

# SYNTHESIS OF QUASI-OPTIMAL MOTION CRANE'S CONTROL IN THE FORM OF FEEDBACK

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## Abstract:

The solution of the problem of optimal crane's control is proposed in this article. The crane's model is adopted as two-mass. The synthesized quasi-optimal control allows one to eliminate vibrations during braking load of the crane. Control is a function of phase coordinates of dynamic system "truck-load" and it's limited in size. One may use for the solution of the problem the method of dynamic programming. The results are illustrated with the help of graphics which are bold on the phase planes.

**Keywords:** quasi-optimal control, crane, load vibrations, dynamic programming, control in the form of feedback

## 1. Introduction

The handling of various cargoes with a help of bridge cranes is widespread. They are used in sea and river ports, factories of chemical and metallurgical industry, mechanical engineering and more. Bridge type cranes often work in unsteady operating modes (start, stop, reverse). It is known that the default mode of motion of the crane may be absent in general case. Dynamic processes occurred during the transient motion of crane mechanisms may determine the efficiency of the crane, as well. The cargo usually is fixed on a rope and its vibrations affect on the performance, reliability and efficiency of bridge crane. The problem of eliminating of load's vibrations for port's reloaders and steel valves is particularly relevant. In the first case, the elimination of load's vibrations increases crane's productivity and reduces the idle of the ship in port in the first case and increases safety of the work in the second one.

Vibrations of the load on a rope appear during transient motion of the crane, continue during its steady movement phase and are present even after crane's stop. It is desirable to eliminate these crane's vibrations as quickly, as it is possible [1]. However, the optimal control of velocity's action to eliminate the load's vibration significantly increases the dynamic load of crane's elements and this crane can quickly fail. One may use other methods of solving this problem. For example, one may use fuzzy-controllers [2–5]. The disadvantage of such methods is that they do not include the restrictions imposed on the drive mechanism of the crane, also that load vibrations may have big amplitude during the transient process.

One may use the passive damping devices for the elimination of load's vibrations. By the way, there are

a number of ways that are patented and used by different companies [6–8]. The main drawback of these methods is that they do not provide optimal control. That's why the problem of finding of the optimal control of crane's load oscillations during its removal is very important.

## 2. References

### 2.1. Problem of Optimal Control

For the research purpose one may take the two-mass dynamic model of the mechanism of movement of the crane which is performed in Fig. 1. This model is common and is used by many researchers [9–11]

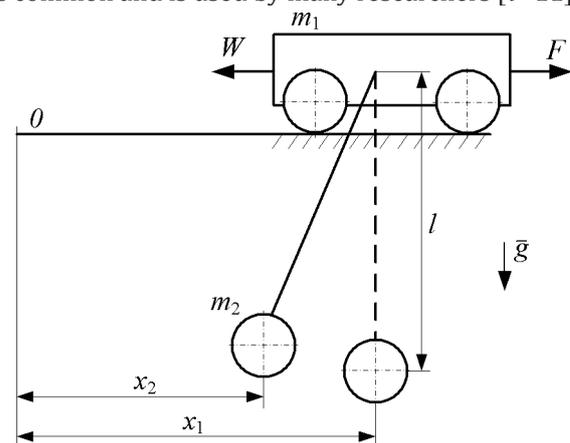


Fig. 1. A dynamic model of the system "crane-load"

The above mentioned calculation model (Fig. 1) is described by a system of differential equations:

$$\begin{cases} m_1 \ddot{x}_1 + m_2 \ddot{x}_2 = F - W \cdot \text{sign}(\dot{x}_1); \\ \ddot{x}_2 + \frac{g}{l}(x_2 - x_1) = 0, \end{cases} \quad (1)$$

where  $m_1$  is the mass given to the translational motion of the drive mechanism and the weight of the crane;  $m_2$  is the mass of cargo;  $x_1, x_2$  are the coordinates of the centers of mass of the crane and of the cargo;  $g$  is the acceleration of the free fall;  $l$  is the length of the flexible suspension;  $F$  is the total traction or braking force which is acting on the crane;  $W$  is the power of resistance given to the movement of the crane. Point over a symbol means differentiation in time.

