinate system (BF'')

Under this assumption and taking into account the matrices: (1), (2), (3), the total transformation matrix form the SF to the BF system may be written:

$$R_B^S(\underline{\Theta}) = C_X(\phi) \cdot C_Y(\theta) \cdot C_Z(\psi) = \quad (4)$$

$$\begin{bmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\phi s\theta c\psi - c\phi s\psi & s\phi s\theta s\psi + c\phi c\psi & s\phi c\theta \\ c\phi s\theta c\psi + s\phi s\psi & c\phi s\theta s\psi - s\phi c\psi & c\phi c\theta \end{bmatrix}, \quad (5)$$

where s and c stands for sin and cos respectively.

In the next step, knowledge about the transformation from BF to the SF $(R_S^B(\underline{\Theta}))$ is required, therefore the inverse of the matrix $R_B^S(\underline{\Theta})$ must be calculated. However, this is an orthonormal matrix, so its inverse is equal to the transpose of this matrix:

$$R_S^B(\underline{\Theta}) = (R_B^S(\underline{\Theta}))^{-1} = (R_B^S(\underline{\Theta}))^T \quad (6)$$

Hence:

$$R_S^B(\underline{\Theta}) = \quad (7)$$

$$\begin{bmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & c\phi s\theta c\psi + s\phi s\psi \\ c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{bmatrix} \quad (8)$$

In the notation above as well as in (5) vector $\underline{\Theta}$ consists of Euler angles:

$$\underline{\Theta} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \quad (9)$$

In the further part of this work, the vector of Euler angles will be used for the purpose of defining the transformation matrices.

4.2. Angular velocities

In the first point of the description of the mathematical model (Section 4.1), the model assumption and a description of Euler's notation (Chapter 3) have been presented. Euler angles also constitutes the base for the notation of angular velocities of the quadrorotor. The Euler rates (vector $\dot{\underline{\Theta}}$) are projections of the robot's angular velocities (in the local coordinate system BF) onto the axes of the SF coordinate system. This is an alternative notation in relation to the notation in the system BF (vector $\underline{\Omega}$). This calculation is necessary, because measurements are performed by gyroscopes, which refer to the robot's frame. Thus, in order to calculate the frequency of Euler's speed (Euler rates) projection of the system BF $(\underline{\Omega})$ on axes of the SF coordinate system must be made. Algorithm for the Euler rates calculation has been defined by [9]. The first step is the theoretical calculation of the angular velocity in the local coordinate system:

$$\underline{\Omega} = \begin{bmatrix} \omega_X \\ \omega_Y \\ \omega_Z \end{bmatrix} = \quad (10)$$

$$\begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + C_X(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + C_X(\phi)C_Y(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}, \quad (11)$$

which may be written in a matrix form:

$$\underline{\Omega} = P_B^S \cdot \dot{\underline{\Theta}}, \quad (12)$$

where:

$$P_B^S = \begin{bmatrix} 1 & 0 & -sin\theta \\ 0 & cos\phi & sin\phi cos\theta \\ 0 & -sin\phi & cos\phi cos\theta \end{bmatrix} \quad (13)$$

The next step is to calculate the inverse of the matrix P_B^S:

$$P_S^B = (P_B^S)^{-1} = \begin{bmatrix} 1 & sin\phi tg\theta & cos\phi tg\theta \\ 0 & cos\phi & -sin\phi \\ 0 & \frac{sin\phi}{cos\theta)} & \frac{cos\phi}{cos\theta} \end{bmatrix} \quad (14)$$

