Analysis of Influence of Drive System Configurations of a Four Wheeled Robot on its Mobility

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Abstract:
The work covers analysis of mobility of a four-wheeled robot based on its dynamics model. Several configurations of robot's drive system are considered: one driven axle, two independently driven axles and drive transmission from one axle to another by means of a toothed belt. The analysis of robot's mobility is limited to cases of its motion with constant velocity on the ground with various inclinations and mechanical characteristics. It is assumed that robot's wheels roll without sliding. In the conducted investigations of robot's mobility, limitations resulting from wheels' interaction with the ground are taken into account. Based on results of the investigations, advantages and drawbacks of each of drive system configurations of the robot are discussed.

***Keywords**: mobile robot, mobility analysis, dynamics model, computer simulation.*

1. Introduction

Motion of a robot on diverse terrain depends on its movement abilities, in the literature often termed as 'mobility'. It can be defined as robot's ability to move with desired parameters of motion in defined conditions of environment, with limitations of the robot itself taken into account [3]. For determination of the mobility, analysed are robot's motion on the ground with various mechanical properties and inclinations and its ability of negotiation of environment obstacles of various shape and height (e.g., kerbs, stairs).

Robot's movement abilities on particular terrain are affected by a number of factors, including:

- geometry and type of a locomotion system (e.g., wheeled, tracked, hybrid, legged, jumping),
- properties of effectors (e.g., tyre type for wheeled robots),
- mass properties of a robot,
- constraints resulting from characteristics of drives (e.g. power, maximum rotational speed, maximum driving torque),
- battery parameters (e.g., maximum continuous discharge current), etc.

So far, a very popular locomotion system intended for moving in diverse terrain was the tracked system. However, observation of mobile robots' market reveals that even more often the tracked locomotion system gives way to the wheeled system [4] and the hybrid system, that is, the one which incorporates features of both continuous and discrete locomotion [5].

The analysis of mobility of a particular robot can be carried out for two principal purposes, that is, for the purpose of:

designing and testing of robot's mechanical structure,

synthesis of robot's control system.

At the stage of mechanical design, the mobility analysis based on robot's dynamics model allows testing of fulfilment of the imposed requirements and design optimization. The analysis can be repeated on the robot's physical prototype in order to verify results of the analysis carried out using the robot's virtual model.

An important assumption associated with the analysis of robot mobility is whether sliding of robot's wheels is taken into account. The problem of modelling of motion of mobile robots including wheel slip was discussed, for instance, in [6]. From previous research it follows that the longitudinal slip of wheels should be taken into account mainly in case of significant accelerations and sometimes also in case of transition from one type of ground's material into another (e.g., from concrete onto ice). On the other hand, side slip of wheels should be considered during negotiation of a curved path always if the robot is of the skid-steered type. Side slips tend to increase with increasing robot's velocity of motion and with decreasing radius of curvature of the path.

In the present work, mobility of a four-wheeled robot is analysed for different configurations of the drive system and including constraints which follow from robot's dynamics and type of terrain on which the motion takes place. Aim of the analysis is discussion of advantages and disadvantages of different configurations of the robot's drive system. The analysis is limited to the case of translational motion of the robot's body with constant velocity. For this reason, the occurrence of wheel slip is neglected.

2. Model of the robot

The subject of this paper is a small four-wheeled mobile robot whose parameters are based on the PIAP SCOUT mobile robot (Fig. 1). This robot was designed for quick reconnaissance of field and places difficult to access, such as, the bottom of vehicle's chassis, spaces under seats in means of transportation, narrow rooms and ventilation ducts.

The robot is equipped with the hybrid locomotion system which combines tracks and wheels. Back wheels of the robot are independently driven with two servomotors. The drive is transmitted from the back wheels to the front wheels via two toothed belts, which also play the role of caterpillar tracks.

In the present work, the following configurations of robot drive are considered:

- single-axle drive,
- drive transmission from one axle to another by means of a toothed belt,
- independent two-axle drive.

