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# Design of Fault-tolerant Structures for Underwater Sensor Networks Based on Markov Chains

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**Abstract:**
*Recently, underwater sensor networks (USN) have gained widespread adoption, proving instrumental in diverse research endeavors within the marine environment and across various scientific and technological domains. Considering the challenging operational conditions, a paramount issue in the present day is the creation of USNs with a predetermined fault tolerance margin. The primary focus of this study is the development and evaluation of a design method for fault-tolerant structures of USNs using Markov chains. The proposed method enables the creation of suitable USNs' structures, ensuring a guaranteed attainment of the desired number of operational cycles before failure at reasonable costs. Utilizing the mathematical framework of Markov chains, coupled with the strategy of a well-selected sequence of hierarchy levels, enables the precise determination of the network's actual number of operation cycles before failure as well as performing efficient reservation without excessive redundancy. To evaluate the efficacy of the proposed method, this study undertakes a specific case – the development of a fault-tolerant structure for a real USN with predetermined parameters. The analysis of the results obtained confirms the high feasibility of utilizing the developed method for crafting USNs for various purposes, ensuring a predefined level of fault tolerance while maintaining acceptable costs.*

**Keywords:** *underwater sensor network, fault tolerance, hierarchical structure, structure designing method, Markov chains*

## 1. Introduction

In the continuum of technological progress, sensor networks (SN) emerge as a pivotal paradigm, ushering in a new era of pervasive data acquisition and dissemination [1–3]. Comprising a distributed ensemble of autonomous sensor nodes endowed with diverse sensing modalities, these networks have witnessed prolific growth across a spectrum of disciplines. From ecological monitoring to industrial process control, healthcare applications to urban infrastructure, sensor networks have become instrumental in furnishing real-time insights that underlie empirical decision-making and propel operational efficiencies across multifarious domains.

The ascendancy of sensor networks can be attributed to their innate adaptability and the expansiveness of their application domain. These networks serve as the indispensable nexus between the corporeal and the digital, engendering the capacity to glean and scrutinize data from hitherto inaccessible or impracticable environs. Their deployment within environments as variegated as urban metropolises, agrarian landscapes, and subaquatic realms attests to their versatility and underscores their transformative potential.

One of the key advantages of sensor networks is their ability to provide real-time, high-resolution data over a wide area [4–6]. They enable data-driven decision-making, allowing for timely responses to changing conditions. Moreover, they can be deployed in harsh or hazardous environments where human intervention may be impractical or risky. Sensor networks also offer scalability, meaning they can be adapted to suit the size and requirements of a specific application. Additionally, they contribute to resource conservation and optimization by enabling targeted interventions based on accurate data.

The future of sensor networks is promising, with ongoing research and development in several key areas. Advancements in energy harvesting technologies are expected to further extend the lifespan of sensor nodes, reducing the need for frequent battery replacements [7, 8]. Integration with artificial intelligence and machine learning will enhance the capabilities of sensor networks for data analysis and decision-making. The emergence of low-power, long-range communication technologies like LoRaWAN and NB-IoT is expanding the reach and applicability of sensor networks. Additionally, advancements in sensing technologies, such as the development of more specialized and accurate sensors, will contribute to the growth and effectiveness of sensor networks in various industries. Overall, sensor networks are a transformative technology with wide-ranging applications and significant advantages in data-driven decision-making and resource optimization. Ongoing research and technological advancements promise to further enhance their capabilities, opening up new possibilities for innovation and impact across various industries.

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In turn, sensor networks have a wide range of applications, including environmental monitoring, industrial automation, healthcare, agriculture, smart cities, and more. In environmental monitoring, they are employed to track parameters like air quality, weather conditions, and pollution levels [9]. In industrial settings, they facilitate real-time data collection for process control and optimization [10]. In healthcare, sensor networks are involved in patient monitoring, fall detection, and other health-related applications [11]. In agriculture, they aid in precision farming by monitoring soil conditions, weather, and crop health [2, 7]. Smart cities use sensor networks for traffic management, waste management, energy conservation, and public safety [12].

The use of underwater sensor networks (USN) deserves special attention [13–15]. USNs represent a transformative technological frontier, designed to navigate and elucidate the depths of our planet's aquatic environments. These networks comprise autonomous sensor nodes, equipped with a diverse array of specialized sensors, collectively tasked with capturing and disseminating invaluable data from underwater domains. Their significance lies in their capacity to augment our understanding of submerged ecosystems, inform resource management strategies, and enable critical applications in fields ranging from marine science to maritime security. The relevance of USNs is manifest across a spectrum of disciplines. In marine biology and oceanography, they serve as indispensable tools for the study of marine life behavior, tracking migratory patterns, and monitoring the health of delicate ecosystems. Furthermore, in the realm of environmental monitoring, USNs play a pivotal role in assessing water quality, tracking pollution levels, and conducting comprehensive studies of aquatic habitats. In maritime industries, they contribute to the optimization of vessel navigation, support offshore resource exploration, and facilitate the maintenance of underwater infrastructure.

Parameters monitored by USNs encompass a multitude of critical environmental indicators [15]. These include temperature, a fundamental parameter influencing marine ecosystems and climate patterns. Salinity, a measure of water's salt content, is essential for understanding oceanographic phenomena and assessing ecological niches. Pressure measurements afford insights into ocean depths and topography, vital for comprehensive marine surveys. Acoustic data, capturing underwater soundscapes, provides a wealth of information on marine life behavior, as well as anthropogenic noise pollution. Additionally, sensors for dissolved oxygen, pH levels, turbidity, and nutrient concentrations round out the suite of instruments utilized in USNs, providing a comprehensive perspective on the aquatic environment. In achieving their mission, USNs employ a variety of specialized sensors tailored to the challenging underwater milieu.

Acoustic sensors, known as hydrophones, are pivotal in capturing underwater sounds, enabling the study of marine life behavior, as well as detecting seismic activity. Temperature sensors, often utilizing thermistors or thermocouples, monitor water temperatures, which play a crucial role in oceanographic studies. Conductivity sensors, along with temperature and pressure sensors, facilitate the calculation of salinity levels. Pressure sensors themselves are instrumental in providing depth measurements, contributing to bathymetric mapping and maritime navigation. Optical sensors, designed to measure parameters like turbidity or light penetration, are employed in environments with sufficient visibility. These diverse sensor modalities collectively empower USNs to unravel the mysteries of our planet's submerged ecosystems and contribute to informed decision-making in a range of critical applications.

Regardless of this, USNs face unique challenges due to the harsh underwater environment, including limited communication range, high propagation delay, and the need for energy-efficient communication protocols. Researchers in this field work on developing specialized hardware, communication protocols, and deployment strategies to overcome these challenges and enhance the capabilities of underwater sensor networks [14,16,17]. However, one of the most important problems in this area today is the creation of underwater sensor networks with a set fault tolerance margin [18, 19]. A successful solution to this problem can be found by designing an optimal network structure which will allow the creation of USNs with a specified operating time before failure. To assess fault tolerance and further synthesize the most suitable USN structure, it is advisable to apply the mathematical apparatus of Markov chains, which is effectively used for calculating the required number of operation cycles before failure of various complex systems [20].

This study is dedicated to the development and research of the designing method of fault-tolerant structures for underwater sensor networks based on Markov chains. The subsequent sections of this paper are organized as follows. Section 2 offers a concise review of pertinent literature in this field, establishing the primary aim of this study. Section 3 presents a step-by-step method for designing the USNs fault-tolerant structures, as proposed in this research. Following that, in Section 4, a study of the effectiveness of the developed method is carried out using the example of the synthesis of a specific network. Finally, Section 5 provides conclusions and outlines potential avenues for future research endeavors.

