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Pearson Correlation and Ordered Weighted Average Operator in the World Stock Exchange Market

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Martha Flores-Sosa, Ernesto Leon-Castro, Jose M. Merigo

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Abstract:
The stock market is of great importance for the financial development of a country due to the large volume of transactions therein. Analyzing the correlation between indices in the world helps us figure out which variables are most impactful. This paper proposes the use of ordered weighted average (OWA) operators in combination with the Pearson coefficient to create a measure of correlation that can analyze a wide range of possible scenarios that go from minimum to maximum. The new frameworks can add additional information to the process of correlation. The work presents an application in ten of the largest stock exchanges in the world. The results suggest a broad positive correlation that is reinforced in times of instability.

Keywords: *Stock market, OWA operator, Pearson coefficient, Financial development*

1. Introduction

The world financial market is essential in the development of economic processes since it contributes to the transfer of financial flows between agents. The stock market establishes a close connection with the productive sector to the extent that each country has developed its financial system [1,2]. In the last decades, there has been a considerable increase in the number of transactions and their values in stock markets. Therefore, many of its aspects have been investigated to search for knowledge and clarify the phenomena in the markets. In this sense, issues such as variables that affect it [3, 4], modeling [5, 6], forecasting [7, 8], and integrations [9, 10] have been studied.

Market integration has allowed many of the stock markets to move in synchrony when fortuitous events occur, and some indices tend to affect others to a great extent. Barunik et al. [11] show that in times of instability, the correlation of the stock market with other indicators, such as gold and oil, becomes stronger. Jung and Chang [12] found that stocks tend to cluster by Pearson correlation and partial correlation. Intending to know the relationship of world stock markets over time, Wang et al. [13] propose a network-based Pearson coefficient to analyze some stock exchanges.

This work proposes a Pearson coefficient with OWA aggregation operators to analyze world stock markets. The OWA operators [14] are a parameterized family of aggregation operators, whose main characteristic is the reordering of the attributes that allow an analysis of multiple scenarios that go from minimum to maximum. One of the most popular extensions is the induced operator IOWA [15]. It uses a more complex reorder using induced variables. For the treatment of uncertain data, operators with additional vectors have been proposed. The POWA operator [16] considers probability, and the ordered weighted averaging-weighted average (OWAWA) [17] operator uses an extra weighting. Note that all these ideas can be unified in a single operator called IPOWAWA [18]. Since its inception, the OWA operator and its extensions have been used successfully in statistical procedures such as regressions issue [19, 20], standard deviation [21], variance, and covariance [22, 23].

This paper uses the IPOWA, IOWAWA, and IPOWAWA operators in the form of variances and covariances to calculate the Pearson correlation coefficient. The new methodology is called PC-IPOWA, PC-IOWAWA and PC-IPOWAWA. The main objective is to obtain a correlation coefficient that, in addition to considering scenarios that go from minimum to maximum, can consider probabilities and weights when environments of uncertainty exist. In order to find important information in financial markets, we analyze the correlation of some of the most representative stock exchanges in the world.

The paper is developed as follows: Section 2 presents a summary of the methodologies used. Section 3 shows the new proposed Pearson coefficient and OWA operators. In Section 4, a generalization of the new structure is presented. Section 5 develops the application of the OWA correlation coefficients in the stock market. Finally, the conclusions of the work are described in Section 6.

2. Preliminaries

Below is a brief description of the approaches used in the proposal of this work. The OWA operator, some of its extensions, and the Pearson coefficient are defined.

2.1. OWA Operator and Extensions

The ordered weighted averaging (OWA) operator [14] provides a method to aggregate several arguments that die between the maximum and minimum. The main characteristic is the reordering of the attribute vector that goes from minimum to maximum (AOWA) or from maximum to minimum (DOWA). The OWA operator is defined as follows:

Definition 1. An OWA operator with dimensions *n* is a model $OWA\colon R^n \rightarrow R$ such that it has associated weights vector W thus $w_i = \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, then:

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (1)$$

where b_j is the jth largest a_i. The OWA operator is a mean operator as it satisfies the conditions:

- Monotonicity: if $a_i \geq \hat{a}_i$ then $F(a_1, \dots, a_n) \geq F(\hat{a}_1, \dots, \hat{a}_n)$ for i.
- Commutativity: The initial indexing of de arguments doesn't matter.
- Idempotent: if $a_j = a$ for all j, so $F(a_i, \dots, a_n) = a$.

If the reordering of the OWA arguments is not considered, then we can use induced variables for it. The induced weighted average operator (IOWA) [15] uses argument pairs called OWA pairs, with the objective of inducing an ordering and aggregation of the second components. It can be defined as follows:

Definition 2. An IOWA operator is a mapping $IOWA\colon R^n \rightarrow R$ of dimension n with an associated weights vector $W = [w_1, w_2, \dots,]$ wnT, such that $0 \leq w_i \leq 1$ and $w_i + \dots + w_n = 1$, with an induced IOWA pair $\langle u_i, a_i \rangle$, where u_i is the variable that induced order and a_i is the argument of the variable, the formula is as follows:

$$IOWA(\langle u_1, a_1 \rangle \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} w_j b_j, \quad (2)$$

where b_j is the value a_i in the IOWA pair that have the jth most extensive u_i. The IOWA operator satisfies the conditions: Monotonicity, Commutativity and Idempotent.

In practice, probability can be of great importance to know the characteristics of a current phenomenon. Merigó [16] proposes the probabilistic OWA (POWA) operator, which provides a unification of the probabilities and the OWA operators. It considers the degree of importance of each one in the aggregation process.

Then:

Definition 3. A POWA operator is a mapping $POWA\colon R^n \rightarrow R$ associated with a weight vector W where its components lie in the unit interval and sum to one. Additionally, it has an associated probability vector P with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$, according to the following equation:

$$POWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \hat{p}_j b_j, \quad (3)$$

where b_j is the jth largest in $a_1, a_2, \dots, a_n$. There is such a relationship between probabilities and weights as $\hat{p}_j = \beta w_j + (1-\beta) p_j$ with $\beta \in [0,1]$. If $\beta = 0$, the PA operator appears, and if $\beta = 1$, the OWA operator is obtained.