Fig. 1. PIAP SCOUT mobile robot

The robot's model (Fig. 2) consists of the frame (0) and driven wheels (1, 2, 3, 4). Additionally, the front wheels can be connected with the back wheels by means of toothed belts. Origin of the coordinate system of the robot is chosen at the point R, which is located in the middle of the distance between the front and back wheels and in the middle of the distance between wheels of the left-hand side and right-hand side. For particular pairs of wheels the following subscripts are introduced: l – wheels of the left-hand side ($l = \{1,3\}$), r – wheels of the right-hand side ($r = \{2,4\}$), f – front wheels ($f = \{1,2\}$), b – back wheels ($b = \{3,4\}$). The following symbols for the i^{th} wheel have been also introduced in the robot's model: A_i – geometrical center, $r_{gi} = r_g$ – geometrical (unloaded) radius, θ_i – rotation (spin) angle. The distance of the front axle from the back axle (wheelbase) is denoted with L, whereas the distance of the left-hand side wheels from the right-hand side wheels (track width) with W.

For the mobility analysis assumed are the following values of robot geometric parameters: $L = 0.35$ [m], $W = 0.42$ [m], $r_g = 0.085$ [m], and mass parameters:

- robot's total mass: $m_R = 13.73$ [kg],
- coordinates of the mass centre: $x_{CM} = 0$ [m], $y_{CM} = 0$ [m],
- mass moment of inertia of the wheel about its spin axis $I_{Wy} = 0.006$ [kg m^2].

a)

b)

c)

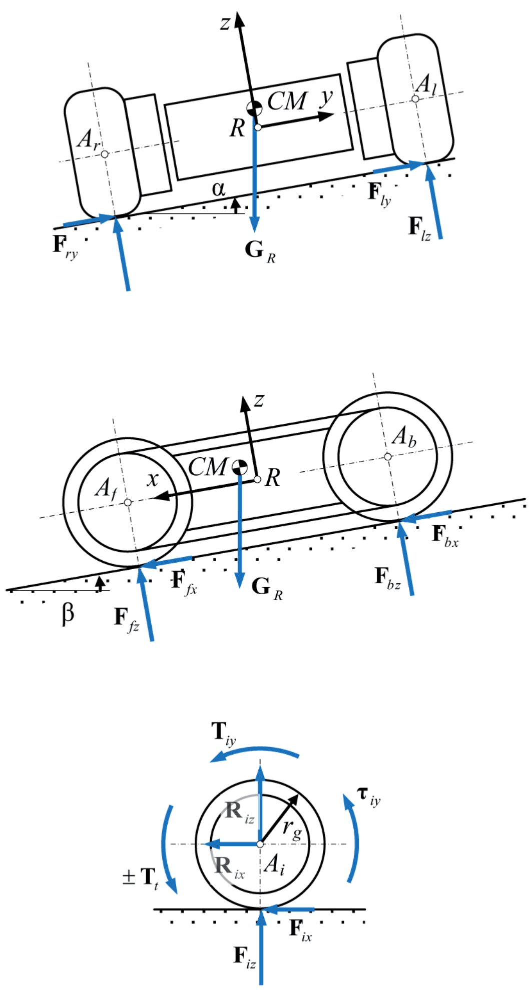

Fig. 2. A simplified model of the robot moving on the inclined terrain resulting in robot's tilt about its x-axis (a) and about its y-axis (b), driving torque as well as reaction forces and moments of forces acting on the wheel (c) ($l = \{1, 3\}$, $r = \{2, 4\}$, $f = \{1, 2\}$, $b = \{3, 4\}$, $L = A_fA_b = 2A_fR = 2A_bR$, $W = A_lA_r = 2A_lR = 2A_rR$)

2.1. Dynamics of the robot

Within the present work robot motion on inclined terrain will be analysed, which results in the robot being tilted about x- or y-axis (see Fig. 2a-b). The analysis will be constrained to the case of translational motion of robot's body. It is assumed that robot's wheels roll without sliding, so the following relationships are satisfied:

$$v_R = v_{Rx} = \dot{\theta}_i r_g \;=>\; \dot{\theta}_i = v_R / r_g = \;\; \ddot{\theta}_i = \dot{v}_R / r_g \;, \qquad (2.1)$$

where: v_R – value of velocity of the characteristic point R of the robot.