## 2. Brief Literature Review

Today, research on underwater sensor networks is intensively carried out in the following main areas.

*Energy-efficient communication protocols.* Ongoing research in underwater sensor networks focuses on the development of energy-efficient communication protocols tailored to the challenges of underwater acoustic channels [21, 22]. Researchers are exploring adaptive protocols that can effectively navigate variable propagation delays and limited bandwidth, ensuring optimal data transmission and prolonged network operation.

*Localization and navigation.* Advancements in accurate localization techniques for underwater sensor nodes are very important [23, 24]. Precise localization is essential for oceanographic studies, environmental monitoring, and navigation of autonomous underwater vehicles. Researchers are developing novel methods to enhance the accuracy of node positioning in dynamic underwater environments.

*Acoustic communication and networking.* The improvement of acoustic communication in USNs is one of the critical areas of exploration [25]. Research efforts are directed toward enhancing the reliability, data rates, and range of acoustic communication. This includes investigating innovative modulation techniques and signal processing algorithms to overcome the challenges posed by underwater acoustic channels.

*Sensing technology and sensor integration.* Research in sensing technology for USNs involves the development of specialized and accurate sensors [26, 27]. This includes advancements in chemical, biological, and optical sensors to expand the data collection capabilities of USNs. Integrating diverse sensors contributes to a more comprehensive understanding of underwater environments.

*Adaptive and cognitive sensor networks.* Research is dedicated to creating adaptive and cognitive sensor networks that can autonomously adjust to changing underwater conditions [28, 29]. This involves exploring self-configuring, self-optimizing, and learning-based approaches, such as cognitive radio techniques, to dynamically adapt to spectrum availability and mitigate interference.

*Security and privacy.* Ensuring the security and privacy of data in USNs is an equally important task when developing these networks. Scientists are designing secure communication protocols and encryption techniques to protect data in transit, addressing the unique challenges posed by the underwater environment [30, 31].

*Biologically inspired approaches for USNs optimization.* Bio-inspired algorithms and strategies are being researched to optimize USN functionality [32, 33]. This includes the use of evolutionary and swarm intelligence algorithms to optimize structures, energy consumption, routing, node aggregation, power control, scheduling of sensor activities, functionality, and more.

Bio-inspired intelligent approaches [34–36], soft computing [37–39] and different machine learning algorithms [40–42] have proven themselves to be very effective tools for solving diverse problems in various fields.

Along with the areas discussed, even more significant is research aimed at increasing the fault tolerance, reliability and survivability of underwater sensor networks [43, 44]. The importance of fault tolerance in underwater sensor networks cannot be overstated, as these networks operate in challenging and dynamic aquatic environments where sensor nodes are susceptible to various forms of failure. Fault tolerance is crucial for ensuring the reliability and continuous operation of USNs, where sensor nodes may experience malfunctions due to factors such as harsh underwater conditions, hardware failures, or energy depletion. Given the remote and inaccessible nature of many underwater deployment sites, timely maintenance and node replacement are often impractical. Fault-tolerant mechanisms allow USNs to detect, isolate, and adapt to faults, ensuring that the network can continue its mission-critical functions even in the presence of node failures. This resilience is vital for applications ranging from environmental monitoring and marine research to underwater infrastructure maintenance and defense operations, where uninterrupted and accurate data collection is paramount. Fault-tolerant USNs contribute to the overall robustness and effectiveness of underwater exploration and monitoring, providing a foundation for sustained and reliable performance in challenging underwater environments.

The quest for enhancing the fault tolerance of underwater sensor networks has spurred a prolific body of research, and a comprehensive review of the methods reveals a dynamic landscape of innovative approaches. Several key strategies have been explored to fortify USNs against the challenges posed by the harsh underwater environment [45, 46]. The utilization of advanced error detection and correction mechanisms, including error-correcting codes and checksums, has shown promise in mitigating the impact of data transmission errors [20, 47]. Additionally, adaptive routing protocols dynamically reroute data paths in response to changing network conditions, offering a proactive approach to fault tolerance [48,49]. Machine learning algorithms are increasingly employed for anomaly detection, allowing USNs to identify and isolate faulty nodes based on deviations from normal behavior [33, 50, 51]. Energy-aware strategies, focusing on efficient power management and the utilization of energy harvesting technologies, contribute not only to prolonging network lifespan but also to bolstering fault tolerance [52]. Moreover, redundancy-based methods, such as node duplication and data replication, have been prominent, providing resilience by ensuring that critical functions persist even in the face of node failures [19, 53, 54].

In addition, Markov chains are quite effectively used to assess the obtained fault tolerance and analyze the reliability of the generated architectures of various sensor networks [55–57].

Despite the constant increasing of hardware reliability, efficiency of routing and data processing, redundancy of sensors, etc., the challenge of creating an architecture guaranteed to provide the required number of cycles before failure for each specific case with the given network parameters remains unsolved.

Hence, it is advisable to develop a method of designing fault-tolerant structures for underwater sensor networks using Markov chains mathematical apparatus. This approach will ensure the creation of underwater sensor networks that exhibit the necessary number of operation cycles before encountering failure for the given USN parameters. Therefore, the main aim of this paper is to develop and investigate a design method of fault-tolerant structures for underwater sensor networks utilizing the principles of Markov chains.

The main contributions of the authors are as follows:

a creation of the step-by-step method for designing the fault-tolerant structures of the underwater sensor networks based on Markov chains;

b conduction of the effectiveness study of the developed method using the example of the synthesis of the specific underwater sensor network with the set parameters.

## 3. Development of the Designing Method of Fault-tolerant Structures for Underwater Sensor Networks Based on Markov Chains

### 3.1. Basic Properties and Generalized Functional Structure of the Underwater Sensor Network

Specialized underwater sensor networks are used to collect various types of information about the state of the underwater environment, transmit this information, and further process it. The primary collection of information is carried out from certain information sources using specialized sensors installed in the underwater environment. In turn, the number of data sources and, accordingly, sensors is equal to $X_0$. Signals from these sensors are transmitted to the main information processing unit using data transmission channels.

Since the number of sensors (initial information sources) is quite large (can number several hundreds or thousands), the transfer of data directly from them to the main information processing node through communication channels is quite complicated. This is explained by the fact that the sensors will need to have a sufficiently high signal transmission power and, accordingly, high energy consumption to ensure the transmission of a large amount of information over long distances.

And the main information processing unit will have very high complexity and cost, as well as fairly low reliability, which, of course, will significantly increase the cost and reduce the reliability of the entire system. Therefore, in order to reduce the overall cost and increase the reliability of the USN, it is advisable to form its structure according to a hierarchical principle. To do this, it is advisable to use intermediate data collection nodes that will perform functions of signal grouping with certain information processing. As a result of such processing, these nodes will transmit further smaller amounts of information while preserving all valuable data. Thus, intermediate nodes of the 1st level of the hierarchy will group and process signals from sensors and send them to nodes of the 2nd level of the hierarchy, and so on to the main node of the system.

In this paper we consider the tree-like or pyramidal architecture that is a specific type of hierarchical architecture. In this structure, nodes are organized in a tree-like fashion, with multiple levels and the main sink node at the root. Each level consists of nodes that communicate with nodes in the level above them, ultimately reaching the sink node. The given architecture is highly effective and can provide reduced energy consumption and improved scalability as well as adaptation to varying node densities. In particular, by organizing nodes hierarchically, the number of hops required for data transmission to the sink node is minimized. This can lead to significant energy savings, especially for nodes located deeper in the water. Moreover, this architecture can be well-suited for larger networks, as it provides a structured way to manage communication and data flow. Finally, in environments with varying node densities, the tree-like structure allows for more efficient data collection and transmission.