In some cases, the important information in decision-making is given by other types of weightings that can capture different phenomena. The OWAWA operator was proposed by Merigó [17], and it uses the OWA operator and weighted average (WA) in the same formulation. The definition is as follows:

Definition 4. An OWAWA operator of dimension *n* is a model $OWAWA\colon R^n \rightarrow R$ associated with a weight vector $W = [w_1, w_2, \dots, w_n]^T$ such that $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$. Additionally, it has an associated weight vector V with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$, so that:

$$OWAWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^{n} \hat{v}_j b_j, \quad (4)$$

where b_j is the jth largest a_i. The weight vector is composed as $\hat{v}_j = \beta w_j + (1-\beta) v_j$. The OWAWA operator has similar properties to the OWA operator.

The POWA operator and OWAWA operator can also use a different reorder of arguments. The IPOWA operator [24] and IOWAWA operator [25] consider induced variables for the reorder process. The formulas are as follows:

$$IPOWA(\langle u_1, a_1 \rangle \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} \hat{p}_j b_j, \quad (5)$$

where b_j is the jth largest value of the u_i. There is a weight vector W such that $w_i = \in [0,1]; w_i + \dots + w_n = 1$, and a probability vector P with $\sum_{i=1}^{n} p_i = 1; p_i \in [0,1]$, the degree of importance is $\hat{p}_j = \beta w_j + (1-\beta) p_j$.

$$IOWAWA(\langle u_1, a_1 \rangle \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \sum_{j=1}^{n} \hat{v}_j b_j, \quad (6)$$

where b_j is the value a_i in the IOWA with the jth largest u_i. The weight vector considers two vectors W such that $w_i = \in [0,1]$; $w_i + \dots + w_n = 1$, and V where $\sum_{i=1}^{n} v_i = 1; v_i \in [0,1]$, the degree of importance is $\hat{v}_j = \beta w_j + (1-\beta) v_j$.

It is possible to put together all the ideas seen above in one formulation. The IPOWAWA operator [18] unifies the IOWA, the weighted average (WA) and the probabilistic aggregation (PA) in one formulation that can deal with risk and uncertainty. It can be defined as follows:

Definition 5. An IPOWAWA operator of dimension n is a mapping $IPOWAWA\colon R^n \rightarrow R$, if it has two associated weighting vectors W and V and probability vector P, where its components lie in the unit interval and sum to one.

Additionally, an induced IOWA pair $\langle u_i, a_i \rangle$ is considered, then:

$$IPOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle)$$
$$= C_1 \sum_{j=1}^{n} w_j b_j + C_2 \sum_{j=1}^{n} v_i a_i + C_3 \sum_{j=1}^{n} p_i a_i, \quad (7)$$

where b_j is the value a_i with the jth largest u_i, and C_1, C_2 and $C_3 \in [0,1]$, with $C_1 + C_2 + C_3 = 1$. The special cases appear: if $C_1 = 1$, we get the IOWA operator. If $C_2 = 1$, the WA is formed. If $C_3 = 1$, the PA is obtained. If $C_1 = O$, we create the probabilistic weighted average (PWA).

2.2. Variances and Covariances OWA

The OWA operator has a multidisciplinary application using the idea of weighting and reordering in other methodologies. The OWA operators with variances (Var-OWA) [26] adapt the arithmetic variance to a vector of parameterized weights, according to the following equation:

Definition 6. A variance OWA of dimension n is a model $OWA\colon R^n \rightarrow R$ with an associated weights vector W thus $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$, then a variance component $D_j = (x_i - \mu)^2$ is associated with a weight value w_j in the following way:

$$Var - OWA(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j D_j, \quad (8)$$

where D_j is the largest of the $(x_i - \mu)^2$, μ is the OWA operator mean. Meanwhile, the covariance is formulated using a similar procedure. Merigó [27] proposed the covariance with OWA operators (Cov-OWA). So:

Definition 7. A covariance OWA is a model $OWA\colon R^n \rightarrow R$ of dimension n, where there is a weights vector $W = [w_1, w_2, \dots, w_n]^T$ thus $0 \leq w_i \leq 1$ and $w_i + \dots + w_n = 1$, then the variance component $K_j = (x_i - \mu)(y_i - v)$ is associated with a weight w_j. The formula is as follows:

$$Cov - OWA(X, Y) = \sum_{j=1}^{n} w_j K_j, \quad (9)$$

where K_j is the jth largest of the $(x_i - \mu)(y_i - v)$, x_i is the argument variable of the set of elements X, y_i is the argument variable of the set Y. μ and v are the OWA means of X and Y, respectively.

2.3. Pearson Coefficient

A common framework for measuring the linear relationship between two variables is the Pearson Correlation (PC) coefficient [28, 29]. It can be an index simple and easy to apply with interesting results in decision-making. Then:

Definition 8. It is a PC coefficient if given a set of variables (x_k, y_k), so the $k = 1, \dots, K\colon x_k \in U^n$, $y_k \in U^n$, we have a model $f_\theta\colon R^n \rightarrow R$. The formula is as follows:

$$PC = \frac{Cov(X,Y)}{\sqrt{var(X) \times var(Y)}}, \quad (10)$$

where $Cov(X,Y)$ is the covariance $(x_i - \overline{x})(y_i - \overline{y})$. Variance X is $(x_i - \overline{x})^2$. Variance Y is $(y_i - \overline{y})^2$. The $\overline{x}$ and $\overline{y}$ are the arithmetic means.

3. Probabilistic Weighted OWA on Pearson Correlation

The relationship of two variables can include several aspects that are not captured by the arithmetic Pearson coefficient. Probability measures the certainty with which an event can occur. In this sense, a Pearson coefficient with probabilistic OWA operators (PC-POWA) offers a correlation coefficient that connects the probability in the calculation of the PC. The PC-POWA can be defined as follows:

Proposition 1. A PC-POWA of dimension n is a model $POWA\colon R^n \rightarrow R$ with two sets of variables $x_k \in U^n$, $y_k \in U^n$ that has an associated weighting vector W with $w_j \in [0,1]$ and $w_i + \dots + w_n = 1$. Then:

$$PC - POWA(a_1, \dots, a_n)$$
$$= \frac{Cov - POWA(X,Y)}{\sqrt{var - POWA(X) \times var - POWA(Y)}}$$
$$= \frac{\sum_{J=1}^{N} w_j (x_i - \mu)(y_i - v),}{\sqrt{[\sum_{j=1}^{n} w_j (x_i - \mu)^2][\sum_{j=1}^{n} w_j (y_i - v)^2]}}, \quad (11)$$

where b_j is the calculation of variances and covariances jth largest. The components $D_j = (x_i - \mu)^2$ and $K_j = (x_i - \mu)(y_i - v)$ in variance and covariance have an associated weight w_j. The PC-POWA has the same proprieties that OWA operators, this is:

- Monotonic. If $a_i \geq \hat{a}_i$ then, we have:

$$F(PC - POWA(a_1, a_2, \dots, a_n))$$
$$\geq F(PC - POWA(\hat{a}_1, \hat{a}_2 \dots, \hat{a}_n)).$$

- Symmetry. If $A = a_1, a_2, \dots, a_n; A' = a'_1 a'_2, \dots, a'_n$, then:

$$F(PC - POWA(a_1, a_2, \dots, a_n))$$
$$= F(PC - POWA(a'_1, a'_2, \dots, a'_n)).$$

- Idempotent. If $a_j = a$, for all $j = 1, \dots, n$, then:

$$F(PC - POWA(a_1, a_2, \dots, a_n) = a.$$

Example 1. Consider a variable (X = 2,4,6) and a variable (Y = 5, 8,3), a weight vector (W = 0.3,0.3,0.4) and a probability vector (P = 0.4,0.4,0.2) and a $\beta = 0.6$.

$$\hat{P} = 0.34, 0.34, 0.32$$

POWA means:

$$\mu = (6 \times 0.34) + (4 \times 0.34) + (2 \times 0.32) = 4.04$$

$$\nu = (8 \times 0.34) + (5 \times 0.34) + (3 \times 0.32) = 5.38$$

Variances and covariances POWA:

$$var - POWA(X) = (6 - 4.04)^2 + (4 - 4.04)^2 + (2 - 4.04)^2 = (4.16 \times 0.34) + (3.84 \times 0.34) + (0.001 \times 0.32) = 2.72$$

$$var - POWA(Y) = (8 - 5.04)^2 + (5 - 5.04)^2 + (3 - 5.04)^2 = (6.86 \times 0.34) + (5.66 \times 0.34) + (0.14 \times 0.32) = 4.30$$

$$Cov - POWA(X,Y) = [(6 - 4.04)(8 - 5.38)] + [(4 - 4.04)(5 - 5.38)] + [(2 - 4.04)(3 - 5.38)] = (5.13 \times 0.34) + (4.85 \times 0.34) + (0.01 \times 0.32) = 3.40$$

$$PC - POWA(a_1, \dots, a_n) = \frac{3.40}{\sqrt{2.72 \times 4.30}} = 0.99$$

Pearson's coefficient can also be calculated by adding additional weight vectors where important information about the correlations can be added. The PC-OWAWA can analyze the correlations in more complex scenarios. It can be defined as follows:

Proposition 2. A PC-OWAWA is a mapping $OWAWA: R^n \rightarrow R$ of dimension n with two sets of variables $x_k; y_k$ that has an associated weighting vector W with components that lie in the unit interval and sum to one. The formulation is as follows:

$$PC - OWAWA(a_1, \dots, a_n) = \frac{Cov - OWAWA(X,Y)}{\sqrt{var - OWAWA(X) \times var - OWAWA(Y)}} = \frac{\sum_{J=1}^{N} w_j (x_i - \mu)(y_i - \nu),}{\sqrt{[\sum_{j=1}^{n} w_j (x_i - \mu)^2][\sum_{j=1}^{n} w_j (y_i - \nu)^2]}}, \quad (12)$$

where $var - OWAWA$ and $Cov - OWAWA$ are calculated as equations ()() by OWAWA operators. The PC-OWAWA shares the proprieties on OWA operators: monotonic, symmetric and idempotent.

It is important to note that the assignment of weights is an essential point in the OWA aggregation operators. So, many ways of measuring the degree of overestimation and underestimation have been proposed. Yager [30] proposes the degree of orness. This is, if $w_1 = 1$, we have a pure "or" operator. The formulation is obtained as follows:

$$\alpha(W) = \sum_{j=1}^{n} w_j^* \left(\frac{n-j}{n-1}\right), \quad (13)$$

where w_j^* is the w_j with the jth largest a_i value.

Additionally, Yager [30] also shares the entropy of dispersion, which captures the variability and the use of the inputs by the weights as follows:

$$H(W) = -\sum_{j=1}^{n} w_j \ln(w_j). \quad (14)$$

The balance [31] measures the degree of selection between favoring the higher valued elements or lower-valued elements, then:

$$BAL(W) = \sum_{j-1}^{n} \left(\frac{n+1-2j}{n-1}\right) w_j. \quad (15)$$

The divergence [32] distinguishes between two OWA weights vectors, so:

$$DIV(W) = \sum_{j=1}^{n} w_j \left(\frac{n-j}{n-1} - \alpha(W)\right)^2. \quad (16)$$

The vector weight measurement can be used to calculate the characteristics of the PC-OWAWA and all the proposals seen here. In some cases, the relationship between two variables may be affected by various elements that change values from one moment to another.

The approaches discussed above can also be extended to use induced variables. The PC-IPOWA can connect the probabilities and the influence of other variables on the study into a coefficient of correlation. The main advantage is that we can analyze situations in a much more complex way as reality can present on some occasions. It can be defined as follows:

Proposition 3. A PC-IPOWA of dimension n is a model $IPOWA: R^n \rightarrow R$ with two sets of variables $x_k \in U^n$, $y_k \in U^n$ that has an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, additionally, an induced IPOWA pair $\langle u_i, a_i \rangle$ and a probability vector P is considered. The formulation can be defined as follows:

$$PC - IPOWA(\langle u_1, a_1 \rangle \langle u_2, a_2 \rangle, \dots, \langle u_n, a_n \rangle) = \frac{Cov - IPOWA(X,Y)}{\sqrt{var - IPOWA(X) \times var - IPOWA(Y)}} = \frac{\sum_{J=1}^{N} w_j (x_i - \mu)(y_i - \nu),}{\sqrt{[\sum_{j=1}^{n} w_j (x_i - \mu)^2][\sum_{j=1}^{n} w_j (y_i - \nu)^2]}}, \quad (17)$$

where b_j is the calculation of variances and covariances with the jth largest u_1. The μ and ν are IPOWA means.