Dynamic equations of motion, for the case when robot is tilted about y-axis with pitch angle $\beta = const$, have the form (see Fig. 2b):

$$m_R \ddot{x}_{CM} = \sum_{i=1}^{4} F_{ix} + m_R g \sin(\beta) = \\ = 2F_{fx} + 2F_{bx} + m_R g \sin(\beta) \qquad (2.2)$$

$$m_R \ddot{z}_{CM} = \sum_{i=1}^{4} F_{iz} - m_R g \cos(\beta) = \\ = 2F_{fz} + 2F_{bz} - m_R g \cos(\beta) = 0 \qquad (2.3)$$

$$I_{Ry} \ddot{\beta} = -\sum_{i=1}^{4} F_{ix} h - \sum_{i=1}^{4} F_{iz} x_i = \\ = -(2F_{fx} + 2F_{bx}) h - 2F_{fz} x_f - 2F_{bz} x_b = 0 \qquad (2.4)$$

where: $h = r_g + z_{CM}$, $x_f = L/2 - x_{CM}$, $x_b = -L/2 - x_{CM}$, x_{CM} and z_{CM} – coordinates of the robot's mass centre in the robot's coordinate system, $G_R = m_R g$, m_R and G_R – respectively mass and value of gravity force for the robot, $g = 9.81$ [m/s^2] – the acceleration of gravity, I_{Ry} – mass moment of inertia about the axis parallel to y and passing through the mass centre of the robot.

Dynamic equations of motion for the robot's wheels can be written as:

$$I_{Wy} \ddot{\theta}_i = \tau_i - F_{ix} r_g + T_{iy} \pm T_t \;, \qquad (2.5)$$

where: I_{iy} – mass moment of inertia of a wheel about its spin axis, τ_i – driving torque, $T_{iy} = -F_{iz} f_r r_g$ – rolling resistance moment, f_r – coefficient of rolling resistance, T_t – moment transmitted to the wheel by the toothed belt.

Moment of force T_t occurs only in case of transmitting drive from one wheel to another by means of the toothed belt. The "+" sign in the above equation pertains to the wheel being driven, and "–" to the driving wheel.

In equation (2.4), the rolling resistance moments of wheels were neglected, because they have only minor influence on dynamics of the robot treated as a whole as compared to other moments of forces.

In turn, dynamic equations of motion of the robot, for the case when it is tilted about x-axis at roll angle $\alpha = const$, have the form (see Fig. 2a):

$$m_R \ddot{y}_{CM} = \sum_{i=1}^{4} F_{iy} - m_R g \sin(\alpha) = 0, \quad (2.6)$$

$$m_R \ddot{z}_{CM} = \sum_{i=1}^{4} F_{iz} - m_R g \cos(\alpha) = 0, \quad (2.7)$$

$$I_{Rx} \ddot{\alpha} = \sum_{i=1}^{4} F_{iy} h + \sum_{i=1}^{4} F_{iz} y_i = 0 \quad (2.8)$$

where: $y_l = W/2$, $y_r = -W/2$, $l = \{1,3\}$, $r = \{2,4\}$, I_{Rx} – mass moment of inertia with respect to the axis parallel to x and passing through the robot's mass centre.

3. Mobility analysis

The mobility analysis of the robot covers two characteristic cases in which it is tilted about x- or y-axis (see Fig. 2a-b). In both cases the robot's body is in translational motion.