Finally, having already P_S^B matrix, which depends on the angular speed of the local coordinate system, Euler rates may be defined:

$$\dot{\underline{\Theta}} = P_S^B \cdot \underline{\Omega} \quad (15)$$

4.3. Angular acceleration

One of the simplifications is the assumption of robot's structure perfect stiffness. On the basis of this simplification, the description of angular acceleration of the robot may be determined in the same way as for the rigid body in space. Mathematical description for this effect is possible thanks to Euler equation of a rigid body motion [1]. The general form of the Euler equation may be defined as:

$$\frac{d\underline{L}}{dt} + \underline{\Omega} \times \underline{L} = \underline{\tau}, \quad (16)$$

where:
$\underline{L}$ - angular momentum vector
$\underline{\tau}$ - vector of input torques
$\underline{\Omega}$ - vector of angular velocity

For a rigid body is also true that:

$$\underline{L} = I \cdot \underline{\Omega}, \quad (17)$$

where I is the matrix of inertia moments of an object [7]:

$$I = \begin{bmatrix} I_X & 0 & 0 \\ 0 & I_Y & 0 \\ 0 & 0 & I_Z \end{bmatrix} \quad (18)$$

Substituting (17) to (16) (assuming the constancy of inertia moment) following equation may be obtained:

$$\underline{\tau} = I \cdot \underline{\dot{\Omega}} + \underline{\Omega} \times (I \cdot \underline{\Omega}) \tag{19}$$

This equation, however, due to the gyroscopic effect caused by the motors and rotors, must be modified.
Gyroscopic effect is described by the basic equation of the gyroscope:

$$\underline{\tau}_{gyro} = \underline{\Omega}_p \times \underline{L}_p, \tag{20}$$

where:
$\underline{L}_p$ – vector of angular momentum of rotors
$\underline{\tau}_{gyro}$ – vector of input torques
$\underline{\Omega}_p$ – precession angular velocity vector

The effect of gyroscopic precession motion manifests itself on the object, which is equipped with a rotating mass. It is therefore true that:

$$\underline{\Omega}_p = \underline{\Omega}, \tag{21}$$

where: $\underline{\Omega}$ is the angular velocity vector of the robot in the local coordinate system BF.
Angular momentum vector is formed by multiplying the moment of inertia of the rotor (I_w) by the rotor's angular velocity about the axis where the rotation occurs ($\underline{\Omega}_w$):

$$\underline{L}_p = I_w \cdot \underline{\Omega}_w = \begin{bmatrix} I_w \cdot \omega_{w_x} \\ I_w \cdot \omega_{w_y} \\ I_w \cdot \omega_{w_z} \end{bmatrix} = \tag{22}$$

$$\begin{bmatrix} 0 \\ 0 \\ I_w \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) \end{bmatrix} \tag{23}$$

It must be noted that the rotor angular velocity vector ($\underline{\Omega}_w$) contains zeros for the components in the axes of X, Y and the sum of the rotor angular velocity ω_i (where i varies in the range of $< 1,4 >$) in Z-axis. This is because rotors axes are parallel to the Z axis in the BF system, which is valid in this description of gyroscopic effect.

Knowing the $\underline{L}_p$ and $\underline{\Omega}_p$, equation (20) may be expanded:

$$\underline{\tau}_{gyro} = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ I_w(\omega_1 + \omega_2 + \omega_3 + \omega_4) \end{bmatrix} = \tag{24}$$

$$\begin{bmatrix} \omega_y I_w(\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ -\omega_x I_w(\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ 0 \end{bmatrix} \tag{25}$$

With the final form of gyroscopic torques vector, Euler's equation for the object (16) may be upgraded:

$$\underline{\tau} = I \cdot \underline{\dot{\Omega}} + \underline{\Omega} \times (I \cdot \underline{\Omega}) + \underline{\tau}_{gyro} \tag{26}$$