We assume that when the crane is moving during braking it does not change its speed, that is  $\text{sign}(\dot{x}_1)=1$ . One may take the system of equations (1) in a canonical form. Let's add one more equation for the function of control's changes:

$$\begin{cases} \dot{y}_0 = y_1; \\ \dot{y}_1 = y_2; \\ \dot{y}_2 = y_3; \\ \dot{y}_3 = u - \Omega^2 y_2; \\ \dot{u} = \varphi, \end{cases} \quad (2)$$

where  $y_0$  is the function proportional to the coordinate the load ( $y_0 = x_2/\Omega_0^2$ );  $\Omega$  is the own frequency of load's vibration relatively movable crane ( $\Omega = \sqrt{(m_1 + m_2)g/m_1 l}$ );  $\Omega_0$  is the own frequency of load's vibration relative to the fixed crane ( $\Omega_0 = \sqrt{g/l}$ );  $u$  is the function of control of dynamic system "crane-load" ( $u = (F - W)/m_1$ );  $\varphi$  is the function of rate of change of control. The restrictions imposed on the control  $u$  are in the form of inequalities:

$$|u| \leq u_{\max} = \frac{F_{\max} - W}{m_1}, \quad (3)$$

where  $F_{\max}$  is the maximum force over the crane, which corresponds to the maximum torque on the motor shaft.

The movement of the crane with a load is characterized by initial conditions which are recorded for the new phase coordinates  $y_0, y_1, y_2, y_3$  as follows:

$$\begin{aligned} y_0(0) &= \frac{x_2(0)}{\Omega_0^2} = \frac{x_1(0) - \Delta x(0)}{\Omega_0^2} \approx \frac{x_1(0) - l\alpha(0)}{\Omega_0^2}; \\ y_1(0) &= \frac{\dot{x}_2(0)}{\Omega_0^2} = \frac{\dot{x}_1(0) - \Delta\dot{x}(0)}{\Omega_0^2} \approx \frac{\dot{x}_1(0) - l\dot{\alpha}(0)}{\Omega_0^2}; \\ y_2(0) &= \frac{\ddot{x}_2(0)}{\Omega_0^2} = \frac{\ddot{x}_1(0) - \Delta\ddot{x}(0)}{\Omega_0^2} \approx \frac{\ddot{x}_1(0) - l\ddot{\alpha}(0)}{\Omega_0^2}; \\ y_3(0) &= \frac{\ddot{x}_2(0)}{\Omega_0^2} = \frac{\ddot{x}_1(0) - \Delta\ddot{x}(0)}{\Omega_0^2} \approx \frac{\ddot{x}_1(0) - l\ddot{\alpha}(0)}{\Omega_0^2}, \end{aligned} \quad (4)$$

here  $\Delta x$  is the difference of coordinates of the crane and load ( $\Delta x = x_1 - x_2$ );  $\alpha$  - is the angle of the rope load with the vertical. The system (4) used an approximate estimation follows  $\Delta x \approx l\alpha$  from the fact that  $\sin\alpha \approx \alpha$ , for the small values of  $\alpha$ . This approach does not give significant errors.

The initial conditions (4) allow one to determine parameters of motion of the crane and of the load which must be measured. This is necessary to determine these conditions and for their default at the crane's system control. One must measure the coordinate of crane's position and its higher derivatives in time up to the third as well, a length of rope and rope angle of the load from the vertical and its higher derivatives in time up to the third as follows from this system. These parameters are measured with a help of the three encoders. One encoder measures the length of rope. It's installed on the cable drum. The second encoder measures the position of the crane relative to zero. The third encoder measures the angle of the rope load from the vertical. Its output shaft is attached to the rope with a help of special fittings.

The following final conditions must be performed in order to eliminate load's oscillations during the moment when crane is putting on the breaks:

$$\begin{cases} y_1(T) = 0; \\ y_2(T) = 0; \\ y_3(T) = 0. \end{cases} \quad (5)$$

The first condition in (5) is equivalent to the situation when the load's speed is equal to zero, the second condition is equivalent to the situation when the difference of coordinates of the crane and cargo is equal to zero, the third condition in (5) is equivalent to the situation when the difference in speed of the crane and load is equal to zero. So, the amount of energy's oscillations of the load and of the crane's movement should be equal to zero just at the moment  $t=T$ . This situation means the crane's stopping and the lack of load's vibration.