In the investigations five types of ground are analysed. The interaction of the robot's tyre with those types of ground is described with coefficients of friction, adhesion and rolling resistance. Values of those coefficients assumed in the work are summarized in Table 1 – for contact of dry surfaces. Coefficients in the table have the following meanings: μ_s – coefficient of static friction, μ_k – coefficient of kinetic friction, μ_p – coefficient of peak adhesion, f_r – coefficient of rolling resistance.

Because of specifics of interaction of the tyre with the ground, that is, existence within the contact area of regions of adhesion (where the coefficient μ_s is valid) and sliding (coefficient μ_k) in the literature usually the coefficient of peak adhesion μ_p is introduced, which reflects maximum value of adhesion for tyre-ground pair. The coefficient is defined as ratio of maximum value of longitudinal component of ground reaction force to value of normal component of ground reaction force at the area of tyre-ground contact. The coefficient of adhesion depends not only on types of contacting surfaces of the tyre and the ground, but also, for example, on tyre tread pattern, tyre pressure, etc. In turn, the coefficient of kinetic friction μ_k can be identified with the coefficient of sliding adhesion, frequently introduced in tyre modelling.

Tab. 1. Coefficients of: sliding friction, adhesion and rolling resistance describing interaction of the tyre with selected ground types according to [1,2,3] other sources and author's own estimations

type of ground	μ_s	μ_p	μ_k	f_r
asphalt and concrete	1.00	0.85	0.75	0.015
unpaved road	0.80	0.68	0.65	0.050
rolled gravel	0.71	0.60	0.55	0.020
compressed snow	0.24	0.20	0.15	0.032
ice	0.12	0.10	0.07	0.010

Due to difficulties with finding in the literature the coefficient of static friction describing the interaction of a rubber tyre with all considered ground types, its value was estimated based on the known coefficients of peak adhesion, with the assumption that $\mu_p = 0.85\ \mu_s$. This relationship is obtained on the basis of values of coefficients in the case of the rubber tyre interaction with dry asphalt, which are easily available.

3.1. Solution for the case of the robot tilted about *x*-axis

For the mobility analysis of the robot, in the case when it is tilted through angle $\alpha = const$ equations (2.6) – (2.8) are used and additional constraints are introduced.

The first constraint follows from the fact, that robot cannot slide sideways, so side components of ground reaction forces are not allowed to exceed the value of developed friction. Taking in account the coefficient of static friction μ_s for the tyre-ground pair, this constraint can be written in the form:

$$-\mu_s F_{iz} \le F_{iy} \le \mu_s F_{iz}. \quad (2.9)$$

One should underline that trivial solution for the range of robot roll angles α:

$$\alpha_{min1} = -\arctan(\mu_s) \le \alpha \le \arctan(\mu_s) = \alpha_{max1}. \quad (2.10)$$

in this case depends only on the value of coefficient μ_s. Limiting values of this angle (i.e., minimum and maximum) for particular types of ground are given in Tab. 2.

Tab. 2. Limiting values of terrain inclination and robot's roll angles for various types of the ground

type of ground	$\alpha_{max1}/\alpha_{min1}$ [°]
asphalt and concrete	±45.00
unpaved road	±38.66
rolled gravel	±35.37
compressed snow	±13.50
ice	±6.843

From the fact that the robot cannot experience rollover follows the other constraint, according to which normal components of ground reaction forces must be positive, that is:

$$F_{iz} \ge 0. \quad (2.11)$$

After taking into consideration critical cases, where $F_{lz} = 0$ (so in consequence $F_{ly} = 0$) and $F_{rz} = 0$ (hence $F_{ry} = 0$), one obtains

$$\alpha_{min2} = -\arctan(W/(2h)) \le \alpha \le$$
$$\le \arctan(W/(2h)) = \alpha_{max2} \quad (2.12)$$

It follows that in this case the range of angles of robot roll α depends only on position of the robot's mass centre and the track width.