In this equation the resulting quantity is the angular acceleration, so after transformations it may be written:

$$\underline{\dot{\Omega}} = I^{-1}(\underline{\tau} - \underline{\Omega} \times (I \cdot \underline{\Omega}) - \underline{\tau}_{gyro}), \tag{27}$$

which, after total multiplication of all components, leads to the form:

$$\underline{\dot{\Omega}} = \begin{bmatrix} \frac{1}{I_X}(\tau_x - \omega_y \omega_z (I_z - I_y)) \\ \frac{1}{I_Y}(\tau_y - \omega_x \omega_z (I_x - I_z)) \\ \frac{1}{I_Z}(\tau_z - \omega_x \omega_y (I_y - I_x)) \end{bmatrix} + \tag{28}$$

$$+ \begin{bmatrix} -\omega_y I_W(\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ -\omega_x I_W(\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ 0 \end{bmatrix} \tag{29}$$

4.4. Linear acceleration

Similar to the case of angular accelerations, linear accelerations are also calculated in the local coordinate system BF. This is because all forces and torques operate in the local coordinate system - directly on the robot.
The general form of the equation for linear acceleration may be written in the form of the second law of dynamics:

$$m\underline{a} = \underline{F} - \underline{F}_g - \underline{F}_{co} - \underline{F}_{cw}, \tag{30}$$

where:
m - total mass of the robot,
$\underline{a}$ - accelerations vector of the robot,
$\underline{F}$ - active force vector in the local coordinate system,
$\underline{F}_g$ - gravitational forces vector in local coordinate system,
$\underline{F}_{co}$ - Coriolis forces vector due to rotations of the entire robot,
$\underline{F}_{cw}$ - Coriolis forces vector due to rotors rotations of the robot,
As may be seen above, this equation consists of a constraint, which is the active force decreased by the gravitational force and the Coriolis force.
The first of Coriolis forces results from the rotation of the entire robot:

$$\underline{F}_{co} = 2m \cdot \underline{\Omega} \times \underline{v} = 2m \cdot \begin{bmatrix} v_z \omega_y - v_y \omega_z \\ v_x \omega_z - v_z \omega_x \\ v_y \omega_x - v_x \omega_y \end{bmatrix}, \tag{31}$$

while the second Coriolis force is due to the rotation of rotors:

$$\underline{F}_{cw} = 2m_w \cdot \underline{\Omega}_w \times \underline{v} = \tag{32}$$

$$2m_w \cdot \begin{bmatrix} -v_y \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ v_x \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ 0 \end{bmatrix}, \tag{33}$$

where mass of the entire object is equal to m, while the mass of propellers and rotors is equal to m_w.

In equation (30) the force of gravity should also be explained - it is a projection of force from the system SF to the system BF:

$$\underline{F}_g = R_B^S(\underline{\Theta}) \cdot \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = \begin{bmatrix} -sin\theta mg \\ sin\phi cos\theta mg \\ cos\phi cos\theta mg \end{bmatrix} \tag{34}$$

Having already all components, the equation (30) may be expanded to its extended final form:

$$\underline{a} = \frac{1}{m}\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} - \begin{bmatrix} -gsin\theta \\ gsin\phi cos\theta \\ gcos\phi cos\theta \end{bmatrix} + \tag{35}$$

$$-2\begin{bmatrix} v_z\omega_y - v_y\omega_z \\ v_x\omega_z - v_z\omega_x \\ v_y\omega_x - v_x\omega_y \end{bmatrix} + \tag{36}$$

$$-2\frac{m_w}{m}\begin{bmatrix} -v_y \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ v_x \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4) \\ 0 \end{bmatrix} \tag{37}$$

5. Forces and Forcing Torques

In the mathematical transformations derived above, two forcing vectors were used, namely the forces vector $\underline{F}$ and the torques vector $\underline{\tau}$. However, these physical quantities were not yet specified. To do so, the lift force of the quadrocopter must be determined. This force is equal to the product of the sum of squared rotors angular velocities and the gain factor b.