In order to create the synthesis of control one must set the criterion of optimality which will determine only the one optimal control of the entire set of alternatives. The criterion of optimality of motion of the crane during its braking may be adopt as such integrated functionality:

$$I = \int_0^T \left( \sum_{i=1}^{n=3} \delta_i y_i^2 + \delta_4 u^2 + \delta_5 \varphi^2 \right) dt \rightarrow \min, \quad (6)$$

where  $\delta_1, \delta_2, \delta_3, \delta_4, \delta_5$  are some coefficients. These coefficients can be calculated as follows:

$$\delta_j = k_j \tilde{I}_j, \quad j = 1, 2, 3, 4, 5, \quad (7)$$

where  $k_j$  - the weight's coefficient that takes into account the respective importance of the  $j$ -th index in the structure of the criterion;  $\tilde{I}_j$  - a factor that brings the dimension of the  $j$ -th index in the structure of the criterion (6) to dimensionless form.

Criterion (6) is an integrated linear-quadratic integral criterion and it reflects both the phase coordinates of the dynamical system and the "costs" to its control as well.

Thus, one staged the task of the optimal control of the dynamic system "truck-load". The problem is that the dynamic system must be converted from the original position which is characterized by initial conditions (4) into the final one which is characterized by finite terms of (5). This optimality criterion (6) should be least. In addition, one imposes the restricts on control in the form of inequality (3) and the end of control  $T$  is unstable.

## 2.2. Synthesis of Optimal Control

We use the method of dynamic programming [12] for solving the problem of optimal control. This method of synthesis of optimal control lets one to know the control as a function of phase coordinates of dynamical systems. This control is in the form of feedback. The basic functional equation for this problem is written as follows:

$$\begin{aligned} \min \left( \sum_{i=1}^{n=3} \delta_i y_i^2 + \delta_4 u^2 + \delta_5 \varphi^2 + \frac{\partial S}{\partial y_0} y_1 + \frac{\partial S}{\partial y_1} \times \right. \\ \left. \times y_2 + \frac{\partial S}{\partial y_2} y_3 + \frac{\partial S}{\partial y_3} (u - \Omega^2 y_2) + \frac{\partial S}{\partial u} \varphi \right) = 0, \end{aligned} \quad (8)$$

where  $S$  - Bellman's function.

The problem will be solved for the case when the control  $u$  is unrestricted (3). This circumstance gives one the possibility to find an analytical solution of the problem. However, we will consider the inequality (2) in future. One may search the minimum of the right side of the equation (8) for the function  $\varphi$ . Let's differentiate it by the function  $\varphi$  and then equate the result to zero:

$$2\delta_5\varphi + \frac{\partial S}{\partial u} = 0. \quad (9)$$

We find from equation (10) function:

$$\varphi = -\frac{1}{2\delta_5} \frac{\partial S}{\partial u}. \quad (10)$$

Let's put the equation (10) into the equation (8). Then we have:

$$\begin{aligned} & \delta_1 y_1^2 + \left( \frac{\partial S}{\partial y_1} + y_2 \delta_2 \right) y_2 + \left( \frac{\partial S}{\partial y_2} + y_3 \delta_3 \right) y_3 + \\ & + \left( \frac{\partial S}{\partial y_3} + u \delta_4 \right) u - \frac{1}{4\delta_5} \left( \frac{\partial S}{\partial u} \right)^2 - \frac{\partial S}{\partial y_3} y_2 \Omega^2 = 0. \end{aligned} \quad (11)$$

Equation (11) is a nonlinear differential equation in partial derivatives. We seek its solution in the form of a quadratic form as one does this usually when solving similar problems [13]:

$$\begin{aligned} S = & A_1 y_1^2 + A_2 y_2^2 + A_3 y_3^2 + A_4 u^2 + A_5 y_1 y_2 + \\ & + A_6 y_1 y_3 + A_7 y_1 u + A_8 y_2 y_3 + A_9 y_2 u + A_{10} y_3 u, \end{aligned} \quad (12)$$

where  $A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}$  – constant coefficients to be determined.