After taking into account the assumed values of the robot's parameters, the following solution is obtained:

$$\alpha_{min2} = -67.96\,[\text{deg}] \le \alpha \le 67.96\,[\text{deg}] = \alpha_{max2} \quad (2.13)$$

Eventually, the allowable range of robot's roll angles depends on ground type as well as on position of the robot's mass centre and robot's track width, and is equal to:

$$\max(\alpha_{min1}, \alpha_{min2}) \le \alpha \le \min(\alpha_{max1}, \alpha_{max2}), \quad (2.14)$$

where angles α_{min1} and α_{min2} are negative, while angles α_{max1} and α_{max2} positive.

3.2. Solution for the case of the robot tilted about *y*-axis

In order to obtain solutions of dynamic equations of motion for the case of robot motion on inclined ground, that is, to determine components of ground reaction forces and driving torques, the following assumptions are introduced:

- for single-axle driver

$$\tau_p = 0,\ T_t = 0, \quad (2.15)$$

- for single-axle drive and transmission of drive to another axle by means of the toothed belt

$$F_{fx}/F_{bx} = F_{fz}/F_{bz},\ \tau_p = 0, \quad (2.16)$$

- for independent drive of each axle

$$F_{fx}/F_{bx} = F_{fz}/F_{bz},\ T_t = 0, \quad (2.17)$$

where: p – passive axle, $p = b$ (for front wheels driven) or $p = f$ (for back wheels driven).

Based on the above assumptions, the following solutions for driving torques and components of ground reaction forces are obtained.

3.2.1. Solution for the case of single-axle drive

Solution for the case of single-axle drive (a – active axle, p – passive axle, $a = f$ and $p = b$ or $a = b$ and $p = f$) has the form:

$$\begin{aligned}&\tau_p = 0,\ \tau_a = (2I_{Wy}/r_g + m_R r_g/2)\dot{v}_R + \\ &+m_R\, g\, r_g (f_r c_\beta - s_\beta)/2,\ T_t = 0\end{aligned} \quad (2.18)$$

$$\begin{aligned}&F_{ax} = \left(2I_{Wy}L/r_g^2 + m_R(L \pm f_r h)\right)\dot{v}_R/(2L) + \\ &-m_R\, g\left(Ls_\beta + f_r(-l_a c_\beta \pm h s_\beta)\right)/(2L)\end{aligned} \quad (2.19)$$

$$\begin{aligned}&F_{px} = -\left(2I_{Wy}L/r_g^2 \pm m_R f_r h\right)\dot{v}_R/(2L) + \\ &-m_R\, g\, f_r(l_a c_\beta \pm h s_\beta)/(2L)\end{aligned} \quad (2.20)$$

where: $s_\beta = \sin(\beta)$, $c_\beta = \cos(\beta)$, $l_f = L/2 - x_{CM}$, $l_b = L/2 + x_{CM}$, $h = r_g + z_{CM}$, and the „+" sign is valid for the case of driving the front wheels, and „–" for the back wheels.

3.2.2. Solution for the case of drive transmission from one axle to another by means of the toothed belt

Solution for the case of driving one axle, and drive transmission to another axle by means of the toothed belt (a – active axle, p – passive axle, $a = f$ and $p = b$ or $a = b$ and $p = f$):

$$\begin{aligned}&\tau_p = 0,\ \tau_a = (2I_{Wy}/r_g + m_R r_g/2)\dot{v}_R + \\ &+m_R\, g\, r_g (f_r c_\beta - s_\beta)/2\end{aligned} \quad (2.21)$$

$$\begin{aligned}&T_t = \pm I_{Wy}\dot{v}_R/r_g + \\ &-m_R \dot{v}_R r_g\left((f_r h \pm l_a)c_\beta - h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &-m_R\, g\, r_g (f_r c_\beta - s_\beta)(\pm l_a c_\beta - h s_\beta)/(2Lc_\beta)\end{aligned} \quad (2.22)$$

$$\begin{aligned}&F_{ax} = m_R \dot{v}_R\left(l_p c_\beta \pm h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &-m_R\, g\, t_\beta (l_p c_\beta \pm h s_\beta)/(2L)\end{aligned} \quad (2.23)$$

$$\begin{aligned}&F_{px} = -m_R \dot{v}_R\left(-l_a c_\beta \pm h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &+m_R\, g\, t_\beta (-l_a c_\beta \pm h s_\beta)/(2L)\end{aligned} \quad (2.24)$$

where: $t_\beta = \tan(\beta) = s_\beta / c_\beta$, and „+" sign is valid for the case of driving the front wheels, and „–" for the back wheels.

