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Controllability, Observability and Transfer Matrix Zeroing of the 2D Roesser Model

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Tadeusz Kaczorek, Krzysztof Rogowski

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Abstract:
The article considers the fundamental properties of the two-dimensional (2D) system described by the Roesser model. The controllability and observability are analyzed and the sufficient conditions under which the transfer matrix is zero are given. It is shown that if the matrix of the state equation A and B or A and C of the Roesser model has full row rank (respectively, full column rank) then there exists a nonsingular matrix of transformation such that the new pair of new matrices is controllable (observable). The numerical examples are given to show the correctness of the obtained conditions.

Keywords: *Controllability, Observability, Roesser model, Transfer matrix, Zeroing*

1. Introduction

Two-dimensional (2D) dynamical systems have been an active area of research for many years, owing to their widespread applications in various fields such as physics, biology, engineering, and economics. Some interesting practical applications of a 2D systems theory may be found in [1–5]. The most popular models of 2D linear systems are the models introduced by Roesser [6], Fornasini-Marchesini [7,8] and Kurek [9]. An overview of 2D linear systems theory is given in [10–13].

In this article, we explore the behaviour of two-dimensional dynamical systems, defined by two differential matrix equations describing the dynamics in horizontal and vertical directions. This approach for 2D systems modelling was proposed first by Roesser in [6] and this type of model is called by his name. This class of systems exhibits complex behaviour, since there occurs interference between these two dimensions. Understanding and analysing the dynamics of such systems is crucial for many real-world applications, including the design and control of physical systems such as thermal processes, distributed parameters systems, digital filters, long transmission lines and many others.

The dynamical properties of the Roesser model have been the subject of many papers. The stability problem has been solved in [14–18]. The asymptotic stability of positive 2D linear systems has been investigated in [19–22] and the robust stability in [23,24].

Controllability and observability are fundamental concepts in the theory of dynamical systems and control theory. Controllability refers to the ability to steer a system from one state to another using a control input, while observability refers to the ability to estimate the system's internal state from the available measurements and known steering. These concepts are essential for understanding and designing control systems for a wide range of applications, including aerospace, robotics, power systems, and biomedical engineering.

Controllability conditions for the 2D Roesser model have been obtained by Kurek [25]. These conditions are based on checking the ranks of a system's matrices. Next, the controllability problem has been considered in [26–30]. Observability is a dual notion in comparison with controllability and may be found in [29–31].

In the context of the Roesser model, controllability and observability are critical concepts that determine the effectiveness of control system design. In many practical cases, if 2D system is unobservable and/or uncontrollable, the most popular control strategies cannot be applied. In particular, a controllable and observable system can be easily stabilized using feedback control techniques. This is because the internal state of the system can be accurately estimated from its output measurements, and an appropriate control input can be designed to stabilize the system. Using the proposed similarity operation on the matrices A and B, we may transform our system into a controllable and observable one. This mathematical operation opens up the possibility of utilizing well-known control algorithms to design a desired dynamic of the system. Finally, through the inverse transformation, we return to the original state-space model. The results presented in the manuscript expand the potential for achieving better dynamical performance of the control 2D system.

Another important concept in control theory is the zeroing of the transfer matrix. The transfer matrix of a system is a mathematical representation that describes the relationship between the input and output of the system. Zeroing the transfer matrix of a system refers to the ability to maintain the output of the system at zero, irrespective of the input.

This concept is particularly relevant in the design of robust control systems, where the objective is to ensure that the system remains stable even in the presence of disturbances. This problem has been considered in [32, 33].

In this article, the controllability, observability, and the transfer matrix zeroing of the Roesser model will be considered. It will be shown that for uncontrollable and unobservable matrices of the system, there exists a nonsingular matrix that transforms these matrices into controllable and observable ones. Sufficient conditions for the existence of such a linear transformation will be given and proved. Moreover, sufficient conditions for zeroing of the transfer matrix of the Roesser model will be established. Numerical examples will illustrate the obtained conditions' utility.