Take the partial derivatives of expression (12) for functions  $y_1, y_2, y_3$  and  $u$ , and:

$$\frac{\partial S}{\partial y_1} = 2A_1 y_1 + A_5 y_2 + A_6 y_3 + A_7 u, \quad (13)$$

$$\frac{\partial S}{\partial y_2} = A_5 y_1 + 2A_2 y_2 + A_8 y_3 + A_9 u, \quad (14)$$

$$\frac{\partial S}{\partial y_3} = A_6 y_1 + A_8 y_2 + 2A_3 y_3 + A_{10} u, \quad (15)$$

$$\frac{\partial S}{\partial u} = A_7 y_1 + A_9 y_2 + A_{10} y_3 + 2A_4 u. \quad (16)$$

Let's substitute expressions (13)–(16) in equation (11) and then remove of common factors of the brackets. We get:

$$\begin{aligned} & y_1^2 \delta_1 - \frac{A_7^2}{4\delta_5} + y_2^2 A_5 + \delta_2 - \frac{A_9^2}{4\delta_5} - A_8 \Omega^2 + y_3^2 \times \\ & \times A_8 + \delta_3 - \frac{A_{10}^2}{4\delta_5} + u^2 A_{10} + \delta_4 - \frac{A_4^2}{\delta_5} + y_1 y_2 \times \\ & \times 2A_1 - \frac{A_7 A_9}{2\delta_5} - A_6 \Omega^2 + y_1 y_3 A_5 - \frac{A_{10} A_7}{2\delta_5} + y_1 \times \\ & \times u A_6 - \frac{A_4 A_9}{\delta_5} + y_2 y_3 A_2 + A_6 - \frac{A_{10} A_9}{2\delta_5} - 2A_3 \times \\ & \times \Omega^2 + y_2 u (A_7 + A_8 - \frac{A_4 A_9}{\delta_5} - A_{10} \Omega^2 + y_3 u (2A_3 + \\ & + A_9 - 2A_3 + A_9 - \frac{A_{10} A_4}{\delta_5} = 0. \end{aligned} \quad (18)$$

Equation (18) is true in the case when the expression in parentheses will be zero because the functions  $y_1, y_2, y_3, u$  can't be zero at the same time. Therefore, equation (18) can be replaced by a system of nonlinear algebraic equations:

$$\begin{aligned} & \delta_1 - \frac{A_7^2}{4\delta_5} = 0; \\ & A_5 + \delta_2 - \frac{A_9^2}{4\delta_5} - A_8 \Omega^2 = 0; \\ & A_8 + \delta_3 - \frac{A_{10}^2}{4\delta_5} = 0; \\ & A_{10} + \delta_4 - \frac{A_4^2}{\delta_5} = 0; \\ & 2A_1 - \frac{A_7 A_9}{2\delta_5} - A_6 \Omega^2 = 0; \\ & A_5 - \frac{A_{10} A_7}{2\delta_5} = 0; \\ & A_6 - \frac{A_4 A_9}{\delta_5} = 0; \\ & A_2 + A_6 - \frac{A_{10} A_9}{2\delta_5} - 2A_3 \Omega^2 = 0; \\ & A_7 + A_8 - \frac{A_4 A_9}{\delta_5} - A_{10} \Omega^2 = 0; \\ & 2A_3 + A_9 - \frac{A_{10} A_4}{\delta_5} = 0. \end{aligned} \quad (19)$$

The system of equations (19) may be solved in analytical. But it is too difficult. So let's simplify it. The expression (10) may be as follows taking into account formula (16):

$$\varphi = -\frac{2A_4 u + A_7 y_1 + A_9 y_2 + A_{10} y_3}{2\delta_5}. \quad (20)$$