3.2.3. Solution for the case of independent driving of two axles

Solution for the case of independent driving of two axles ($T_t = 0$) has the form:

$$\begin{aligned}&\tau_f = I_{Wy}\dot{v}_R/r_g + \\ &-m_R \dot{v}_R r_g\left(-(f_r h - l_b)c_\beta + h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &+m_R\, g\, r_g (f_r c_\beta - s_\beta)(l_b c_\beta + h s_\beta)/(2Lc_\beta)\end{aligned} \quad (2.25)$$

$$\begin{aligned}&\tau_b = I_{Wy}\dot{v}_R/r_g + \\ &-m\dot{v}_R r_g\left(-(f_r h + l_f)c_\beta + h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &+m_R\, g\, r_g (f_r c_\beta - s_\beta)(l_f c_\beta - h s_\beta)/(2Lc_\beta)\end{aligned} \quad (2.26)$$

$$\begin{aligned}&F_{fx} = m_R \dot{v}_R\left(l_b c_\beta + h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &-m_R\, g\, t_\beta (l_b c_\beta + h s_\beta)/(2L)\end{aligned} \quad (2.27)$$

$$\begin{aligned}&F_{bx} = m_R \dot{v}_R\left(l_f c_\beta - h(2s_\beta - \dot{v}_R/g)\right)/(2Lc_\beta) + \\ &-m_R\, g\, t_\beta (l_f c_\beta - h s_\beta)/(2L)\end{aligned} \quad (2.28)$$

3.2.4. Relationships common for all cases

The characteristic of the presented dynamic equations of motion is that for each discussed configuration of the robot one obtains the same solution for the normal components of ground reaction forces, that is of the form:

$$F_{fz} = m_R\left(g(l_b c_\beta + h s_\beta) - \dot{v}_R h\right)/(2L) \quad (2.29)$$

$$F_{bz} = m_R\left(g(l_f c_\beta - h s_\beta) + \dot{v}_R h\right)/(2L) \quad (2.30)$$

It should be also noted that this solution is independent of the type of terrain on which the robot moves. However, the type of terrain will affect values of the tangent components of ground reaction forces, and as a result, the values of driving torques.

3.3. Results of mobility analysis for the case of robot tilted about *y*-axis and moving with constant velocity

The analysis of robot's mobility, for the case in which it is tilted at the pitch angle β, will cover all possible options of drive system configuration. For each option, considered are equations (2.1) – (2.5) and the constraints:

$$F_{iz} \geq 0\,, \quad -\mu_p F_{iz} \leq F_{ix} \leq \mu_p F_{iz}\,, \tag{2.31}$$

that is all robot wheels must be in contact with the ground at all times and the value of longitudinal component of ground reaction force cannot exceed the value of developed friction force.

Limiting values of pitch angle β are determined after considering dynamic equations of motion and particular constraints. In case when for the given constraint is obtained a solution which violates any other constraint, the solution is discarded. Also, presented are values of driving torques and ground reaction forces corresponding to those angles, which may be important from the point of view of required drives' capabilities and mechanical strength of the robot's structure. In particular, the values of driving torques can become a decisive factor at the choice of specific robot's drive system configuration.

Because of very complex form of general solution, the following analysis will be conducted for specific robot parameters. In the present work discussed are results of mobility analysis of the robot for the case of motion with constant velocity, and:

- front-wheel drive only (Tab. 3),
- transmission of drive from back axle to front axle by means of toothed belt (Tab. 4),
- independent two-axle drive (Tab. 5).