The generalized functional structure of the hierarchical underwater sensor network is presented in Fig. 1.

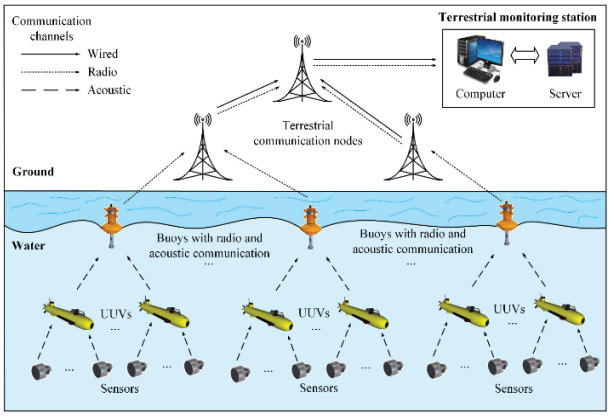



**Figure 1.** Generalized functional structure of the hierarchical underwater sensor network.

In this system, the lowest (zero) level is the level of underwater sensors, which are used to measure various parameters (pressure, temperature, salinity, pH, current speed, etc.). Typically, these sensors have built-in power sources and contactlessly transmit measured information to higher levels using acoustic signals. The possible operating time of these sensors depends on the capacity of the power source, the required frequency of measurement and information transmission, as well as the signal transmission power to ensure data transfer over specified distances. In some cases, sensors can be powered by wire, for example, in the presence of nearby sources of alternative energy (wind, wave, etc.). In turn, the data transmission can also be carried out in the same way, but wired connections significantly complicate the initial deployment and further maintenance of the network in the event of a breakdown.

The next (first) level of the network is implemented using unmanned underwater vehicles (UUVs), which receive information from sensors, process it in a certain way and transmit it further to higher levels of the hierarchy. Moreover, UUVs also transmit data using acoustic signals; however, due to some filtering, processing and structuring, smaller volumes of higher quality information are transmitted.

This makes it possible to significantly save data transmission energy without overloading the network with redundant unnecessary information. These underwater vehicles must be replaced periodically to recharge power supplies and perform maintenance. At higher levels of the USN's hierarchy there are surface buoys that receive acoustic signals underwater from the UUVs and, using radio signals, transmit information above the water to terrestrial communication nodes. These buoys with radio and acoustic communication can have either autonomous power sources or be powered by nearby alternative energy sources.

In turn, the terrestrial communication nodes transmit information using radio signals or by wires. As a result, all data flows through the main node at the highest level of the hierarchy to the terrestrial monitoring station, where the information is analyzed on computers and further stored on servers. In addition, the information is also processed and structured on the buoys and terrestrial communication nodes, as on the UUVs. The number of hierarchy levels implemented on UUVs, buoys and terrestrial communication nodes may vary depending on specific conditions, the selected network topology, the required data transmission distance, the distance of data collection from the shore and the monitoring station and many other factors. Herewith, the fewer the hierarchy levels and, accordingly, data transmission stages, the more powerful the signal transmitters, power supplies and computing capabilities of nodes must be to simultaneously process a large amount of information and send it over long distances. This will substantially affect the overall cost of the network and the significance of the loss of its individual elements. On the other hand, with a smaller number of sequentially connected levels, reliability and fault tolerance will increase in a certain way. This paper does not consider cost optimization of the network structure. However, it should be borne in mind that using a large number of simple and cheap elements will cost less than a smaller number of expensive ones, since the cost of nodes increases exponentially with the increase in the number of input signals and the required data transmission range. This pattern is even more pronounced when it is necessary to reserve elements of certain levels of the hierarchy.

When creating an USN, its structure can be denoted by the vector **X** with dimension $i_{max} + 1$, which corresponds to the total (pre-selected) number of hierarchy levels and a zero level. Each variable of this vector $X_i$ $(i = 0, 1, \dots, i_{\max})$ corresponds to the selected number of nodes on the *i*-th level of the hierarchy. In turn, the number of nodes $X_i$ at the *i*-th level of the hierarchy can be chosen according to the condition (1),

$$X_i \leq \text{floor}\left(\frac{X_{i-1}}{2}\right), \tag{1}$$

since each node of a given hierarchy level (except zero) must have at least 2 inputs $(n_i \geq 2, i = 1, \dots, i_{\max})$, so that the dimension of the levels decreases as information moves to the upper level. At the same time, $X_{i\ \max} = 1$, since there is only one main node of the system at the last (highest) level of the hierarchy. Also, at the penultimate level of the hierarchy $(i_{\max} - 1)$, the number of nodes must be at least 2 $(X_{i\ \max-1} \geq 2)$.

If $X_i$ nodes are selected at each *i*-th level of the structure hierarchy, the number of inputs $n_i$ for each node can be calculated based on the dependence (2)

$$n_i = \frac{X_{i-1}}{X_i}. \tag{2}$$

If the calculated value of $n_i$ is an integer, then all nodes at a given level of the hierarchy will have the given number of inputs. Otherwise, the number of node inputs is calculated as follows. At a given level of hierarchy there will be nodes with the number of inputs equal to the largest integer less than $n_i$, and with the number of inputs equal to the smallest integer greater than $n_i$. In this case, the number of nodes with the number of inputs equal to the smallest integer greater than $n_i$, as a percentage of the total number of nodes of a given hierarchy level, will be equal to the fractional part of the calculated number $n_i$.

For example, the structure of the USN for the number of initial sensors (information sources) $X_0 = 13$ and three hierarchy levels $(i_{\max} = 3)$ can be schematically depicted in Fig. 2.

The nodes of the system are sequentially connected by communication channels, starting from the 1st level of the hierarchy to the last $i_{\max}$. At the same time, nodes of the level $(i - 1)$ are connected only to nodes of the level *i*, taking into account the number of inputs of the latter. The output of each node of the level $(i - 1)$ can be connected to only one input of the node of the level *i*.

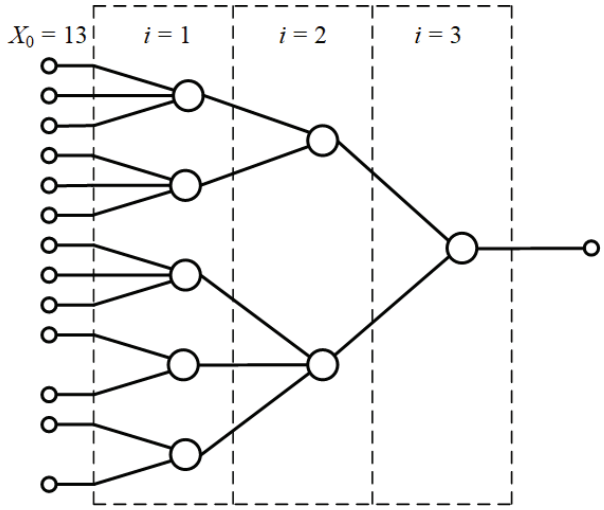



**Figure 2.** Schematic hierarchical structure of the USN with $X_0 = 13$ and $i_{\max} = 3$.

For a given network, the vector **X** will have the form

$$\mathbf{X} = \{13, 5, 2, 1\}. \tag{3}$$

Since the main components of the network operate in harsh underwater environments, where various types of malfunctions and failures occur quite often, the most important task is providing the specified fault-tolerant qualities to extend the USN life and maximize the effective realization of its internal potential. To ensure the required fault tolerance margin of an underwater sensor network, it is necessary to synthesize its hierarchical structure using the advanced method proposed by the authors, the main aspects of which are discussed in the next subsection.