Thus, in order to find the unknown function  $\varphi$  which is the first derivative of the function control of dynamic system one must find only four coefficients  $A_4, A_7, A_9, A_{10}$ . It's necessary to form four equations in order to know these coefficients. The first and fourth equation of (19) contains only the coefficients  $A_4, A_7, A_9, A_{10}$  so we will use them. One can get from equa-

tions (19) the third and fourth equation in which coefficients  $A_4, A_7, A_9, A_{10}$  are unknown. We obtain third equation when rewrite the second equation of (19) taking into account the third and sixth equations of the system. We get the fourth equation when rewrite the ninth equation of system (19) taking into account the third equation of the last system. As a result, we have:

$$\begin{aligned} \delta_1 - \frac{A_7^2}{4\delta_5} &= 0; \\ A_{10} + \delta_4 - \frac{A_4^2}{\delta_5} &= 0; \\ \frac{A_{10}A_7}{2\delta_5} + \delta_2 - \frac{A_9^2}{4\delta_5} - \frac{A_{10}^2}{4\delta_5} - \delta_3 \Omega^2 &= 0; \\ A_7 + \frac{A_{10}^2}{4\delta_5} - \delta_3 - \frac{A_4A_9}{\delta_5} - A_{10}\Omega^2 &= 0. \end{aligned} \tag{21}$$

The first equation of (21) is independent of others and we can immediately write:

$$A_7 = 2\sqrt{\delta_1\delta_5}. \tag{22}$$

Negative root is rejected because it can lead to unstable dynamical system. We can express the unknown coefficients  $A_{10}$  and  $A_9$  by the coefficient  $A_4$ :

$$A_{10} = \frac{A_4^2}{\delta_5} - \delta_4, \tag{23}$$

$$\begin{aligned} A_9 = \pm 4 \sqrt{\frac{A_4^2}{\delta_5} - \delta_4} \sqrt{\delta_1\delta_5} - \\ - \frac{A_4^2}{\delta_5} - \delta_4 \Omega^2 + 4\delta_5(\delta_2 + \delta_3\Omega^2). \end{aligned} \tag{24}$$

The system of equations (21) leads to one algebraic equation of eighth degree relative when one takes into account expressions (22)–(24):

$$A_4^8 + B_1A_4^6 + B_2A_4^4 + B_3A_4^2 + B_4 = 0. \tag{25}$$

The last one is reduced to the equation of fourth degree when we will use replacement  $A_4^2 = \tilde{A}_4$ :

$$\tilde{A}_4^4 + B_1\tilde{A}_4^3 + B_2\tilde{A}_4^2 + B_3\tilde{A}_4 + B_4 = 0. \tag{26}$$

Equation (26) may be solved by Descartes-Euler’s method. We will not bring solutions of these equations because they have significant volume. We note only that equation (26) has two real and two complex solutions. One can find eight roots of the equation (25) turning to the reverse substitution  $A_4 = \pm\sqrt{\tilde{A}_4}$ . Thereafter, we choose only one – the real and positive root. Furthermore, we choose sign “+” before the root in expression (24) for the unambiguous determination of the coefficient  $A_9$ . Thus, all complex and negative values of coefficients  $A_4, A_9$  that satisfy the system of equations (21) are rejected because they can lead to the instability of dynamical system “crane-load”.

The expression (20) may be used to find a function  $\varphi$  that is the first derivative of the control’s function  $u$  over time. We need to get just the same control’s function in a such manner  $u = u(y_0, y_1, y_2, y_3)$ .

One must to integrate the expression (20) for this purpose:

$$\begin{aligned} u = \int \varphi dt = -\frac{A_4}{\delta_5} \int u dt - \frac{A_{10}}{2\delta_5} \int y_3 dt - \frac{A_9}{2\delta_5} \int y_2 dt - \frac{A_7}{2\delta_5} \times \\ \times \int y_1 dt = -\frac{A_4}{\delta_5} y_3 - \frac{A_{10}}{2\delta_5} y_2 - \frac{A_9}{2\delta_5} y_1 - \frac{A_7}{2\delta_5} y_0 + C, \end{aligned} \tag{27}$$

where  $C$  – is the constant of integration. In order to find the constant of integration it is necessary to solve the following equation  $u(0) = u_0$  which in expanded form will take such a form:

$$-\frac{A_4}{\delta_5} y_3(0) - \frac{A_{10}}{2\delta_5} y_2(0) - \frac{A_9}{2\delta_5} y_1(0) - \frac{A_7}{2\delta_5} y_0(0) + C = u_0. \tag{28}$$