In this sense, the IOWAWA operator can also be used to calculate the Pearson coefficient. The PC-IOWAWA is a correlation coefficient that combines some characteristics: 1) induced criteria for reordering arguments and 2) an additional weighted vector that is considered whit the weighted vector OWA. It is developed as the following definition:

Proposition 4. A PC-IOWAWA is a model $IOWAWA: R^n \rightarrow R$ with two sets of variables $x_k;\ y_k$ with two weighting vectors W and V such that both have $0 \leq w_i \leq 1$ and $\sum_{i=1}^{n} w_i = 1$, additionally, an induced IPOWA pair $\langle u_i, a_i \rangle$. So that:

$$PC - IOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \frac{Cov - IOWAWA(X,Y)}{\sqrt{var - IOWAWA(X) \times var - IOWAWA(Y)}} = \frac{\sum_{J=1}^{N} w_j(x_i - \mu)(y_i - \nu),}{\sqrt{[\sum_{j=1}^{n} w_j(x_i - \mu)^2][\sum_{j=1}^{n} w_j(y_i - \nu)^2]}}, \quad (18)$$

where b_j are the calculation of variances and covariances with the jth largest u_1. The μ and ν are IOWAWA means.

Example 2. Consider the data previously seen: the variable (X = 2,4,6) and the variable (Y = 5, 8,3), a weight vector (W = 0.3,0.3,0.4) a weighted vector (V = 0.2,0.3,0.5), and a $\beta = 0.6$. Additionally, an induced vector (U = 10, 15, 12).

$$\hat{V} = 0.26, 0.3, 0.44$$

IOWAWA means:

$$\mu = (4 \times 0.26) + (6 \times 0.3) + (2 \times 0.44) = 3.72$$

$$\nu = (8 \times 0.26) + (3 \times 0.3) + (5 \times 0.44) = 5.18$$

Variances and covariances IOWAWA:

$$var - IOWAWA(X) = (4 - 3.72)^2 + (6 - 3.72)^2 + (2 - 3.72)^2 = (5.19 \times 0.26) + (2.95 \times 0.3) + (0.07 \times 0.44) = 2.26$$

$$var - IOWAWA(Y) = (8 - 5.18)^2 + (3 - 5.18)^2 + (5 - 5.18)^2 = (4.75 \times 0.26) + (0.03 \times 0.3) + (7.95 \times 0.44) = 4.74$$

$$Cov - IOWAWA(X,Y) = [(4 - 3.72)(8 - 5.18)] + [(6 - 3.72)(3 - 5.18)] + [(2 - 3.72)(5 - 5.18)] = (-4.97 \times 0.26) + (0.30 \times 0.3) + (0.78 \times 0.44) = -0.85$$

$$PC - IOWAWA(a_1, \dots, a_n) = \frac{-0.85}{\sqrt{2.26 \times 4.74}} = -0.26$$

One can observe that the results can vary in quantity and sign when we use induced operators comparing exercises 1 and 2.

The Pearson coefficient can also consider very complex scenarios where uncertainty and risk are present. The PC-IPOWAWA is a correlation coefficient that uses induced components, weighted means and probability to measure the relationship of two variables. Within these characteristics, it can collect a series of factors that affect the variables and preferences or probabilities of each data. The PC-IPOWAWA can be defined as follows:

Proposition 5. A PC-IPOWAWA of dimension n is a mapping $IPOWAWA: R^n \rightarrow R$ if it has two sets of variables $x_k;\ y_k$ and three weighting vectors W, P and V such that have components ranging from zero to one and the sum is one, so an induced IPOWA pair $\langle u_i, a_i \rangle$ is used. Then:

$$PC - IPOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \dots, \langle u_n, a_n\rangle) = \frac{Cov - IPOWAWA(X,Y)}{\sqrt{var - IPOWAWA(X) \times var - IPOWAWA(Y)}} = \frac{\sum_{J=1}^{N} w_j(x_i - \mu)(y_i - \nu),}{\sqrt{[\sum_{j=1}^{n} w_j(x_i - \mu)^2][\sum_{j=1}^{n} w_j(y_i - \nu)^2]}}, \quad (19)$$

where μ and ν are IPOWAWA means. The component with the weight w_j is the one that has the largest u_1. The weight vector can be calculated as $\hat{C}_1 = C_1 w_1 + C_2 p_1 + C_3 v_1$, where $C_1 + C_2 + C_3 = 1$.

Example 3. Consider the data used in previous examples: the variable (X = 2,4,6) and the variable (Y = 5, 8,3), a weight vector (W = 0.3,0.3,0.4), probability vector (P = 0.4,0.4,0.2) a weighted vector (V = 0.2,0.3,0.5), and a C = 0.3, 0.4, 0.2. The induced vector is (U = 10, 15, 12).

$$\hat{C} = 0.33, 0.35, 0.32$$

IPOWAWA means:

$$\mu = (4 \times 0.33) + (6 \times 0.35) + (2 \times 0.32) = 4.06$$

$$\nu = (8 \times 0.33) + (3 \times 0.35) + (5 \times 0.32) = 5.29$$

Variances and covariances IPOWAWA:

$$var - IPOWAWA(X) = (4 - 4.06)^2 + (6 - 4.06)^2 + (2 - 4.06)^2 = (3.76 \times 0.33) + (4.24 \times 0.35) + (0.003 \times 0.32) = 2.72$$

$$var - IPOWAWA(Y) = (8 - 5.29)^2 + (3 - 5.29)^2 + (5 - 5.29)^2 = (5.24 \times 0.33) + (0.08 \times 0.35) + (7.34 \times 0.32) = 4.10$$

$$Cov - IPOWAWA(X,Y) = [(4 - 4.06)(8 - 5.29)] + [(6 - 4.06)(3 - 5.29)] + [(2 - 4.06)(5 - 5.29)] = (-4.44 \times 0.33) + (0.59 \times 0.35) + (-0.16 \times 0.32) = -1.30$$

$$PC - IPOWAWA(a_1, \ldots, a_n) = \frac{-1.30}{\sqrt{2.72 \times 4.10}} = -0.39$$

In this case, vector C indicates that the combination of probability and weights brings us closer to arithmetic means.