2. Controllability and Observability of the Roesser Model

Consider the discrete 2D Roesser-type model described by the state-space equations of the form

$$\begin{aligned} \begin{bmatrix} x^h_{i+1,j} \\ x^v_{i,j+1} \end{bmatrix} &= A \begin{bmatrix} x^h_{ij} \\ x^v_{ij} \end{bmatrix} + Bu_{ij}, \quad i,j = 0,1,\dots \\ y_{ij} &= C \begin{bmatrix} x^h_{ij} \\ x^v_{ij} \end{bmatrix}, \end{aligned} \tag{1a}$$

where $x^h_{ij} \in \mathbb{R}^{n_1}$ is the horizontal state vector, $x^v_{ij} \in \mathbb{R}^{n_2}$ is the vertical state vector, $u_{ij} \in \mathbb{R}^m$ is the input vector, $y_{ij} \in \mathbb{R}^p$ is the output vector and takes the following forms

$$\begin{aligned} &A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \\ &A_{11} \in \mathbb{R}^{n_1 \times n_1}, \quad A_{12} \in \mathbb{R}^{n_1 \times n_2}, \quad B_1 \in \mathbb{R}^{n_1}, \\ &A_{21} \in \mathbb{R}^{n_2 \times n_1}, \quad A_{22} \in \mathbb{R}^{n_2 \times n_2}, \quad B_2 \in \mathbb{R}^{n_2}, \\ &C = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, \quad C_1 \in \mathbb{R}^{p \times n_1}, C_2 \in \mathbb{R}^{p \times n_2}. \end{aligned} \tag{1b}$$

Definition 1. *The Roesser model (1) is called controllable in the rectangle* $[(0,0),(k_1,k_2)]$ *if for every boundary condition* $x^h_{0,j}$, $x^v_{i,0}$, $i \in [0,k_1]$, $j \in [0,k_2]$ *and every vector* $x_f \in \mathbb{R}^n$ *there exists a sequence of inputs* u_{ij}, $(0,0) \le (i,j) < (k_1-1,k_2-1)$ *such that* $x_{k_1,k_2} = x_f$.

Theorem 1. *The Roesser model (1) is controllable in the rectangle* $[(0,0),(k_1,k_2)]$ *if and only if*

$$\operatorname{rank} \begin{bmatrix} \mathbb{I}_{n_1} z_1 - A_{11} & -A_{12} & B_1 \\ -A_{21} & \mathbb{I}_{n_2} z_2 - A_{22} & B_2 \end{bmatrix} = n \tag{2}$$

for $n = n_1 + n_2$ *and all* $z_1, z_2 \in \mathbb{C}$.

The proof is given in [31].

Definition 2. *The Roesser model (1) is called observable in the rectangle* $[(0,0),(k_1,k_2)]$ *if knowing sequences of its inputs* u_{ij} *and outputs* y_{ij} *for* $(i,j) \in [(0,0),(k_1,k_2)]$ *it is possible to compute its unique initial state* $x_{00} \in \mathbb{R}^n$.

Theorem 2. *The Roesser model (1) is observable in the rectangle* $[(0,0),(k_1,k_2)]$ *if and only if*

$$\operatorname{rank} \begin{bmatrix} \mathbb{I}_{n_1} z_1 - A_{11} & -A_{12} \\ -A_{21} & \mathbb{I}_{n_2} z_2 - A_{22} \\ C_1 & C_2 \end{bmatrix} = n \tag{3}$$

for all $z_1, z_2 \in \mathbb{C}$.

The proof is given in [31].

In the next theorem, we will introduce the linear transformation of the state equations (1).

Theorem 3. *If*

$$\operatorname{rank} \begin{bmatrix} A & B \end{bmatrix} = n \tag{4a}$$

and

$$\operatorname{rank} \begin{bmatrix} \overline{A}A^T + \overline{B}B^T \end{bmatrix} = n \tag{4b}$$

then there exists a nonsingular matrix $M \in \mathbb{R}^{n \times n}$ *such that the pair*

$$(\overline{A}, \overline{B}), \quad \overline{A} = MA, \quad \overline{B} = MB \tag{5}$$

is controllable.