### 3.2. Basic Aspects of the Designing Method of Fault-tolerant Structures for Underwater Sensor Networks

Fault tolerance of an underwater sensor network is the ability to maintain its full or partial functionality after the failure of one or more of its components [56, 58]. Fault tolerance is determined by the number of single failures of the component elements (malfunctions) of the system, after the occurrence of which the operability of the system as a whole is still maintained. The basic level of fault tolerance implies protection against failure of any element. Therefore, the main way to increase fault tolerance is redundancy. This approach has been successfully used to manage fault tolerance and survivability both in USNs and in a number of other critical infrastructure facilities [59, 60]. Redundancy is most effectively implemented in hardware, through reservation. In this case, failures due to wear and aging are not considered, since for the main elements of the system, the energy reserve of the power sources is used up much earlier than the intended operating life. The main reason for system failure before the power supply runs out is random failures caused by harsh network operating environment.

To determine the fault tolerance of a USN with a branched hierarchical structure, it is necessary to calculate the maximum number of cycles of its operation $N_{\max}$ before failure occurs in conditions of periodic malfunctions of its individual elements. If it is necessary to determine the maximum operating time of the system before failure, the operating cycles can be quite easily translated into time, knowing the duration of one cycle. By the USN failure we mean the inability to transmit a certain number of signals (a certain percentage of information) through its nodes and channels. For example, a system can be considered operational as long as it transmits signals from more than 50% of the initial measurement sensors. If information is transmitted from no more than 50% or less of the sensors, then it can be considered that a system failure has occurred. Since the levels of the USN hierarchy are connected to each other in series, it is obvious that the failure of the entire system can occur as a result of the failure of at least one of its hierarchy levels. In turn, this can happen when 50% of the nodes or channels (or nodes and channels) of a given hierarchy level are down or damaged due to disturbances.

Failure or damage of any one node (or channel) of any hierarchy level will be called a malfunction in the operation of this hierarchy level. Thus, a failure of a certain hierarchy level (that corresponds to the failure of the whole system) can occur as a result of sequential malfunctions, the number of which is equal to the (prespecified) critical number of nodes $X_{ic}$ of a given $i$-th hierarchy level ($i = 0, 1, \ldots, i_{\max}$). For example, when selecting a critical number of 50% of signals from all sensors, for a level with 10 nodes, the failure will be considered the malfunctions of 5 or more nodes ($X_{ic} \geq 0.5X_i \geq 5$).

The probability of a malfunction occurring at each specific level of the USN $p_{\mathrm{M}i}$ can be preset or calculated based on the average value of the number of operation cycles between single malfunctions $N_{\mathrm{M}i}$ (determined experimentally) in accordance with the expression (4)

$$p_{\mathrm{M}i} = \frac{1}{N_{\mathrm{M}i}}. \tag{4}$$

In turn, the probability $p_{\mathrm{M}i}$ can differ significantly at different hierarchical levels of the USN, which is caused by different properties of the elements and different environmental conditions in which these elements are located and operate. For instance, the lowest (zero) level of the hierarchy, which includes sensors for the initial measurement of quantities, has the highest probability of a malfunction occurring, since, due to the harsh conditions of being in the underwater environment, its sensors can quite often fail due to mechanical damage or loss connections with nodes of higher levels of the system. The levels with underwater vehicles and buoys are also located in aggressive underwater and surface environments, respectively; however, they have lower probability values $p_{\mathrm{M}i}$ due to their higher inherent reliability.

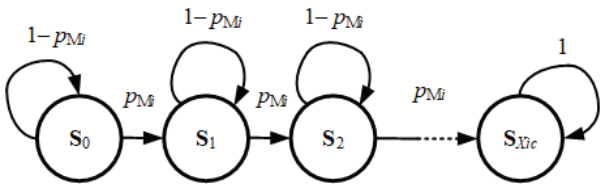



**Figure 3.** A homogeneous Markov chain for calculating the probability of failure of the i-th level of the USN.

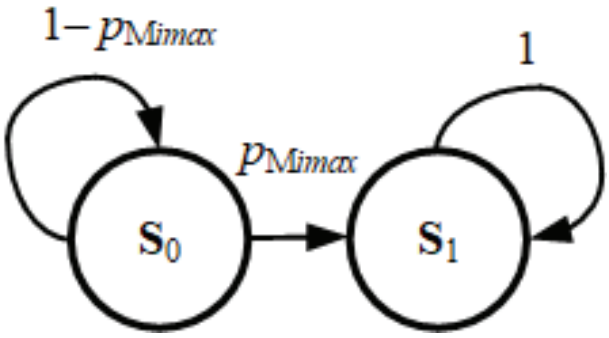



**Figure 4.** A homogeneous Markov chain for calculating the probability of failure of the last (highest) level of the USN.

As for the levels with the terrestrial communication nodes including the highest level with the main node, their probability values $p_{Mi}$ are much lower, since they are on the ground and have a fairly high reliability.

To calculate the probability of failure $p_{Fi}$ of the $i$-th level of the USN, it is advisable to use a simple homogeneous Markov chain, the structure of which is shown in Fig. 3 [61].

The state $\mathbf{S}_0$ corresponds to the absence of a malfunction when performing the next operating cycle. If the first malfunction (a failure of one node) occurs during the operation of the system at the $i$-th level of the hierarchy, then the transition to the state $\mathbf{S}_1$ is performed. The probability of this transition is $p_{Mi}$. On the next cycle, with the probability $p_{Mi}$, a second malfunction (which corresponds to the inoperative state of two nodes of this level) may occur at this level of the hierarchy (transition to the state $\mathbf{S}_2$) or with the probability $1 - p_{Mi}$ no malfunction will occur, which will correspond to the preservation of the state $\mathbf{S}_1$. In turn, returning from the state $\mathbf{S}_1$ to the previous state $\mathbf{S}_0$ is impossible. A failure of the whole level corresponds to the $\mathbf{S}_{Xic}$ state, into which the system will fall after $X_{ic}$ malfunctions occur at the $i$-th hierarchy level (i.e., a critical number of nodes $X_{ic}$ fail sequentially for a given hierarchy level). Transitions from the state $\mathbf{S}_{Xic}$ to the other states are not possible. For the calculation, we assume that the system moves from the state $\mathbf{S}_{Xic}$ to this state with the probability 1.

Herewith, for the last (highest) level of the USN's hierarchy, on which there is a single main node ($X_i = 1$), the Markov chain will have the form presented in Fig. 4.