One may find the solution of equation (28) and then substitute it in the expression (27). We will have finally such control’s function  $u$ :

$$\begin{aligned} u = u_0 + \frac{A_4}{\delta_5} (y_3(0) - y_3) + \frac{A_{10}}{2\delta_5} (y_2(0) - y_2) + \\ + \frac{A_9}{2\delta_5} (y_1(0) - y_1) + \frac{A_7}{2\delta_5} (y_0(0) - y_0). \end{aligned} \tag{29}$$

So we got control’s function which depends on the initial control and on phase coordinates as well. We can set arbitrarily the initial control’s function. In the particular case  $u_0 = 0$ . This means no dynamic efforts over the crane’s drive at the beginning of its inhibition, in practice. The risk of significant current in electric and dynamic loads of the mechanical part of the crane’s drive and its metal faucet is eliminated as well.

Let’s build a graph (Fig. 2) for the resulting control’s law. There is also the three-dimensional phase portrait of dynamical system (Fig. 3). The gray point in Fig. 3 marks origin of the coordinate system.

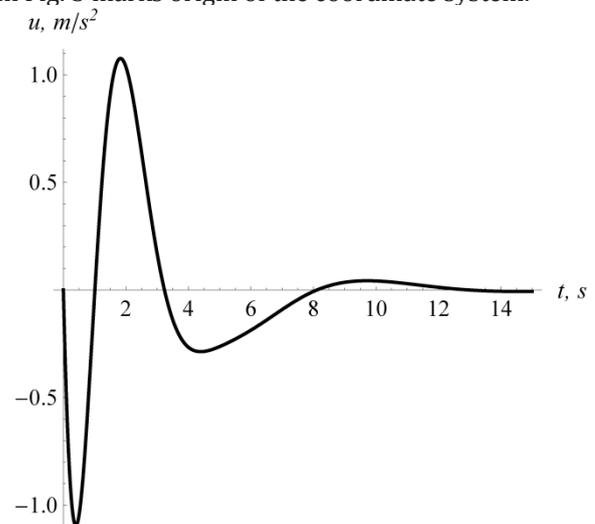


Fig. 2. Graph of the function of optimal control of dynamic system “crane-load”

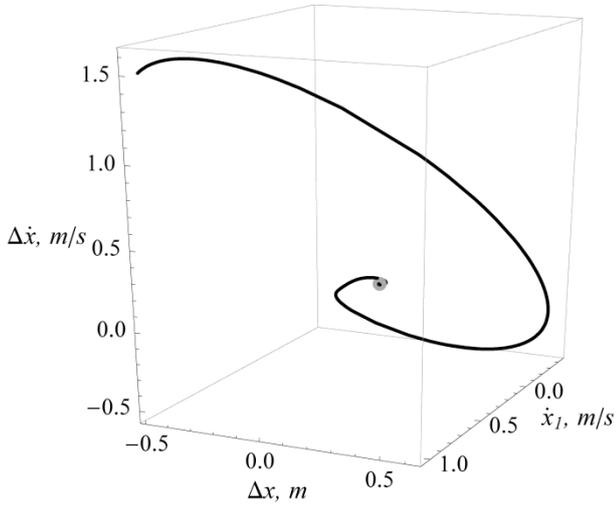


Fig. 3. Three-dimensional phase portrait of motion of dynamic system "crane-load"

The dynamic system has zero energy of motion in the origin of coordinate system, i.e., crane stopped and load's oscillations on the rope are absent. Thus the problem of optimal control can be considered as a solved problem. However, we do not take into account the constraints (3) when solved this problem. These constraints are usually imposed on control. Physically, this means that electric drive will occasionally transshipped and will not be able to realize the optimal control. It is therefore necessary to take into account these constraints (3).