$$F = b(w_1^2 + w_2^2 + w_3^2 + w_4^2) \tag{38}$$

Because this force acts only in the Z axis direction

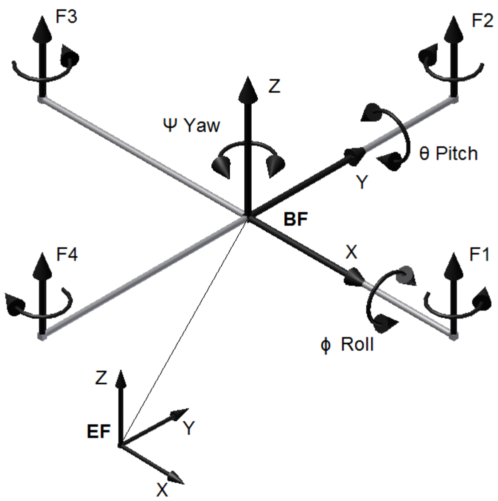

Fig. 4. Coordinate systems on the robot

(Figure 4) of the BF local system, therefore it may be written as:

$$\underline{F} = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} \tag{39}$$

Another issue is the generation of moments of forces, which arise only in the case of unbalanced rotation speeds of rotors. That is when the:

$$(w_1 + w_2 + w_3 + w_4) \neq 0 \tag{40}$$

The equation of generated moments may be written as:

$$\underline{\tau} = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} l \cdot b(\omega_4^2 - \omega_2^2) \\ l \cdot b(\omega_3^2 - \omega_1^2) \\ d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \end{bmatrix}, \tag{41}$$

where l is equal to arm force (length of the quadrocopter arm), while d refers to the reaction torque gain coefficient in the Z axis direction.

6. The Complete Model

Having the successive elements of model and forces, one may write a complete model of a flying object. The first part refers to a linear acceleration, which is defined in the BF local coordinate system:

$$\underline{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \tag{42}$$

$$\begin{bmatrix} gsin\theta \\ -gsin\phi cos\theta \\ -gcos\phi cos\theta \end{bmatrix} + \begin{bmatrix} -2(v_z\omega_y - v_y\omega_z) \\ -2(v_x\omega_z - v_z\omega_x) \\ -2(v_y\omega_x - v_x\omega_y) \end{bmatrix} + \tag{43}$$

$$+\begin{bmatrix} 2\frac{m_w}{m}(v_y \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ -2\frac{m_w}{m}(v_x \cdot (\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ 0 \end{bmatrix} + \tag{44}$$

$$+\begin{bmatrix} 0 \\ 0 \\ \frac{1}{m}b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \end{bmatrix} \tag{45}$$

The second part are Euler rates, which will be used in order to determine changes of robot's BF local system orientation ralative to the auxiliary system SF:

$$\underline{\dot{\Theta}} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \tag{46}$$

$$\begin{bmatrix} \omega_x + \omega_y sin\phi tg\theta + \omega_z cos\phi tg\theta \\ \omega_y cos\phi - \omega_z sin\phi \\ \omega_y \frac{sin\phi}{cos\theta} + \omega_z \frac{cos\phi}{cos\theta} \end{bmatrix} \tag{47}$$

The last part refers directly to angular accelerations:

$$\underline{\dot{\Omega}} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \tag{48}$$

$$\begin{bmatrix} \frac{1}{I_X} l \cdot b(\omega_4^2 - \omega_2^2) \\ \frac{1}{I_Y} l \cdot b(\omega_3^2 - \omega_1^2) \\ \frac{1}{I_Z} d(\omega_2^2 + \omega_4^2 - \omega_1^2 - \omega_3^2) \end{bmatrix} \tag{49}$$

$$\begin{bmatrix} -\frac{1}{I_X}(\omega_y\omega_z(I_z - I_y)) \\ -\frac{1}{I_Y}(\omega_x\omega_z(I_x - I_z)) \\ -\frac{1}{I_Z}(\omega_x\omega_y(I_y - I_x)) \end{bmatrix} \tag{50}$$

$$\begin{bmatrix} -\frac{1}{I_X}(\omega_y I_w(\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ -\frac{1}{I_Y}(\omega_x I_w(\omega_1 + \omega_2 + \omega_3 + \omega_4)) \\ 0 \end{bmatrix} \tag{51}$$

In this model, both the vector $\underline{a}$ as well as the vector $\underline{\dot{\Omega}}$ describe the relationships which exist in the local coordinate system BF. Euler rates vector $\underline{\dot{\Theta}}$ describes the relationship between systems: BF and SF.