To describe the probabilities of transitions from state to state at the $i$-th level of the hierarchy ($i$ = 0, 1, ..., $i_{max}$), it is advisable to use matrices of transition probabilities $R_{Xic}$. As an example, for hierarchy levels with a critical number of nodes $X_{ic}$ = 2, $X_{ic}$ = 3, $X_{ic}$ = 4, $X_{ic}$ = 5, these matrices have the form:

$$R_2 = \begin{vmatrix} 1-p_{Mi} & p_{Mi} & 0 \\ 0 & 1-p_{Mi} & p_{Mi} \\ 0 & 0 & 1 \end{vmatrix}; \quad (5)$$

$$R_3 = \begin{vmatrix} 1-p_{Mi} & p_{Mi} & 0 & 0 \\ 0 & 1-p_{Mi} & p_{Mi} & 0 \\ 0 & 0 & 1-p_{Mi} & p_{Mi} \\ 0 & 0 & 0 & 1 \end{vmatrix}; \quad (6)$$

$$R_4 = \begin{vmatrix} 1-p_{Mi} & p_{Mi} & 0 & 0 & 0 \\ 0 & 1-p_{Mi} & p_{Mi} & 0 & 0 \\ 0 & 0 & 1-p_{Mi} & p_{Mi} & 0 \\ 0 & 0 & 0 & 1-p_{Mi} & p_{Mi} \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}; \quad (7)$$

$$R_5 = \begin{vmatrix} 1-p_{Mi} & p_{Mi} & 0 & 0 & 0 & 0 \\ 0 & 1-p_{Mi} & p_{Mi} & 0 & 0 & 0 \\ 0 & 0 & 1-p_{Mi} & p_{Mi} & 0 & 0 \\ 0 & 0 & 0 & 1-p_{Mi} & p_{Mi} & 0 \\ 0 & 0 & 0 & 0 & 1-p_{Mi} & p_{Mi} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix}. \quad (8)$$

The probability of failure $p_{Fi}$ of the $i$-th level of the USN is the probability of transition from the initial state $\mathbf{S}_0$ to the state $\mathbf{S}_{Xic}$, i.e. is equal to the conditional probability located in the last column of the first row (upper right corner of the matrix of transition probabilities) [61]. At the beginning of the operation of the USN (at the "zero" operating cycle), this probability is zero.

Analyzing expressions (5)-(8) it can be concluded that for a certain level of the hierarchy, the matrix of transition probabilities will have an order of one greater than the critical number of nodes in a given hierarchy level ($X_{ic}$ + 1). Thus, for the last (highest) level of the USN with a single main node ($X_{imax}$ = 1), the matrix of transition probabilities will be of 2nd order:

$$R_1 = \begin{vmatrix} 1-p_{Mi\,max} & p_{Mi\,max} \\ 0 & 1 \end{vmatrix}. \quad (9)$$

In this case, for the matrix (9) already at the beginning of operation $p_{Fimax} = p_{Mimax}$.

In turn, the probability of failure of the entire system $P_{F\Sigma}$ can be calculated based on the expression (10), since all the levels of the hierarchy are connected in series.

$$P_{F\Sigma} = 1 - (1-p_{F_0}) \cdot (1-p_{F_1}) \cdot (1-p_{F_2}) \times \ldots \cdot (1-p_{Fi}) \cdot \ldots \cdot (1-p_{Fimax}), \quad (10)$$

where $p_{Fi}$ is the probability of failure of the $i$-th level of the USN; $p_{Fi\,max}$ is the probability of failure of the last (highest) level of the USN.

To find the probability of failure of the $i$-th level of the system ($i$ = 0, 1, ..., $i_{\max}$) on the $N$-th cycle, it is necessary to raise its matrix of transition probabilities to the $N$-th integer power: $R_{Xic}^{N}$.

Thus, to calculate the maximum number of operating cycles $N_{\text{i max}}$ of the $i$-th level of the USN before failure occurs, it is necessary to determine the power to which the matrix of transition probabilities of a given level must be raised so that the probability in the last column of the first row of the matrix is equal to one ($p_{Fi}$ = 1). To reduce calculations, instead of one in the upper right corner of the matrix, we can use a probability of 0.999 ($p_{Fi} \geq 0.999$). For example, if the probability of a malfunction at the $i$-th level of the hierarchy is $p_{Mi}$ = 0.7, then the value of the maximum number of operation cycles of this level for the given matrices $R_1$, $R_2$, $R_3$, $R_4$, $R_5$, respectively, will be: $N_{\max}$ = 6, $N_{\max}$ = 9, $N_{\max}$ = 11, $N_{\max}$ = 13, $N_{\max}$ = 15.

In turn, to calculate the maximum number of cycles of operation before failure of the entire system $N_{\Sigma\max}$, it is necessary to determine the power to which the matrices of transition probabilities of all levels must be raised so that the probability of failure of the entire system is greater than or equal to 0.999 ($P_{F\Sigma} \geq 0.999$).

Analyzing the obtained values $N_{i\max}$, it can be argued that for the same probability value $p_{Mi}$, the greater the critical number of nodes $X_{ic}$ at the $i$-th level of the hierarchy, the greater the value of the maximum number of cycles before failure $N_{imax}$ of this level. In addition, increasing the number of levels of the hierarchy $i_{max}$ will significantly increase the probability of failure of the entire system $P_{F\Sigma}$.

Thus, to increase the number of cycles to failure, two main ways can be distinguished: 1) increasing the critical number of nodes at each specific level of the hierarchy; 2) reducing the number of levels of the system hierarchy.

Herewith, the second way in some cases may not give the desired results due to the presence of initial restrictions on the distance of signal transmission and on the types of network's elements used. For example, if there is only a certain type of elements (sensors, UUVs, buoys and other nodes) and a given distance for transmitting information to a terrestrial monitoring station, then to create a network it will be necessary to form a given number of hierarchy levels, which cannot be reduced.

Therefore, to ensure a given number of the USN operation cycles (or operating time) under different initial conditions, the most effective way is to increase the critical number of nodes at specific network's levels through redundancy.

Taking into account the expression (10), the probability of failure of the entire system $P_{F\Sigma}$ is always greater than the probability of failure of each individual level $p_{Fi}$, and, accordingly, the level, for which this value is the greatest. Therefore, to increase the number of operation cycles (or operating time) before failure of the entire system, it is first necessary to reserve elements of the level with the smallest value of this quantity.

At the same time, since it is unknown which node will fail, in order to ensure a stable increase in the critical number of nodes at a certain level, it is necessary to reserve all nodes at this level. For example, if there are 10 nodes at a certain USN's level and the critical number of nodes is 5 (when choosing 50%), then for redundancy at the initial stage it is necessary to duplicate all 10 nodes. Then, there will already be 20 nodes at this level, and the critical number of nodes is guaranteed to increase to 10. If this number will be still not enough to ensure the given number of operation cycles, then it is necessary to continue to sequentially reserve nodes, with step by step increasing their number to 30, 40, and so on.

Next, based on the above principles, we formulate the method for development of the fault-tolerant structures for USNs.

### 3.3. Main steps of the design method of fault-tolerant structures for underwater sensor networks

The authors' proposed design method of fault-tolerant structures for USNs consists of the following sequential steps.

**Step 1.** Initialization. At this step, the initial USN structure without redundancy is formed. The specified number of initial sensors $X_0$ at the zero level is set. Depending on the characteristics (signal transmission range, frequency, etc.) of the used sensors and other basic network nodes, as well as the specified information transmission range to the terrestrial monitoring station, the number of levels of the network's hierarchy $i_{\max}$ is determined and the number of nodes $X_i$ at each $i$-th level ($i$ = 1, ..., $i_{\max}$) is set. In turn, to build a network with a hierarchical tree-like structure the number of nodes $X_i$ at the $i$-th level of the hierarchy should be chosen according to the condition (1). Moreover, the number of faulty channels from initial sensors (in percentage) is selected, at which a network's failure will be considered. Further, based on this number the critical number of nodes $X_{ic}$ for each $i$-th hierarchy level ($i$ = 0, 1, ..., $i_{\max}$) is defined. Also, the probability $p_{Mi}$ of a malfunction occurring at each $i$-th level ($i$ = 0, 1, ..., $i_{\max}$) of the USN is set, that can be calculated based on expression (4). Finally, the required number of the USN's operation cycles before failure $N_{\Sigma\max D}$, which must be ensured, is set. If the required USN's operation time before failure is set, then it is also necessary to set the duration of one cycle.