### 2.3. Analysis of the Results (Synthesis of Quasi-optimal Control)

An easy way to take into account constraints (such as (4)) is to miss the optimal control signal through a nonlinear element such as "saturation". Such control will be called as quasi control because it consists of the pieces of optimal control and of the pieces of maximum and minimum values of control. Analytically this is expressed in the following form:

$$u^* = \begin{cases} u, & \text{if } u_{\min} \leq u \leq u_{\max}; \\ u_{\min}, & \text{if } u < u_{\min}; \\ u_{\max}, & \text{if } u > u_{\max}, \end{cases} \quad (30)$$

where  $u^*$  – quasi-optimal control that satisfies constraints (3);  $u_{\min}, u_{\max}$  – respectively the minimum and maximum control. Here are the graphs similar to the above in Fig. 4 and Fig. 5 for the case  $u_{\min} = -0,4 \text{ m/s}^2$  and  $u_{\max} = 0,4 \text{ m/s}^2$ . One may see from the resulted graphs that the control does not go to the upper limit. Let us narrow the limits of permissible values of controls:  $u_{\min} = -0,2 \text{ m/s}^2$  and  $u_{\max} = 0,2 \text{ m/s}^2$ . Physically, this means that the drive motor power is reduced by the half. So it is possible to project the crane's motor of less power. However, the duration of the transition process is increasing as seen from Fig. 6 and Fig. 7. Thus, one can reduce the crane's drive power when the duration of the transition process is increased.

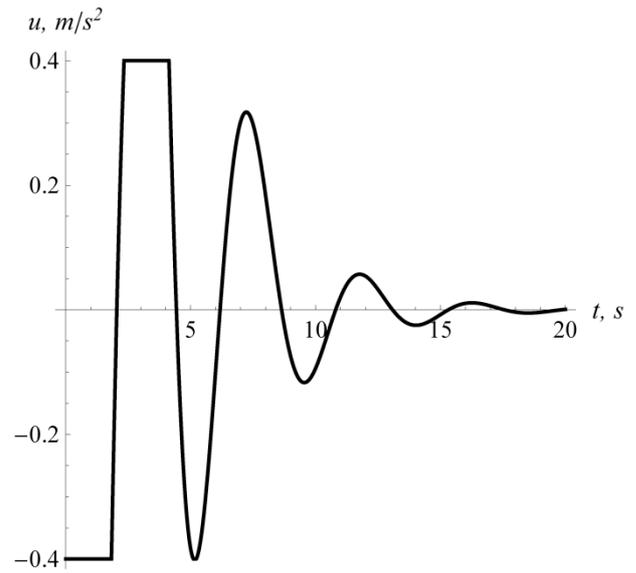


Fig. 4. Graph of the function of the quasi dynamic control of the system "crane-load" while respecting the constraints (3)  $u_{\min} = -0,4 \text{ m/s}^2$  and  $u_{\max} = 0,4 \text{ m/s}^2$

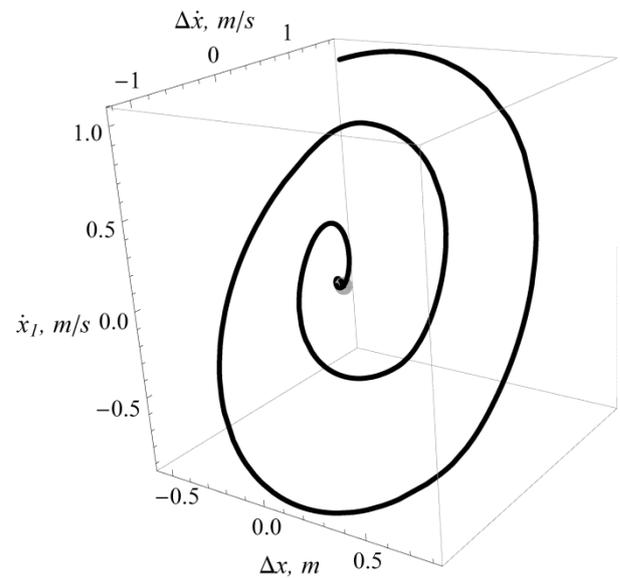


Fig. 5. Three-dimensional phase portrait of the motion of the dynamic system "crane-load" while control is (30) ( $u_{\min} = -0,4 \text{ m/s}^2, u_{\max} = 0,4 \text{ m/s}^2$ )