7. The Results of Simulation Tests

The mathematical model of the quadrocopter, described above, was implemented in the MATLAB environment for testing and to assess the impact of the dynamics on the behavior of real four-rotor flying robot. Simulation tests allowed to verify the flight trajectory of the robot at various control signal levels. The most important advantage in this case was the ability to analyze the object stability to the different external disturbances, which in the work with a real robot equipped with several sensors to record flight, in general might easily expose him to crash. Implementation and simulation tests were preceded by the identification of physical quantities characterizing parameters of the robot from the Figure 2 in the general model (42) - (51). According to the notation (42) - (51), transparency of the model is guaranteed by the knowledge of: a mass of the propeller-rotor (m_w), total mass of the robot (m), moment of inertia in X, Y, Z axes (respectively I_x, I_y, I_z, I_w), as well as rotor coefficients b and d and the arm length l. For the purpose of further simulations, appropriate measurements were done and numerical values of coefficients were obtained:

- total mass of the robot: $m = 2.3\ [kg]$
- mass of the rotor: $m_w = 0.01\ [kg]$
- arm length of the robot: $l = 0.2825\ [m]$
- moment of inertia in the X axis: $I_x = 0.0250\ [kg{\cdot}m^2]$
- moment of inertia in the Y axis: $I_y = 0.0230\ [kg{\cdot}m^2]$
- moment of inertia in the Z axis: $I_z = 0.0475\ [kg{\cdot}m^2]$
- moment of inertia of the rotor: $I_w = 0.000065\ [kg \cdot m^2]$
- coefficient: $b = 0.01149$
- coefficient: $d = 0.001$

For the quadrocopter mathematical model with parameters given above, a number of simulations was undertaken. During analysis and results verification, particular attention was paid to Euler angles. They provide the most important information about the behavior of the orientation while maneuvering in the air. This information is key and essential knowledge for the development of an efficient algorithm of quadrocopter control. For tests purposes of implemented mathematical robot's model, authors have proposed the use of PID controllers due to clear, intuitive operation and simplicity of parameters tuning (with heuristic method). Each controller has been dedicated to one of the Euler angles. For proper settings of the controllers parameters (selected by use of Swarm algorithm), graphs shown in Figures 5, 6, 7, 8 and 9, have been generated.

In Figures 5, 6 and 7 Euler angles change over time is presented for the given, external disturbance. The controller response is directly visible on courses shown in Figure 8 and 9 - it is noted that every change of angular velocity begins and ends at the speed of 21 radians/s. This value corresponds to the generated thrust, which provides the compensation of gravity forces by the drivetrain of a quadrocopter. Figures: 8 and 9 purposely grouped rotors angular velocities