In tables 3-5 are given the limiting values of terrain longitudinal inclination, driving torques and ground reaction forces for given constraints. Because of the fact that the robot's mass centre is located at the geometric centre of the body, robot's mobility is affected mainly by the coefficients of peak adhesion μ_p. With increasing pitch angle, robot can at first undergo downward slide (in case when constraint $-\mu_p F_{iz} \leq F_{ix} \leq \mu_p F_{iz}$ is not satisfied), and only then experience overturn (when constraint $F_{iz} \geq 0$ is not satisfied). Analogous conclusions also pertain to the other considered robot's drive system configurations.

4. Conclusions and future work

From the conducted analysis of robot's mobility for the case of its motion with constant velocity can be drawn the following conclusions.

The worst of the analysed options with respect to robot's mobility is driving of only one axle, that is, only front or back wheels. In this case, range of angles of longitudinal terrain inclination, at which motion is possible, is the narrowest. Moreover, the range is non-symmetric, so for instance in the case of front-wheel-drive when going up the hill the angle is smaller, than while going down.

The allowable range of longitudinal terrain inclination angles is the same for the case of drive transmission from back to front axle and for independent two-axle drive. Additionally, in the case of symmetric robot's mass distribution between axles, the range is symmetric.

After assuming the same total robot's mass in all analysed cases, it is evident that for single-axle drive and for back-to-front drive transmission, driving torque necessary for motion is equal to the sum of driving torques required for independent driving of both axles. The advantage of independent two-axle drive configuration as compared to the other considered solutions are the smaller required values of driving torques per single drive.

The advantage of solution with back-to-front-axle-transmission drive is the analogous robot's mobility with respect to the independent two-axle drive, but with smaller number of applied drives. This solution is associated with minor increase in complexity of design, because of application of additional toothed belts. On the other hand, the belts can enhance robot mobility in case of motion on uneven terrain if used as a caterpillar track – example of this is the PIAP Scout robot of PIAP Poland. After taking this into consideration, design solution like that can be optimal in the analysed case and become a reasonable trade-off between single-axle drive and independent two-axle drive. However, it should be emphasized that in order to point-out the true optimal solution for the robot's drive system configuration, a more comprehensive research is required.

In accordance, during future research will be analysed robot's mobility including:

- more advanced cases of motion (e.g., negotiation of a curved path),

Tab. 3. Results of robot's mobility analysis for the case of motion with constant velocity and the front-wheel driv

type of ground	constraint	β [°]	τ_f [Nm]	F_{fx} [N]	F_{bx} [N]	F_{fz} [N]	F_{bz} [N]
asphalt and concrete	$F_{fx} \leq \mu_p F_{fz}$	–19.0	1.95	22.52	–0.56	26.50	37.17
	$F_{fx} \geq -\mu_p F_{fz}$	28.5	–2.65	–31.79	–0.33	37.40	21.80
unpaved road	$F_{fx} \leq \mu_p F_{fz}$	–15.0	1.76	19.24	–1.84	28.30	36.76
	$F_{fx} \geq -\mu_p F_{fz}$	23.3	–2.00	–25.43	–1.22	37.40	24.45
rolled gravel	$F_{fx} \leq \mu_p F_{fz}$	–14.1	1.51	17.19	–0.73	28.65	36.65
	$F_{fx} \geq -\mu_p F_{fz}$	19.8	–1.84	–22.34	–0.52	37.23	26.12
compressed snow	$F_{fx} \leq \mu_p F_{fz}$	–4.5	0.64	6.45	–1.15	32.27	34.86
	$F_{fx} \geq -\mu_p F_{fz}$	6.9	–0.51	–7.08	–1.01	35.39	31.47
ice	$F_{fx} \leq \mu_p F_{fz}$	–2.5	0.31	3.29	–0.34	32.92	34.36
	$F_{fx} \geq -\mu_p F_{fz}$	3.2	–0.26	–3.45	–0.33	34.54	32.70