**Step 2.** Calculation of the real value of the maximum number of operation cycles before failure $N_{\Sigma\max R}$ for the USN with a given structure. At this step, Markov chains and corresponding matrices of transition probabilities $R_{Xic}$ are constructed for each $i$-th level of the system ($i$ = 0, 1, ..., $i_{max}$). Herewith, each matrix has an order of one greater than the critical number of nodes in a corresponding hierarchy level ($X_{ic}$ + 1).

Next, the power is determined, to which it is necessary to raise the given matrices of all levels so that the probability of failure of the entire system becomes greater than or equal to 0.999 ($P_{F\Sigma} \geq 0.999$). The value of this power is the real value of the maximum number of operation cycles before failure $N_{\Sigma \text{maxR}}$ for the USN with a given structure. In turn, the probability of failure of the entire system $P_{F\Sigma}$ is calculated based on the expression (10).

**Step 3.** Checking the completion of the process of designing fault-tolerant structure of the USN. At this step, the value calculated in the previous step $N_{\Sigma \text{maxR}}$ is compared with the set in the first step value $N_{\Sigma \text{maxD}}$. If the condition $N_{\Sigma \text{maxR}} \geq N_{\Sigma \text{maxD}}$ is satisfied, then go to step 12. Otherwise, go to the next step.

**Step 4.** Calculation of the maximum number of operation cycles before failure $N_{\text{i max}}$ for each individual *i*-th level of the USN. At this step, for each individual *i*-th level ($i$ = 0, 1, ..., $i_{\text{max}}$), the power is determined, to which it is necessary to raise its matrix of transition probabilities $R_{\text{Xic}}$ so that the probability of failure of this level becomes greater than or equal to 0.999 ($p_{Fi} \geq 0.999$). The value of this power is the value of the maximum number of operation cycles before failure $N_{\text{i max}}$.

**Step 5.** Selecting the USN's levels whose maximum number of operation cycles before failure $N_{\text{i max}}$ is less than the number $N_{\Sigma \text{maxD}}$ for the entire system set in step 1. At this step, the calculated in the previous step values $N_{\text{i max}}$ ($i$ = 0, 1, ..., $i_{\text{max}}$) are alternately compared with the set in the first step value $N_{\Sigma \text{maxD}}$. Those levels, for which the condition $N_{\text{i max}} < N_{\Sigma \text{maxD}}$ is met, are selected for further reservation in the next step. If this condition is satisfied for at least one level, then go to step 6. Otherwise, go to step 10.

**Step 6.** Reserving of the USN levels' elements selected at step 5 until each of them has a maximum number of operation cycles before failure $N_{\text{i max}}$ greater than the number $N_{\Sigma \text{maxD}}$ for the entire system set in step 1. At this step, a stage-by-stage reserving of all elements of each selected level is carried out to increase the critical number of nodes $X_{ic}$. First, all elements of the selected level are duplicated to increase the critical number of nodes $X_{ic}$ by 2 times. If this is not enough, then one extra is added again to each node to increase the critical number of nodes $X_{ic}$ by 3 times, and so on. After all levels have a maximum number of operation cycles before failure $N_{\text{i max}}$ greater than the number $N_{\Sigma \text{maxD}}$ for the entire system set in step 1, the transition to the next step is carried out.

**Step 7.** Calculation of the real value of the maximum number of operation cycles before failure $N_{\Sigma \text{maxR}}$ for the USN with a given structure. This step is performed in the same way as step 2.

**Step 8.** Checking the completion of the process of designing fault-tolerant structure of the USN. This step is carried out in the same way as step 3. If the condition $N_{\Sigma \text{maxR}} \geq N_{\Sigma \text{maxD}}$ is satisfied, then go to step 12. Otherwise, go to the next step.

**Step 9.** Calculation of the maximum number of operation cycles before failure $N_{\text{i max}}$ for each individual *i*-th level of the USN. This step is performed in the same way as step 4.

**Step 10.** Selecting the USN's level whose maximum number of operation cycles before failure is the smallest among all the levels.

**Step 11.** Reserving of elements of the USN's level selected at step 10. At this step, one extra node is added to each existing node for increasing the critical number of nodes $X_{ic}$. After this, go to step 7.

**Step 12.** Completion of the process of designing fault-tolerant structure of the USN. After this, the implementation and configuration of the USN with designed structure can be carried out.

The block diagram of the proposed designing method of fault-tolerant structures for USNs is shown in Fig. 5.

To study the effectiveness of the proposed method, the next section presents the example of designing the fault-tolerant structure of the specific underwater sensor network.

## 4. Designing the Fault-Tolerant Structure of the Specific Underwater Sensor Network

To conduct a numerical experiment in this section, a specific USN was selected consisting of a zero level (initial sensors' level) and four main hierarchy levels (unmanned underwater vehicles' level, buoys' level and 2 levels of terrestrial communication nodes). At the initialization stage of the proposed method (step 1), the USN's initial structure was formed without redundancy and the main parameters were set as follows. In turn, the number of initial sensors $X_0$ = 16, the number of levels of the network hierarchy $i_{max} = 4$. The given system is considered operational as long as it transmits signals from more than 50% of the initial sensors. The required number of the USN's operation cycles before failure $N_{\Sigma \text{maxD}} = 720$. In this particular example, the signal transmission during the network operation is carried out in such a way that all the sensors are polled once per hour. Thus, this required number of working cycles before failure (720) will allow the network to operate uninterruptedly for 30 days (1 month). For convenience, the main parameters of the formed initial network's structure, including the total number $X_i$ and critical number $X_{ic}$ of nodes, as well as the probabilities $p_{Mi}$ of a malfunction occurring at each *i*-th level of the hierarchy, are presented in Table 1.

Thus for the USN with the given initial structure, the vector **X** has the form

$$\mathbf{X} = \{16, 8, 4, 2, 1\}. \tag{11}$$

At the second step the calculation of the real value of the maximum number of operation cycles before failure $N_{\Sigma maxR}$ was performed for the USN with a given structure.