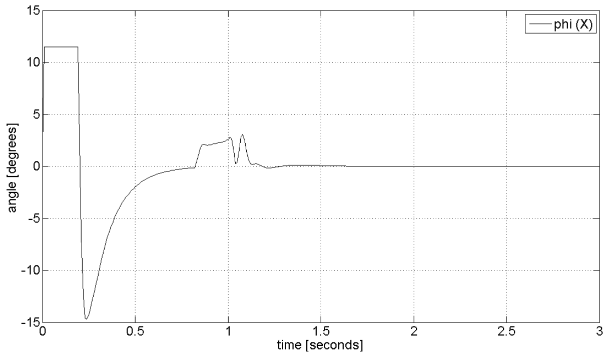

Fig. 5. Euler angle ϕ during the simulated flight

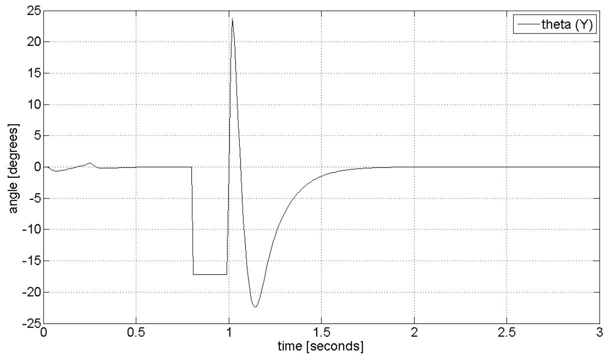

Fig. 6. Euler angle θ during the simulated flight

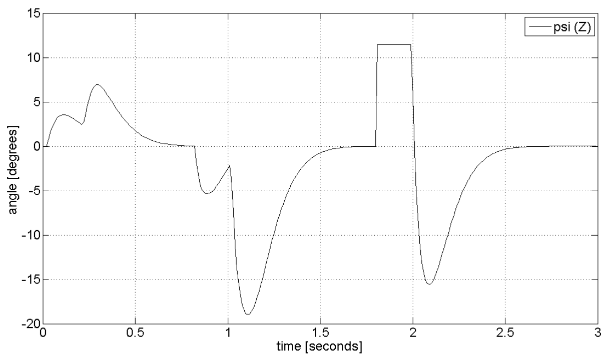

Fig. 7. Euler angle ψ during the simulated flight

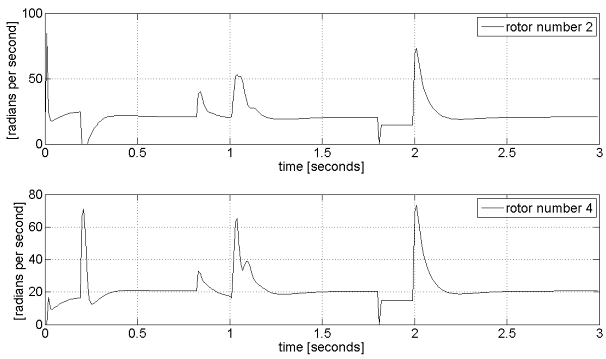

Fig. 8. Angular velocities of the robot's rotors (pair of rotors no.2 and no.4) associated with the Euler angles

pairs as: 1-3 and 2-4. It should be emphasized that this grouping, results from quadrocopter's architecture - each pair of rotors is associated precisely with one axis in the system coordinates of the robot (BF,

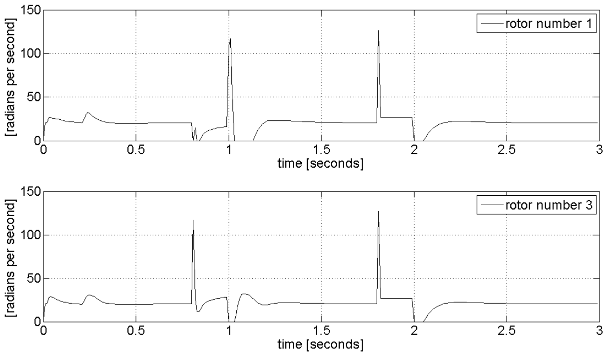

Fig. 9. Angular velocities of the robot's rotors (pair of rotors no.1 and no.3) associated with the Euler angles

local coordinate system). The next part of the experiment was to find the differences that may result from an incomplete physical model. For this purpose, authors removed the gyroscopic effect from the equation (28). This effect refers only to the axis X (angle ϕ) and is due to the rotation of the entire robot:

$$\tau = -\omega_y \omega_z (I_z - I_y) \tag{52}$$

The results are shown in Figures: 10, 11 and 12.