4. Generalized the Induced Pearson Coefficient

A technique that can be used for complex analysis and generating additional scenarios is the generalized or quasi-arithmetic mean. We can generalize the new proposals previously seen in the quasi-PC-IPOWA, the quasi-PC-IOWAWA, and the quasi-PC-IPOWAWA. They are defined as follows:

Proposition 6. A quasi-PC-IPOWA of dimension *n* is a model $IPOWA: R^n \to R$ with a set of variables $x_k \in U^n$ and a second set $y_k \in U^n$ which have an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$ and an associated probability vector P with $\sum_{i=1}^{n} p_i = 1$ and $p_i \in [0,1]$. Additionally, an induced IPOWA pair $\langle u_i, a_i \rangle$ is considered. Then:

$$Quasi - PC - IPOWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \frac{Quasi - Cov - IPOWA(X,Y)}{\sqrt{Quasi - var - IPOWA(X) \times Quasi - var - IPOWA(Y)}} \quad (20)$$

where the quasi-variance and covariance IPOWA are calculated as follows:

$$var_{Q-IPOWA}(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = g^{-1} \sum_{j=1}^{n} \widehat{p_j} g(D_j), \quad (21)$$

$$Cov_{Q-IPOWA}(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = g^{-1} \sum_{j=1}^{n} \widehat{p_j} g(K_j), \quad (22)$$

D_j and K_j are the variance and covariance with the *j*th element with the largest value of u_i; u_i is the induced order of variables; $g(D_j)$ and $g(K_j)$ are continuous strictly monotonic functions.

Proposition 7. A quasi-PC-IOWAWA is a mapping $IOWAWA: R^n \to R$ with a set of variables $x_k \in U^n$ and a set $y_k \in U^n$ such as an associated weighting vector W and a probability vector P, which components are ranging from zero to one and the sum is one. Additionally, an induced IPOWA pair $\langle u_i, a_i \rangle$ is considered. So:

$$Quasi - PC - IOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \frac{Quasi - Cov - IOWAWA(X,Y)}{\sqrt{Quasi - var - IOWAWA(X) \times Quasi - var - IOWAWA(Y)}} \quad (23)$$

where the variances and covariance are calculated in a quasi-form as () ().

Proposition 8. A quasi-PC-IPOWAWA is a mapping $IPOWAWA : R^n \to R$ with a set of variables $x_k; y_k$ such as an associated weighting vector W with $w_j \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, a probability vector P with $\sum_{i=1}^{n} p_i = 1$ and weighted vector V with $\sum_{i=1}^{n} v_i = 1$ and $v_i \in [0,1]$. Additionally, an induced IPOWA pair $\langle u_i, a_i \rangle$ is considered. The formulation is as follows:

$$Quasi - PC - IOWAWA(\langle u_1, a_1\rangle\langle u_2, a_2\rangle, \ldots, \langle u_n, a_n\rangle) = \frac{Quasi - Cov - IOWAWA(X,Y)}{\sqrt{Quasi - var - IOWAWA(X) \times Quasi - var - IOWAWA(Y)}} \quad (24)$$

Additionally, the families of the generalized PC-IPOWA, PC-IOWAWA, and PC-IPOWAWA operator can be seen in Tables 1–3.

Table 1. Families of generalized PC-IPOWA

Particular case	Quasi-PC-IPOWA
$u_i = \frac{1}{n}, for\ all\ i$	Quasi-arithmetic Pearson coefficient induced probabilistic ordered weighted (Quasi-PC-IPOWA)
$g(b) = b^{\lambda}$	Generalized PC-IPOWA
$g(b) = b$	PC-IPOWA
$g(b) = b^2$	Pearson coefficient induced probabilistic ordered weighted quadratic average (PC-IPOWQA)
$g(b) \to b^{\lambda}, for\ \lambda \to 0$	Pearson coefficient induced probabilistic ordered weighted geometric average (PC-IPOWGA)
$g(b) = b^{-1}$	Pearson coefficient induced probabilistic ordered weighted harmonic average (PC-IPOWHA)
$g(b) = b^3$	Pearson coefficient induced probabilistic ordered weighted cubic average (PC-IPOWCA)
$g(b) \to b^{\lambda}, for\ \lambda \to \infty$	Maximum
$g(b) \to b^{\lambda}, for\ \lambda \to -\infty$	Minimum

Table 2. Families of generalized PC-IOWAWA

Particular case	Quasi-PC-IOWAWA
$w_i = \frac{1}{n}, for\ all\ i$	Quasi-arithmetic Pearson coefficient induced ordered weighted averaging-weighted (Quasi-PC-IOWAWA)
$g(b) = b^\lambda$	Generalized PC-IOWAWA
$g(b) = b$	PC-IOWAWA
$g(b) = b^2$	Pearson coefficient induced ordered weighted averaging-weighted quadratic average(PC-IOWAWQA)
$g(b) \to b^\lambda,\ for\ \lambda \to 0$	Pearson coefficient induced ordered weighted averaging-weighted geometric average (PC-IOWAWGA)
$g(b) = b^{-1}$	Pearson coefficient induced ordered weighted averaging-weighted harmonic average (PC-IOWAWHA)
$g(b) = b^3$	Pearson coefficient induced ordered weighted averaging-weighted cubic average (PC-IOWAWCA)
$g(b) \to b^\lambda,\ for\ \lambda \to \infty$	Maximum
$g(b) \to b^\lambda,\ for\ \lambda \to -\infty$	Minimum

Table 3. Families of generalized PC-IPOWAWA

Particular case	Quasi-PC-IPOWAWA
$w_i = \frac{1}{n}, for\ all\ i$	Quasi-arithmetic Pearson coefficient induced probabilistic ordered weighted averaging-weighted (Quasi-PC-IPOWAWA)
$g(b) = b^\lambda$	Generalized PC-IPOWAWA
$g(b) = b$	PC-IPOWAWA
$g(b) = b^2$	Pearson coefficient induced probabilistic ordered weighted averaging-weighted quadratic average (PC-IPOWAWQA)
$g(b) \to b^\lambda,\ for\ \lambda \to 0$	Pearson coefficient induced probabilistic ordered weighted averaging-weighted geometric average (PC-IPOWAWGA)
$g(b) = b^{-1}$	Pearson coefficient induced probabilistic ordered weighted averaging-weighted harmonic average (PC-IPOWAWHA)
$g(b) = b^3$	Pearson coefficient induced probabilistic ordered weighted averaging-weighted cubic average (PC-IPOWAWCA)
$g(b) \to b^\lambda,\ for\ \lambda \to \infty$	Maximum
$g(b) \to b^\lambda,\ for\ \lambda \to -\infty$	Minimum