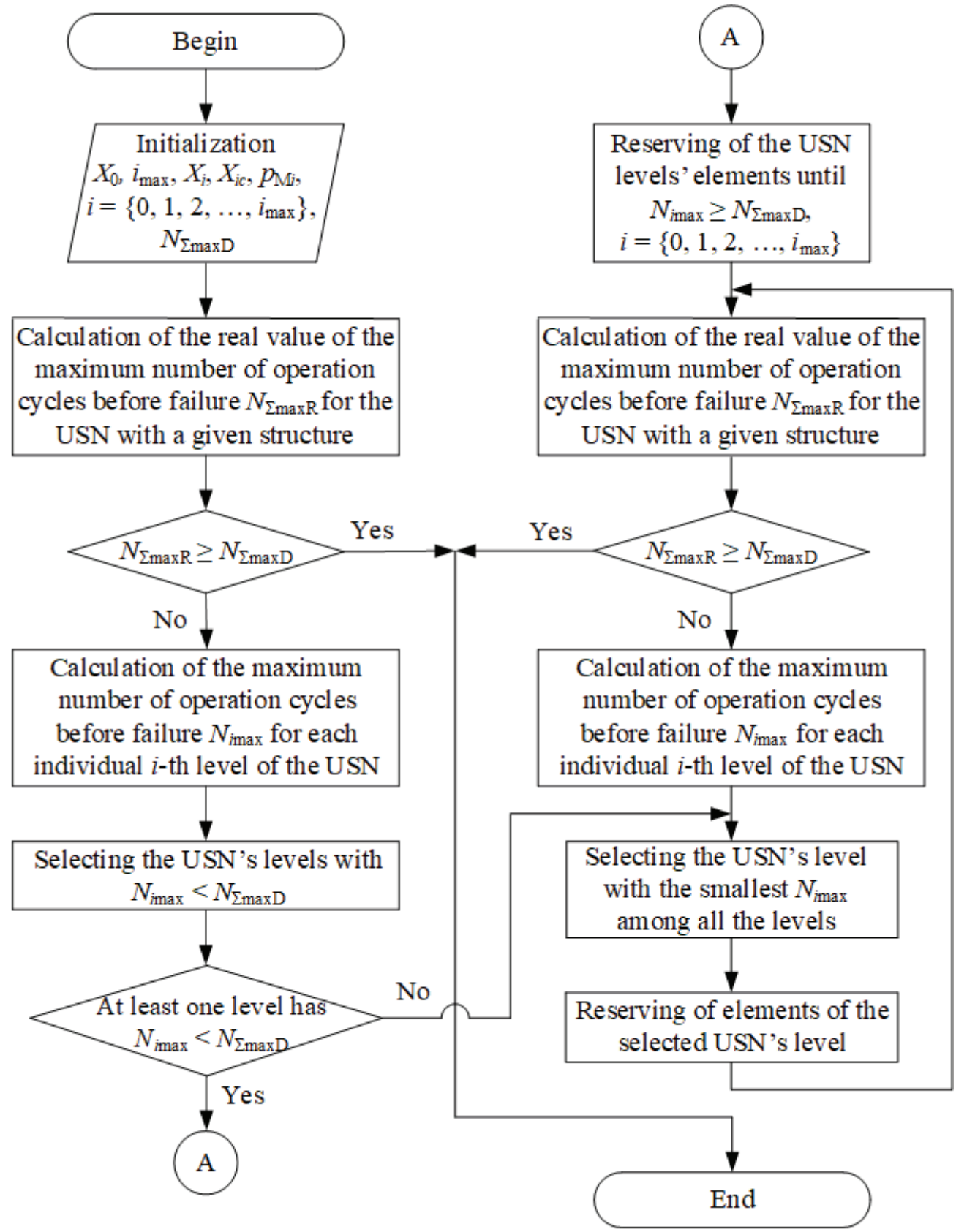



**Figure 5.** A block diagram of the designing method of fault-tolerant structures for USNs.

In particular, the Markov chains and corresponding matrices of transition probabilities $R_8$, $R_4$, $R_2$, $R_1$, and $R_{1'}$ were constructed for each $i$-th level of the system ($i$ = 0, 1, ..., 4). Herewith, matrices $R_1$ and $R_{1'}$ are the second-order matrices for the 3rd and 4th levels with the corresponding probability values $p_{M3}$ and $p_{M4}$. Moreover, each of the formed matrices had an order of one greater than the critical number of nodes in the corresponding levels ($X_{ic}$ + 1) presented in Table 1.

**Table 1.** Main parameters of the initial USN's structure

| *i* | Type of hierarchy level | $p_{Mi}$ | $X_i$ | $X_{ic}$ |
|---|---|---|---|---|
| 0 | Initial sensors | 0.025 | 16 | 8 |
| 1 | UUVs | 0.0125 | 8 | 4 |
| 2 | Buoys | 0.01 | 4 | 2 |
| 3 | Terrestrial communication nodes | 0.001 | 2 | 1 |
| 4 | Main terrestrial communication node | 0.0005 | 1 | 1 |

**Table 2.** Main parameters of the USN's structure at the 1st iteration

| *i* | Type of hierarchy level | $p_{Mi}$ | $X_i$ | $X_{ic}$ | $N_{\text{i max}}$ |
|---|---|---|---|---|---|
| 0 | Initial sensors | 0.025 | 16 | 8 | 781 |
| 1 | UUVs | 0.0125 | 8 | 4 | 1045 |
| 2 | Buoys | 0.01 | 4 | 2 | 924 |
| 3 | Terrestrial communication nodes | 0.001 | 2 | 1 | 6955 |
| 4 | Main terrestrial communication node | 0.0005 | 1 | 1 | 13914 |

Then, the power was determined, to which it was necessary to raise the given matrices of all levels so that the probability of failure of the entire system reached the value of 0.999 ($P_{F\Sigma} \geq 0.999$). In turn, using the expression (10) and all constructed matrices ($R_8$, $R_4$, $R_2$, $R_1$, and $R_{1'}$), this value was found to be $N_{\Sigma\text{maxR}} = 439$.

Next, at the third step, a comparison was made of the real value of the maximum number of operation cycles $N_{\Sigma\text{maxR}}$ with the required value $N_{\Sigma\text{maxD}}$. Since, the real value calculated at the previous step was less than the required value ($N_{\Sigma\text{maxR}} < N_{\Sigma\text{maxD}}$), the transition to the next step was performed (step 4).

At step 4 the calculation of the maximum number of operation cycles before failure $N_{\text{i max}}$ was performed for each individual *i*-th level of the USN with a given structure. In this case, for each individual *i*-th level ($i$ = 0, 1, ..., $i_{\text{max}}$), the power was determined, to which it was necessary to raise its matrix $R_{Xic}$ so that the probability of failure of this level reached the value of 0.999 ($p_{Fi} \geq 0.999$). In turn, using separately taken constructed matrices ($R_8$, $R_4$, $R_2$, $R_1$, and $R_{1'}$), these values were found and summarized in Table 2.

At step 5 the selection of the USN's levels was carried out, whose maximum number of operation cycles before failure $N_{\text{i max}}$ was less than the required number $N_{\Sigma\text{maxD}}$ for the entire system. Since, each level initially already had a value $N_{\text{i max}}$ greater than $N_{\Sigma\text{maxD}}$ (condition $N_{\text{i max}} < N_{\Sigma\text{maxD}}$ was not met), then the transition to step 10 was performed.

Next, at the 10th step the USN's level was selected with the smallest value of maximum number of operation cycles before failure among all the levels. This level was the zero level ($i$ = 0) of the initial sensors with $N_{0\,\text{max}} = 781$ (Table 2).

**Table 3.** Main parameters of the USN's structure at the 2nd iteration

| *i* | Type of hierarchy level | $p_{Mi}$ | $X_i$ | $X_{ic}$ | $N_{\text{i max}}$ |
|---|---|---|---|---|---|
| 0 | Initial sensors | 0.025 | 32 | 16 | 1245 |
| 1 | UUVs | 0.0125 | 8 | 4 | 1045 |
| 2 | Buoys | 0.01 | 4 | 2 | 924 |
| 3 | Terrestrial communication nodes | 0.001 | 2 | 1 | 6955 |
| 4 | Main terrestrial communication node | 0.0005 | 1 | 1 | 13914 |

**Table 4.** Main parameters of the USN's structure at the 3rd iteration

| *i* | Type of hierarchy level | $p_{Mi}$ | $X_i$ | $X_{ic}$ | $N_{\text{i max}}$ |
|---|---|---|---|---|---|
| 0 | Initial sensors | 0.025 | 32 | 16 | 1245 |
| 1 | UUVs | 0.0125 | 8 | 4 | 1045 |
| 2 | Buoys | 0.01 | 8 | 4 | 1307 |
| 3 | Terrestrial communication nodes | 0.001 | 2 | 1 | 6955 |
| 4 | Main terrestrial communication node | 0.0005 | 1 | 1 | 13914 |

Then, at step 11 the reserving of elements of the USN's zero level was carried out. In particular, one extra sensor was added to each existing sensor to increase the critical number of nodes $X_{0c}$ from 8 to 16. After that, a transition to the next iteration of the method was performed and, accordingly, a return to step 7.