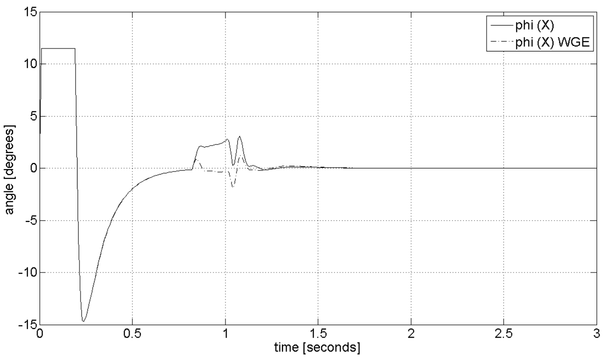

Fig. 10. Euler angle ϕ with and without (WGE) the gyroscopic effect

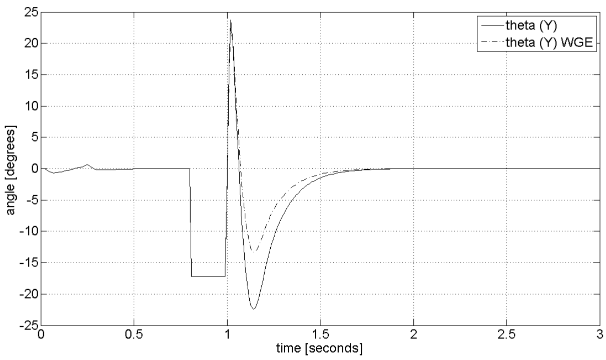

Fig. 11. Euler angle θ with and without (WGE) the gyroscopic effect

It may be noted that the gyroscopic effect is a result of unbalanced of inertia movements about for axes Z and Y (52). As it turns out, this is a very important effect, which in dynamic situations may strongly influence the orientation of the robot. The omission of this effect is therefore unacceptable.

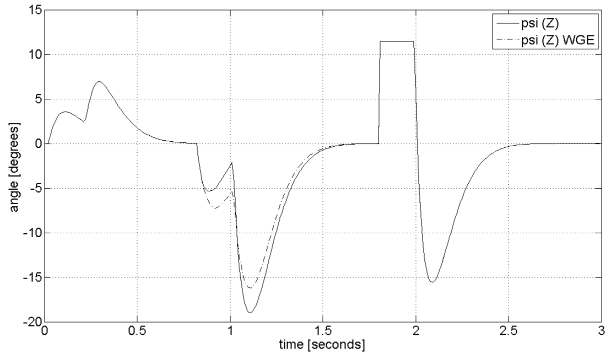

Fig. 12. Euler angle ψ with and without (WGE) the gyroscopic effect

8. Summary

The ways how to assess the efficiency and validation of the mathematical model in science and engineering are multiple, but physical forces impose somehow the use of certain measures of assessment. In the case of such a complex object as four-rotor flying robot, a reasonable way to assess the adequacy of its mathematical model, is to compare responses of both: real object and model - for the forces set in deterministic way. It should be remembered that certain factors can not be measured and predicted in a real system as they have a random character. Also, the same measurement method may be unreliable, having consequences on the dynamics of the flying robot under control. Data acquisition system with sensors or onboard vision system which might be used for experimentation should have to be quick and precise. Such instrumentation increases significantly the costs of experiment in the modeling phase. Besides, not all behaviors of quadrocopter can be checked on the real object - as the risk of robot damaging is high (for example a rapid set value in control signal). Therefore, in searching for a compromise, it was decided to use the mathemathical analysis of Euler angles while considering the orientation of robot model in the context of the gravity force. The obtained results confirm that the mathematical model adequately describes the behavior of a simulated robot, and allows for its implementation in the simulator (computer program) reflecting the physical aspects of the environment. The obtained simulation tool enables a better understanding of robot navigation aspects, which can not be directly tested on a the real robot. Tests in the simulator will be the next step undertaken by the authors in order to develop and propose further efficient control algorithms of quadrocopter subjected to the external disturbances (wind, turbulence, air temperature variation).

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