Tab. 4. Results of robot's mobility analysis for the case of motion with constant velocity and transmission of drivefrom back axle to the front axle via toothed belt

type of ground	constraint	β [°]	τ_b [Nm]	T_t [Nm]	F_{fx} [N]	F_{bx} [N]	F_{fz} [N]	F_{bz} [N]
asphalt and concrete	$F_{fx} \le \mu_p F_{fz}$	–40.4	3.77	1.11	12.80	30.81	15.06	36.25
	$F_{fx} \ge -\mu_p F_{fz}$	40.4	–3.64	–2.57	–30.81	–12.80	36.25	15.06
unpaved road	$F_{fx} \le \mu_p F_{fz}$	–34.2	3.46	1.16	12.68	25.19	18.65	37.04
	$F_{fx} \ge -\mu_p F_{fz}$	34.2	–2.98	–1.98	–25.19	–12.68	37.04	18.65
rolled gravel	$F_{fx} \le \mu_p F_{fz}$	–31.0	3.04	1.08	12.28	22.37	20.46	37.29
	$F_{fx} \ge -\mu_p F_{fz}$	31.0	–2.85	–1.84	–22.37	–12.28	37.29	20.56
compressed snow	$F_{fx} \le \mu_p F_{fz}$	–11.3	1.30	0.59	5.96	7.25	29.81	36.23
	$F_{fx} \ge -\mu_p F_{fz}$	11.3	–0.94	–0.52	–7.25	–5.96	36.23	29.81
ice	$F_{fx} \le \mu_p F_{fz}$	–5.7	0.63	0.30	3.19	3.51	31.88	35.13
	$F_{fx} \ge -\mu_p F_{fz}$	5.7	–0.51	–0.27	–3.51	–3.19	35.13	31.88

Tab. 5. Results of robot's mobility analysis for the case of motion with constant velocity and the independent two-axle drive

type of ground	constraint	β [°]	τ_f [Nm]	τ_b [Nm]	F_{fx} [N]	F_{bx} [N]	F_{fz} [N]	F_{bz} [N]
asphalt and concrete	$F_{fx} \le \mu_p F_{fz}$	–40.4	1.11	2.67	12.80	30.81	15.06	36.25
	$F_{fx} \ge -\mu_p F_{fz}$	40.4	–2.57	–1.07	–30.81	–12.80	36.25	15.06
unpaved road	$F_{fx} \le \mu_p F_{fz}$	–34.2	1.11	2.30	12.68	25.19	18.65	37.04
	$F_{fx} \ge -\mu_p F_{fz}$	34.2	–2.57	–1.00	–25.19	–12.68	37.04	18.65
rolled gravel	$F_{fx} \le \mu_p F_{fz}$	–31.0	1.08	1.97	12.28	22.37	20.46	37.29
	$F_{fx} \ge -\mu_p F_{fz}$	31.0	–1.84	–1.01	–22.37	–12.28	37.29	20.46
compressed snow	$F_{fx} \le \mu_p F_{fz}$	–11.3	0.59	0.71	5.96	7.25	29.81	36.23
	$F_{fx} \ge -\mu_p F_{fz}$	11.3	–0.52	–0.43	–7.25	–5.96	36.23	29.81
ice	$F_{fx} \le \mu_p F_{fz}$	–5.7	0.30	0.33	3.19	3.51	31.88	35.13
	$F_{fx} \ge -\mu_p F_{fz}$	5.7	–0.27	–0.24	–3.51	–3.19	35.13	31.88

- motion with variable speed (accelerating, braking),
- motion with occurrence of wheel sliding,
- traversing various obstacles of environment, e.g. kerbs.

Also planned are investigations which will consist in analysis of various robot's drive system configurations from the point of view of ensuring the best accuracy of realisation of motion in the presence of wheels' slip.

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