Calculated at step 7, the real value of the maximum number of operation cycles for the entire USN with the new structure was $N_{\Sigma\text{maxR}} = 535$, which was less than the required value $N_{\Sigma\text{maxD}}$. Therefore, further steps of the method were implemented.

At step 9 the calculation of the $N_{\text{i max}}$ value was performed for each individual *i*-th level of the USN with a given structure. These values, as well as new USN's parameters, were summarized in Table 3.

For the given network's configuration (Table 3), the 2nd level (buoys level) had the lowest value of $N_{max}$. After defining this level at the 10th step, its elements were reserved (one extra buoy was added to each existing buoy) at step 11. Then, a transition to the next (3rd) iteration of the method was performed and, accordingly, a return to step 7.

The calculated value at step, 7 real $N_{\Sigma\text{maxR}}$ for the entire USN with the new structure was 635, which was less than the required value $N_{\Sigma\text{maxD}}$. Therefore, further steps of the 3rd iteration were implemented.

At step 9 the calculation of the $N_{\text{i max}}$ value was performed for each individual *i*-th level of the USN with a given structure. These values, as well as new USN's parameters were summarized in Table 4.

**Table 5.** Main parameters of the USN's structure at the 4th iteration

| *i* | Type of hierarchy level | $p_{Mi}$ | $X_i$ | $X_{ic}$ | $N_{\text{i max}}$ |
|---|---|---|---|---|---|
| 0 | Initial sensors | 0.025 | 32 | 16 | 1245 |
| 1 | UUVs | 0.0125 | 16 | 8 | 1569 |
| 2 | Buoys | 0.01 | 8 | 4 | 1307 |
| 3 | Terrestrial communication nodes | 0.001 | 2 | 1 | 6955 |
| 4 | Main terrestrial communication node | 0.0005 | 1 | 1 | 13914 |

For the given network's configuration (Table 4), the 1st level (UUVs' level) had the lowest value of $N_{\max}$. After defining this level at the 10th step, its elements were reserved (one extra UUV was added to each existing UUV) at step 11. Then, a transition to the next (4th) iteration of the method was performed and, accordingly, a return to step 7.

The calculated at step 7 real value $N_{\Sigma\text{maxR}}$ for the entire USN with the new structure was 761, therefore the condition $N_{\Sigma\text{maxR}} \geq N_{\Sigma\text{maxD}}$ was satisfied. Thus, the transition to step 12 was performed and the process of designing fault-tolerant structure of the USN was completed.

Thus for the USN with the designed fault-tolerant structure, the vector **X** has the form

$$\mathbf{X} = \{32, 16, 8, 2, 1\}. \quad (12)$$

In turn, for clarity, the main parameters of the developed fault-tolerant network's structure are presented in Table 5.

Further, to analyze the results obtained, Table 6 shows how the vector **X** of the USN's structure changed over the course of iterations during the implementation of the method, as well as the real value of the maximum number of operation cycles before failure $N_{\Sigma\text{maxR}}$.

Also, the graph of changes in the real value $N_{\Sigma\text{maxR}}$ from iterations is shown in Fig. 6.

As can be seen from Table 6 and Fig. 6, the fault-tolerant structure with the required number of operation cycles before failure ($N_{\Sigma\text{maxD}}$ = 720) for the given specific USN was built in 4 iterations. This fact confirms that the proposed method allows the development of fault-tolerant structures with a guaranteed achievement of the desired number of operation cycles before failure at relatively low computational costs. Herewith, reserving the elements of the corresponding hierarchical levels in the proper order made it possible to systematically increase the real value of the maximum number of operation cycles before failure $N_{\Sigma\text{maxR}}$ from iteration to iteration until the required value $N_{\Sigma\text{maxD}}$ is achieved without excessive redundancy. This will minimize the cost of the constructed network with the necessary reservation of elements.

**Table 6.** Changing of the vector **X** during the implementation of the method

| Number of iteration | Vector X | $N_{\Sigma\text{maxR}}$ |
|---|---|---|
| 1 | {16, 8, 4, 2, 1} | 439 |
| 2 | {32, 8, 4, 2, 1} | 535 |
| 3 | {32, 8, 8, 2, 1} | 635 |
| 4 | {32, 16, 8, 2, 1} | 761 |

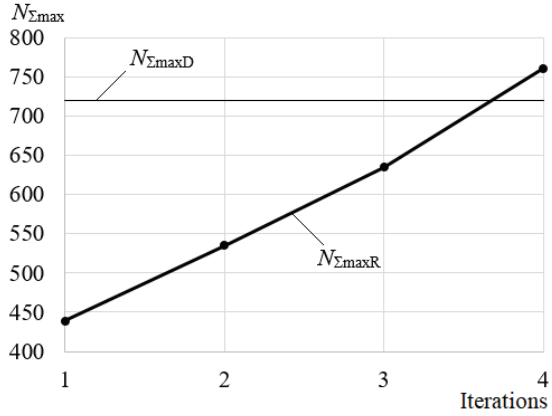



**Figure 6.** Changing of the real value $N_{\Sigma maxR}$ during the implementation of the method.

Thus, the strategy of sequential reserving of elements in levels with the least number of operation cycles before failure, which is used in the proposed method, has a fairly high efficiency.

Overall, the analysis of the results obtained from the conducted numerical experiment indicates that the proposed method enables the design of underwater sensor networks' structures with the required fault tolerance and minimal redundancy. This affirms its viability for utilization in the development of USNs for diverse purposes, aiming to enhance reliability and reduce costs.

## 5. Conclusion

The development and effectiveness study of the method of designing the fault-tolerant structures for underwater sensor networks based on Markov chains is presented in this paper. The proposed by the authors designing method makes it possible to create the proper USNs' structures with a guaranteed provision of the desired number of operation cycles before failure at reasonable costs. The employment of the mathematical apparatus of Markov chains in conjunction with the strategy of correct selection of the hierarchy levels' sequence allows to quite accurately determine the network actual number of operation cycles before failure as well as perform the efficient reservation without excessive redundancy.

The effectiveness of the proposed method was assessed through a specific case study, namely when creating a fault-tolerant structure of a real underwater sensor network with the predetermined parameters.

The analysis of the obtained results from the conducted numerical experiment reveals that reserving the elements of the corresponding hierarchical levels in the proper order in accordance with the method's main steps makes it possible to systematically increase the real value of the maximum number of operation cycles before failure from iteration to iteration up to the attainment of the required value without excessive redundancy. Moreover, for this particular example, only 4 iterations were needed to find the USN's fault-tolerant structure with the desired number of operation cycles before failure, which confirms the high efficiency of the method at relatively low computational costs. Thus, the research conducted in this work substantiates the high practicality of employing the developed method for the creation of underwater sensor networks tailored to diverse purposes, ensuring a predetermined level of fault tolerance while maintaining moderate costs.

In further research, it is planned to consider the issues of optimizing the implementation costs of underwater sensor networks while adhering to specified fault tolerance margins.

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