Table 4. Data analysis

Fecha	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronext	FTSE	BSE	S&P-tsx	S&P-ASX
dic-20	14,524.80	12,888.28	3,473.07	27,231.13	27,444.17	1,103.54	6,460.52	47,751.33	17,433.36	6,587.10
nov-20	14,006.46	12,198.74	3,391.76	26,341.49	26,433.62	1,088.73	6,266.19	44,149.72	17,190.25	6,517.80
oct-20	12,429.28	10,911.59	3,224.53	24,107.42	22,977.13	930.91	5,577.27	39,614.07	15,580.64	5,927.60
sep-20	12,701.89	11,167.51	3,218.05	23,459.05	23,185.12	958.98	5,866.10	38,067.93	16,121.38	5,815.90
ago-20	13,045.60	11,775.46	3,395.68	25,177.05	23,139.76	979.97	5,963.57	38,628.29	16,514.44	6,060.50
jul-20	12,465.05	10,745.27	3,310.01	24,595.35	21,710.00	954.26	5,897.76	37,606.89	16,169.20	5,927.80
jun-20	11,893.78	10,058.76	2,984.67	24,427.19	22,288.14	976.54	6,169.74	34,915.80	15,515.22	5,897.90
may-20	11,802.95	9,489.87	2,852.35	22,961.47	21,877.89	930.25	6,076.60	32,424.10	15,192.83	5,755.70
abr-20	11,372.34	8,889.55	2,860.08	24,643.59	20,193.69	899.87	5,901.21	33,717.62	14,780.74	5,522.40
mar-20	10,301.87	7,700.10	2,750.30	23,603.48	18,917.01	858.11	5,671.96	29,468.49	13,378.75	5,076.80
feb-20	12,380.97	8,567.37	2,880.30	26,129.93	21,142.96	1,021.98	6,580.61	38,297.29	16,263.05	6,441.20
ene-20	13,614.10	9,150.94	2,976.53	26,312.63	23,205.18	1,120.23	7,286.01	40,723.49	17,318.49	7,017.20

5. Pearson Correlation with OWA Operators in Stocks Market

Due to the significant growth of markets worldwide, it is common for turmoil in some financial markets to affect others. The impacts of stock market interdependence become clearer in instability [33–35].

Since Markowitz [36] considers the interdependence of markets as a trigger for risk, many studies have emerged to measure the existing relationship. In this sense, several studies have been proposed as the interrelation of markets in emerging economies [37,38], the relationship with other prices [39,40], and with economic growth [41,42].

The year 2020 was a period of instability where the COVID pandemic had a relevant impact on world stock markets [43,44]. Given this scenario, it is interesting to know the correlation observed between some of the most influential stock exchanges.

Table 5. vector weights

P	W	WA
0.10	0.09	0.07
0.10	0.08	0.07
0.09	0.07	0.08
0.09	0.10	0.09
0.09	0.10	0.09
0.08	0.07	0.09
0.08	0.08	0.10
0.08	0.08	0.10
0.08	0.09	0.10
0.08	0.08	0.09
0.07	0.10	0.06
0.06	0.06	0.06

Therefore, this research considers an application of the methodology of Pearson correlation with OWA operators in ten of the most extensive stock indexes in the world. The period studied is from January to

Table 6. OWA means

Operator	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronex	FTSE	BSE	S&P-tsx	S&P-ASX
PC-IPOWA	12,565.84	10,381.15	3,121.33	24,924.19	22,788.05	984.64	6,123.05	38,067.40	15,964.67	6,035.11
PC-IOWAWA	12,495.84	10,303.48	3,108.35	24,849.42	22,642.42	980.22	6,113.15	37,729.71	15,903.92	6,009.81
PC-IPOWAWA	12,524.54	10,347.54	3,115.39	24,870.22	22,714.88	981.71	6,112.77	37,869.29	15,924.98	6,017.50
PC	12,544.92	10,295.29	3,109.78	24,915.82	22,709.56	985.28	6,143.13	37,947.09	15,954.86	6,045.66

Table 7. Variances OWA

Operator	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronext	FTSE	BSE Sensex	S&P-tsx	S&P-ASX
IPOWA	1,253,619.41	2,377,883.41	60,189.55	1,667,143.11	5,435,703.09	6,007.60	168,708.67	23,615,954.13	1,218,042.80	231,782.27
IOWAWA	1,217,761.06	2,296,761.75	59,351.04	1,605,934.35	5,110,951.39	5,820.68	166,414.62	22,705,072.60	1,214,007.37	230,581.58
IPOWAWA	1,241,214.82	2,318,300.05	59,651.30	1,630,779.00	5,271,417.08	5,934.38	167,021.18	23,285,814.35	1,223,214.08	231,049.86