Proof. It will be shown that if the pair of matrices (A, B) satisfies the condition (4a) then there exists a nonsingular matrix M such that the pair (5) is controllable. From (5) we have

$$M \begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} \overline{A} & \overline{B} \end{bmatrix}. \tag{6}$$

Postmultiplying (6) by the matrix $\begin{bmatrix} A & B \end{bmatrix}^T$ we obtain

$$M \begin{bmatrix} AA^T + BB^T \end{bmatrix} = \begin{bmatrix} \overline{A}A^T + \overline{B}B^T \end{bmatrix} \tag{7}$$

From (4a) it follows that the matrix $\begin{bmatrix} AA^T + BB^T \end{bmatrix}$ is nonsingular and from (7) we have

$$M = \begin{bmatrix} \overline{A}A^T + \overline{B}B^T \end{bmatrix} \begin{bmatrix} AA^T + BB^T \end{bmatrix}^{-1}. \tag{8}$$

The matrix M given by (8) is nonsingular since $\operatorname{rank} \begin{bmatrix} \overline{A} & \overline{B} \end{bmatrix} = n$. □

Example 1. *Consider the Roesser model (1) with the matrices*

$$\begin{aligned} A &= \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{cc|c} 1 & 0 & 0 \\ -2 & 1 & 2 \\ \hline 1 & 0 & 1 \end{array}\right], \\ B &= \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right] = \left[\begin{array}{c} 0 \\ 1 \\ \hline 0 \end{array}\right]. \end{aligned} \tag{9}$$

The pair (A, B) *given by (9) is uncontrollable but it satisfies the condition (4a) since*

$$\operatorname{rank} \begin{bmatrix} A & B \end{bmatrix} = rank \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ -2 & 1 & 2 & 1 \\ \hline 1 & 0 & 1 & 0 \end{array}\right] = 3 = n. \tag{10}$$

We are looking for the nonsingular matrix $M \in \mathbb{R}^{3\times 3}$ such that the corresponding pair (5) is controllable in the form

$$\overline{A} = \left[\begin{array}{cc|c} 1 & 0 & 1 \\ -2 & 1 & 0 \\ \hline 1 & 0 & 2 \end{array}\right], \quad \overline{B} = \left[\begin{array}{c} 0 \\ 1 \\ \hline 2 \end{array}\right]. \tag{11}$$

Note, that condition (4b) is met, since

$$\operatorname{rank}\left[\overline{A}A^T + \overline{B}B^T\right] = \operatorname{rank}\begin{bmatrix} 1 & 0 & 2 \\ -2 & 6 & -2 \\ 1 & 4 & 3 \end{bmatrix} = 3. \tag{12}$$

Using (8), (9) and (11) we obtain

$$M = \left[\overline{A}A^T + \overline{B}B^T\right]\left[AA^T + BB^T\right]^{-1}$$
$$= \left\{\begin{bmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right\}$$
$$\times \left\{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right\}^{-1}$$
$$= \begin{bmatrix} 1 & 0 & 2 \\ -2 & 6 & -2 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 10 & 0 \\ 1 & 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{bmatrix}. \tag{13}$$

The matrix (13) is nonsingular since

$$\det M = \begin{vmatrix} 0 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 1 & 0 \end{vmatrix} = -1. \tag{14}$$

Taking into account the duality of controllability and observability property, a similar approach may be applied to the pair of matrices (A, C).

Theorem 4. *If*

$$\operatorname{rank}\begin{bmatrix} A \\ C \end{bmatrix} = n \tag{15a}$$

and

$$\operatorname{rank}\left[A^T\overline{A} + C^T\overline{C}\right] = n. \tag{15b}$$

then there exists a nonsingular matrix $N \in \mathbb{R}^{n\times n}$ such that the pair

$$(\hat{A}, \hat{C}), \quad \hat{C} = AN, \quad \hat{C} = CN \tag{16}$$

is observable.