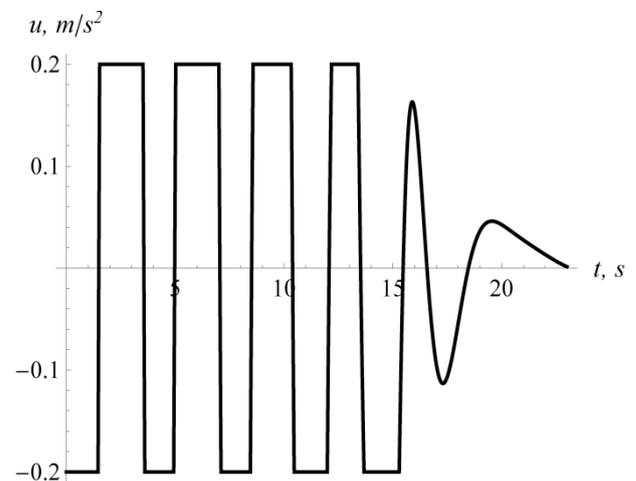
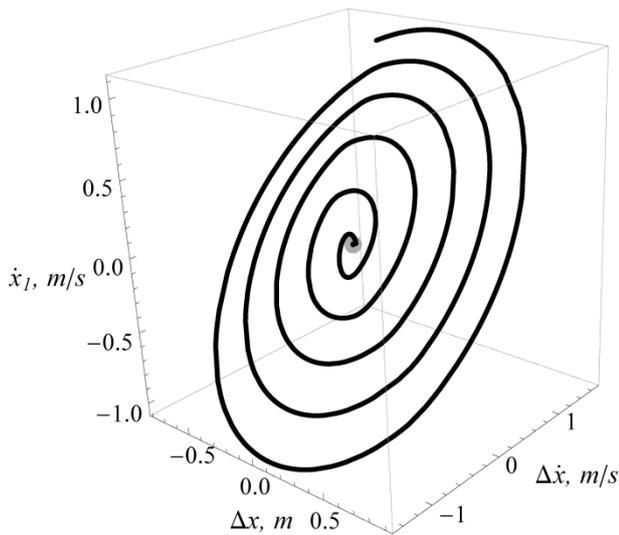


Fig. 6. Graph of the function of the quasi dynamic control system "crane-load" while control is respected the constraints (3) ( $u_{\min} = -0,2 \text{ m/s}^2$  and  $u_{\max} = 0,2 \text{ m/s}^2$ )



**Fig. 7. Three-dimensional phase portrait of the motion of the dynamic system "crane-load" while control is (30) ( $u_{min} = -0,2 \text{ m/s}^2$ ,  $u_{max} = 0,2 \text{ m/s}^2$ )**

Fig. 7 shows that the lack of narrowing of the range of allowable values of control is the changing of the sign of the crane's speed. One can also specify another disadvantage of the optimal control as the quasi-optimal function. The control is too small value when the phase coordinates of the dynamical systems "crane-load" are small as well. It means that at the end of transition period control is "weak." The possible way to solve this problem is to change the variety coefficients  $k_j$ , which are included in the structure of the optimization criterion of the transition process.

### 3. Conclusion

One may use the method of dynamic programming which allows to synthesize the optimal control in the form of feedback without restrictions on the amount of control. The use of nonlinear elements such as "saturation" provides a quasi-optimal control that satisfies the limits imposed on the control just at the every time's moment. This quasi-optimal control in the form of feedback consists of pieces of optimal control and of the boundary limits of the acceptable area. The variation of the coefficients in the structure's optimization criterion is the possible way to solve the problem of the synthesis of the optimal control which would always be in the acceptable limits even when these limits are the functions of the time and of the phase coordinates of the dynamical system "crane-load".

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