Table 8. Covariances OWA

Index	IPOWA	IOWAWA	IPOWAWA
NYSE-NASDAQ	1,420,761.60	1,369,462.42	1,397,785.81
NYSE-Shanghai	223,038.91	217,219.39	220,772.19
NYSE-Hang Seng	1,090,369.62	1,036,272.22	1,070,034.55
NYSE-Nikkei	2,449,791.00	2,334,511.16	2,398,855.45
NYSE-Euronext	78,632.37	76,069.66	77,772.98
NYSE-FTSE	247,577.71	242,843.97	246,813.95
NYSE-BSE	5,248,162.50	5,063,720.48	5,180,949.00
NYSE-SPtsx	1,197,194.21	1,179,006.60	1,195,023.72
NYSE-SPasx	468,873.66	462,083.56	468,668.54
Nasdaq-Shanghai	364,482.94	354,667.81	357,444.98
Nasdaq-HangSeng	828,054.43	764,718.82	816,877.34
Nasdaq-Nikkei	3,208,890.03	3,044,326.90	3,113,711.73
Nasdaq-Euronext	65,197.97	62,124.95	64,475.87
Nasdaq-FTSE	1,473.62	−725.05	6,632.93
Nasdaq-BSE	6,034,995.70	5,774,961.33	5,936,667.89
Nasdaq-SPtsx	1,242,778.30	1,218,941.50	1,240,650.40
Nasdaq-SPasx	344,268.59	336,927.67	348,779.65
Shanghai-HangSeng	147,431.15	140,688.58	147,218.42
Shanghai-Nikkei	469,635.10	447,455.62	456,869.96
Shanghai-Euronext	10,139.66	9,768.71	10,066.92
Shanghai-FTSE	-506.87	−845.86	39.18
Shanghai-BSE	966,632.33	938,312.79	959,113.19
Shanghai-SPtsx	200,213.34	198,313.27	200,902.78
Shanghai-SPasx	56,338.32	55,713.49	57,184.98
HangSeng-NIKKEI	1,925,843.16	1,776,578.12	1,871,784.73
HangSeng-Eoronext	85,958.32	81,995.21	84,019.34
HangSeng-FTSE	367,438.98	354,653.40	358,042.91
HangSeng-BSE	5,055,363.52	4,819,467.72	4,956,712.76
HangSeng-SPtsx	1,067,432.81	1,028,650.24	1,051,813.62
HangSeng-Spasx	508,666.96	492,262.43	498,656.48
Nikkei-Euronext	146,297.86	139,021.48	143,711.23
Nikkei-FTSE	341,939.13	331,515.95	343,435.07
Nikkei-BSE	10,471,773.31	9,894,887.23	10,221,837.83
Nikkei-SPtsx	2,168,373.99	2,097,252.93	2,145,873.09
Nikkei-SPasx	798,709.21	774,345.84	795,745.22
Euronext-FTSE	25,901.07	25,434.69	25,670.00
Euronext-BSE	325,961.07	313,369.57	320,803.36
Euronext-SPtsx	78,609.13	77,038.87	78,191.75
Euronext-Spasx	35,878.12	35,268.23	35,625.91
FTSE-BSE	914,131.76	888,651.46	899,959.46
FTSE-SPtsx	286,259.84	282,922.94	285,150.85
FTSE-SPasx	164,744.14	163,239.63	163,107.66
BSE-SPtsx	4,874,567.26	4,771,972.00	4,849,515.93
BSE-SPasx	1,929,363.85	1,888,809.62	1,918,687.43
SPtsx-SPasx	490,978.77	488,863.74	492,055.27

Table 9. Pearson coefficient with arithmetic means

	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronext	FTSE	BSE	S&P- tsx	S&P-ASX
NYSE	1	0.787	0.782	0.761	0.930	0.903	0.550	0.962	0.969	0.869
NASDAQ		1	0.960	0.386	0.880	0.488	−0.045	0.787	0.688	0.408
Shanghai			1	0.443	0.807	0.483	−0.042	0.798	0.703	0.429
Hang Seng				1	0.633	0.861	0.691	0.805	0.039	0.818
Nikkei					1	0.787	0.342	0.919	0.831	0.692
Euronext						1	0.827	0.850	0.920	0.964
FTSE							1	0.448	0.646	0.845
BSE								1	0.905	0.811
S&P-tsx									1	0.925
S&P-ASX										1

Table 10. PC-IPOWA results

	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronext	FTSE	BSE	S&P- tsx	S&P-ASX
NYSE	1	0.823	0.812	0.754	0.938	0.906	0.538	0.965	0.969	0.870
NASDAQ		1	0.963	0.416	0.893	0.545	0.002	0.805	0.730	0.464
Shanghai			1	0.465	0.821	0.533	−0.005	0.811	0.739	0.477
Hang Seng				1	0.640	0.859	0.693	0.806	0.039	0.818
Nikkei					1	0.810	0.357	0.924	0.843	0.712
Euronext						1	0.814	0.865	0.919	0.961
FTSE							1	0.458	0.631	0.833
BSE Sensex								1	0.909	0.825
S&P- tsx									1	0.924
S&P-ASX										1

Table 11. PC-IOWAWA results

	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei 225	Euronext	FTSE	BSE	S&P- tsx	S&P-ASX
NYSE	1	0.819	0.808	0.741	0.936	0.904	0.539	0.963	0.970	0.872
NASDAQ		1	0.961	0.398	0.889	0.537	−0.001	0.800	0.730	0.463
Shanghai			1	0.456	0.812	0.526	−0.009	0.808	0.739	0.476
Hang Seng				1	0.620	0.848	0.686	0.798	0.038	0.809
Nikkei					1	0.806	0.359	0.919	0.842	0.713
Euronext						1	0.817	0.862	0.916	0.963
FTSE							1	0.457	0.629	0.833
BSE Sensex								1	0.909	0.825
S&P- tsx									1	0.924
S&P-ASX										1

Table 12. PC-IPOWAWA results

	NYSE	NASDAQ	Shanghai	Hang Seng	Nikkei	Euronext	FTSE	BSE Sensex	S&P- tsx	S&P-ASX
NYSE	1	0.824	0.811	0.752	0.938	0.906	0.542	0.964	0.970	0.875
NASDAQ		1	0.961	0.420	0.891	0.550	0.011	0.808	0.737	0.477
Shanghai			1	0.472	0.815	0.535	0.000	0.814	0.744	0.487
Hang Seng				1	0.638	0.854	0.686	0.804	0.745	0.812
Nikkei 225					1	0.813	0.366	0.923	0.845	0.721
Euronext						1	0.815	0.863	0.918	0.962
FTSE							1	0.456	0.631	0.830
BSE								1	0.909	0.827
S&P- tsx									1	0.926
S&P-ASX										1

December 2020. The process for obtaining results is described in the following steps:

Step 1. The data studied are defined in terms of index and period.

Step 2. OWA vectors are described. Vectors of weights, probabilities, and induced.

Step 3. Calculation of OWA means.

Step 4. Calculation of variances and covariances with the different OWA operators.

Step 5. Results. The Pearson correlation with OWA operators is defined in the ten stock exchanges.

5.1. The Process in Pearson Correlation with OWAs

In order to analyze the correlation between some stock exchanges, the following has been carried out:

Step 1. Ten representative indexes of influential stock exchanges have been selected: NYSE, NASDAQ, Hang Seng, Nikkei 225 (Nikkei), Euronext 100 (Euronext), FTS 100 (FTS), BSE Sensex (BSE), S&P-tsx, S&Pasx200 (S&P-asx). The data are monthly for the year 2020. Table 4 shows the information.