Proof. The proof is similar (dual) to the proof of Theorem 2. □

Remark 1. *In a similar way as in the proof of Theorem 2 it can be shown that there exists a nonsingular matrix $\overline{M}$ satisfying the equality*

$$\overline{M}\begin{bmatrix} A & B \end{bmatrix} = \begin{bmatrix} \hat{A} & \hat{B} \end{bmatrix}, \tag{17}$$

where the pair (A, B) is controllable and the pair $(\hat{A}, \hat{B})$ is uncontrollable.

3. Zeroing of the Transfer Matrix of the Roesser Model

Consider the Roesser model (1) with the transfer matrix

$$T(z_1, z_2) =$$
$$\begin{bmatrix} C_1 & C_2 \end{bmatrix}\begin{bmatrix} \mathbb{I}_{n_1}z_1 - A_{11} & -A_{12} \\ -A_{21} & \mathbb{I}_{n_2}z_2 - A_{22} \end{bmatrix}^{-1}\begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \tag{18}$$

Sufficient conditions will be established for zeroing the transfer matrix (18).

Theorem 5. *The transfer matrix (18) of the Roesser model (1) is a zero matrix if the following conditions are satisfied:*

1) *the pair (A, B) is uncontrollable and the pair (A, C) is unobservable;*
2) *the product of the matrices C and B is zero matrix, i.e.*

$$CB = 0. \tag{19}$$

Proof. Similarly to the standard linear system [33] for the proof of the Roesser model we have the following properties:

1) If the pair (A, B) is uncontrollable and the pair (A, C) is unobservable then at least one entry of the adjoint matrix

$$\operatorname{adj}\begin{bmatrix} \mathbb{I}_{n_1}z_1 - A_{11} & -A_{12} \\ -A_{21} & \mathbb{I}_{n_2}z_2 - A_{22} \end{bmatrix} \tag{20}$$

is zero.

2) The product CB chooses in the matrix (20) the zero entries.

Therefore, if conditions 1) and 2) are satisfied then the transfer matrix (18) is a zero matrix. □

Example 2. *Consider the Roesser model (1) with the matrices*

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{cc|c} 1 & 2 & 2 \\ -1 & -2 & -2 \\ \hline 1 & 1 & 0 \end{array}\right],$$
$$B = \left[\begin{array}{c} B_1 \\ \hline B_2 \end{array}\right] = \left[\begin{array}{c} 1 \\ -1 \\ \hline 1 \end{array}\right], \tag{21}$$
$$C = \left[\begin{array}{c|c} C_1 & C_2 \end{array}\right] = \left[\begin{array}{cc|c} 1 & 1 & 0 \end{array}\right].$$

Note that the pair (A, B) is uncontrollable since

$$\operatorname{rank}\left[\begin{array}{cc|c} \mathbb{I}_{n_1}z_1 - A_{11} & -A_{12} & B_1 \\ -A_{21} & \mathbb{I}_{n_2}z_2 - A_{22} & B_2 \end{array}\right]$$
$$= \operatorname{rank}\left[\begin{array}{ccc|c} z_1 - 1 & -2 & -2 & 1 \\ 1 & z_1 + 2 & 2 & -1 \\ -1 & -1 & z_2 & 1 \end{array}\right] < 3 = n \tag{22}$$

and the pair (A, C) is unobservable since

$$\operatorname{rank}\left[\begin{array}{cc} \mathbb{I}_{n_1}z_1 - A_{11} & -A_{12} \\ -A_{21} & \mathbb{I}_{n_2}z_2 - A_{22} \\ \hline C_1 & C_2 \end{array}\right]$$
$$= \operatorname{rank}\left[\begin{array}{ccc} z_1 - 1 & -2 & -2 \\ 1 & z_1 + 2 & 2 \\ -1 & -1 & z_2 \\ \hline 1 & 1 & 0 \end{array}\right] < 3 = n. \tag{23}$$

Therefore, the Roesser model with (21) is uncontrollable and unobservable.