Step 2. OWA weights vectors. To estimate the means, variances and covariances with the proposed

Table 13. PC-IPOWAWA correlations by ranges

0.9 to 1	0.7-0.89	0.5-0-69	0.30-0.49	minor 0.29 and negative
NYSE-Nikkei	NYSE-NASDAQ	NYSE-FTSE	NASDAQ-Hangseng	NASDAQ-FTSE
NYSE-Euronext	NYSE-Shanghai	NASDAQ-Euronext	NASDAQ-S&P-asx	Shanghai-FTSE
NYSE-BSE	NYSE-Hangseng	Shanghai-Euronext	Shanghai-Hangseng	
NYSE- S&P-tsx	NNYSE-S&P-asx	Hangseng-Nikkei	Shanghai-S&P-asx	
NASDAQ-Shanghai	NASDAQ-Nikkei	Hangseng-FTSE	Nikkei-FTS	
Nikkei-BSE	NASDAQ-BSE	FTSE-S&P-tsx	FTSE-BSE	
Euronext-S&P-tsx	NASDAQ-S&P-tsx			
Euronext-S&Ptasx	Shanghai-Nikkei			
BSE-S&P-tsx	Shanghai-BSE			
S&Ptxs-S&P-asx	Shanghai.S&P-tsx			
	Hangseng-Euronext			
	Hangseng-BSE			
	Hangseng-S&P-tsx			
	Hangseng-S&Ptasx			
	Nikkei-Euronext			
	Nikkei-S&P-tsx			
	Nikkei-S&P-asx			
	Euronext-FTSE			
	Euronext-BSE			
	FTSE-S&P-asx			
	BSE-S&P-asx			

OWA extensions, a series of additional vectors are necessary. The probability vector (P) was established with a criterion that close values are more likely to occur. The OWA vector (W) is a random selection.

The weighted vector (WA) valued more the months when COVID started. For practical purposes, the induced vector (I) in each case orders the data by date from the closest to the furthest. Table 5 shows the information:

Step 3. The calculation of the correlation with OWA operators implies that the means OWA are considered to replace the arithmetic means. Table 6 shows the means OWA of each of the indices.

Step 4. Variances and covariances OWA calculation. Previously seen means are substituted for variances and covariances. Table 7 shows the variances for each of the indicators depending on the OWA operator used.

Note that the IPOWA operator overestimates the variances. The IOWAWA operator is the one with the smallest variances, which indicates that the months of the start of the covid did not influence the variation of the indices until months later. The covariances are described in Table 8.

The idea about the estimation previously seen applies the same in the covariances and the chosen OWA operator.

Step 5. Using the variances and covariances for each OWA operator in the Pearson coefficient formula, the results are obtained. In order to make a comparison with arithmetic calculations, the results are first presented without OWA operators. Table 9 shows the data.

The indices with the most correlation are NASDAQ-Shanghai, Nikkei-BSE, NYSE-BSE, NYSE-S&P-tsx, BSE-S&P-tsx, Euronext-S&P-asx and S&P-tsx-S&P-asx. Additionally, there is a negative correlation between NASDAQ-FTSE and Shanghai-FTSE. With these results, we went on to analyze the information with OWA operators. Table 10 presents the results of PC-IPOWA.

Note that when we use probabilities and sub estimate the months with more variations, the correlation increases slightly for indices with a correlation greater than 0.9. An interesting issue is that the NASDAQ-FTSE correlation turns positive. In the use of the weighted vector, Table 11 shows the PC-IOWAWA results.

When a criterion that takes the months of the onset of COVID into account, the result is very similar to the arithmetic average. One can observe only a slight increase in the correlations. Then we connect the last two proposals and calculate the PC-IPOWAWA. Table 12 presents the information.

Note that when a more complex OWA operator is used, the negative values disappear. However, the correlations continue to retain similar or slightly higher values. In order to know to what extent each of the indices correlates, we place them in different ranges. Table 13 shows the order.

One can see that the most common correlation of the indices is between 0.7 to 0.89. Almost 50% of the correlations are in this range. Only the NASDAQ-FTSE and Shanghai-FTSE correlate less than 0.3. Within the correlations greater than 0.9, the NYSE correlation with other indices such as Nikkei, Euronext, BSE, S&P-tsx and how these are also strongly related to each other.

6. Conclusion

Stock markets are essential in developing countries, given the number of participants, the movements, and the variables that cause them to become of great interest. With the increasing integration of markets, it is evident that the indices of stock exchanges with similar characteristics tend to move together.

How important is this relationship, and what questions that become important for decision-making in the financial area?

This work proposes a Pearson coefficient that uses OWA aggregation operators in its formulation. In order to analyze stock indices and other complex data, induced aggregation operators (IOWA), probabilistic (IPOWA), and weighted (IOWAWA) are used. The main advantage is to obtain a correlation coefficient that can be overestimated or underestimated by the decision-maker according to the information available. In this sense, the Pearson coefficient results with OWA operators can be analyzed in a wide range of scenarios.

The new methodology is applied to ten indices of major stock exchanges in the world. The main results show that these indices tend to have a positive correlation to different degrees. The correlations increase in times when the variances are higher. In the first year of COVID-19, the correlation between indices increased slightly. Even correlations that were slightly negative turn positive when considering probability and weight in the months after the onset of the pandemic. The highest correlations are found between the indices NYSE-Nikkei-Euronext, BSE, and S&P-tsx.

AUTHORS

Martha Flores-Sosa – Universidad Autónoma de Occidente, Blvd Lola Beltran s/n, 80020, Sinaloa, Mexico, e-mail: martha.flores@uadeo.mx.

Ernesto Leon-Castro* – Faculty of Economics and Administrative Sciences, Universidad Católica de la Santísima Concepción, Concepción, Chile, Instituto Tecnologico de Sonora, Unidad Navojoa, Ramon Corona sin numero Colonia ITSON, Sonora, Mexico, C.P.85860, e-mail: eleon@ucsc.cl.

Jose M. Merigo – School of Information, Systems & Modelling, Faculty of Engineering and Information Technology, University of Technology Sydney, 81 Broadway, Ultimo, 2007, NSW, Australia, e-mail: jose.merigo@uts.edu.au.

*Corresponding author

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