The condition (19) is also satisfied since

$$CB = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = 0. \tag{24}$$

The transfer function of the Roesser model with the matrices (21) is zero since

$$\begin{aligned} T(z_1, z_2) &= \\ &= \begin{bmatrix} 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} z_1 - 1 & -2 & -2 \\ 1 & z_1 + 2 & 2 \\ -1 & -1 & z_2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \\ &= 0, \end{aligned} \tag{25}$$

since

$$\begin{bmatrix} z_1 - 1 & -2 & -2 \\ 1 & z_1 + 2 & 2 \\ -1 & -1 & z_2 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{z_1 z_2 + 2z_2 + 2}{z_1 z_2 (1+z_1)} & \frac{z_2 + 1}{z_1 z_2 (1+z_1)} & \frac{2}{z_1 z_2 + z_2} \\ -\frac{z_2 + 2}{z_1 z_2 (1+z_1)} & \frac{z_1 z_2 - z_2 - 2}{z_1 z_2 (1+z_1)} & \frac{-2}{z_1 z_2 + z_2} \\ \frac{1}{z_1 z_2} & \frac{1}{z_1 z_2} & \frac{1}{z_2} \end{bmatrix}. \tag{26}$$

This simple example confirms the Theorem 5.

Theorem 6. *The transfer matrix (18) of the Roesser model (1) is zero if*

1)

$$\operatorname{rank} \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \end{bmatrix} < n, \quad \operatorname{rank} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ C_1 & C_2 \end{bmatrix} < n; \tag{27}$$

2) the condition (19) is satisfied, i.e.

$$\begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = C_1 B_1 + C_2 B_2 = 0. \tag{28}$$

Proof. From (2) and (3) it follows that if the conditions (27) are satisfied then the Roesser model (1) is uncontrollable and unobservable. Therefore, by Theorem 5 the transfer matrix (18) of the Roesser model (1) is zero. □

Example 3. *Consider the Roesser model (1) with the matrices (21). The model satisfies the condition (27) since*

$$\begin{aligned} \operatorname{rank} \begin{bmatrix} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \end{bmatrix} &= \operatorname{rank} \begin{bmatrix} 1 & 2 & 2 & 1 \\ -1 & -2 & -2 & -1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\ &= 2 < n = 3, \\ \operatorname{rank} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ C_1 & C_2 \end{bmatrix} &= \operatorname{rank} \begin{bmatrix} 1 & 2 & 2 \\ -1 & -2 & -2 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \\ &= 2 < n = 3. \end{aligned} \tag{29}$$

From (24) it follows that the condition $CB = 0$ is also satisfied. Therefore, by Theorem 6 the transfer matrix of the Roesser model with the matrices (21) is zero.

4. Concluding Remarks

The Roesser model is one of the most popular mathematical models used in control systems engineering to describe the dynamics of processes taking place in two dimensions. This article explores the concepts of controllability, observability, and zeroing of the transfer matrix in the context of the Roesser model.

Zeroing of the transfer matrix refers to the ability to drive the output of the system to zero, irrespective of the input. This concept is particularly relevant in the design of robust control systems, where the objective is to ensure that the system remains stable even in the presence of disturbances.

In this article, we have shown that for uncontrollable and unobservable matrices of the system, there exists a nonsingular matrix that transforms these matrices into controllable and observable ones. Sufficient conditions for the existence of such linear transformation have been given and proved. Moreover, sufficient conditions for the transfer matrix zeroing of the Roesser model have been established. Numerical examples have been presented that show the usefulness of the introduced conditions.

AUTHORS

Tadeusz Kaczorek – Faculty of Electrical Engineering, Bialystok University of Technology, Wiejska 45D, 15-351 Bialystok, Poland, e-mail: t.kaczorek@pb.edu.pl.

Krzysztof Rogowski* – Faculty of Electrical Engineering, Bialystok University of Technology, Wiejska 45D, 15-351 Bialystok, Poland, e-mail: k.rogowski@pb.edu.pl, www: www.we.pb.edu.pl/kair/krzysztof-rogowski/.

*Corresponding author

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