Method of Automatic Parking of a Car – Theoretical Considerations and Simulation Studies

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Abstract: The paper is concerned with the problem of automatic parking of a car. Current state of the art in self-driving cars and methods of automatic parking were discussed. For simulation studies, the kinematic bicycle model of the vehicle was assumed. A method of solving the problem of automatic parking was proposed. The idea of this method consists in planning characteristic poses of the vehicle and realization of motion between these poses using pose controller. Results of simulation research were presented and discussed, which confirmed the correctness of the approach.

Keywords: self-driving car, automatic parking assist system, kinematic modeling, simulation studies

1. Introduction

The beginning of the XXI century is a period of intensification of research in field of self-driving cars (also known as autonomous car, driverless car or robotic car). Important achievement in this domain was Stanley vehicle designed by a team of prof. S. Thrun, which won DARPA Grand Challenge in 2005 [8].

Generally speaking, there are used two different approaches concerning development of self-driving cars. The first promoted by most of automotive companies assumes gradual reaching of higher and higher levels of automation of car driving, starting from single assistance systems and ending on fully automatic car. The example of semi-automatic electric car is Tesla Model S P85D [15].

The second approach promoted by Google company assumes at once development and commercialization of fully automatic car, which even does not include steering wheel that provides possibility of driving by human being. Within Google self-driving car project fully automatic solution of a car was designed, which is based on detailed maps and Street View technology [13].

The increasing use steering-by-wire and throttle-by-wire technologies in cars facilitates the development of self-driving cars. An example of this is Open Source Car Control (OSCC) project [14].

The automatic parking assist system (APAS) is a kind of advanced driver assistance systems (ADAS) supporting driving of a car, which enables performing parallel, perpendicular (forward and reverse) or angle parking. This kind of system helps human in realization of parking maneuvers, which can be troublesome and associated with stress.

The actual state of the art in APAS is described in works [7, 9, 11]. Generally speaking, APAS can be divided into two categories: fully-APAS and semi-APAS in which velocity of a car is controlled by human being. Based on the complexity of parking maneuvers, there exist one trial parking and back-and-forth several trials parking maneuvers. According to the velocity of vehicle, there are variable speed and constant speed parking.

According to [7], typical structure of APAS consists of the following modules: environment recognition, path planning and path tracking control. However, if time of motion is taken into account, then instead of path planning and path tracking control one should talk about motion planning and trajectory tracking control. Environment recognition module can contain ultrasonic, radar and laser sensors, cameras, GPS receivers and INS. The module responsible for environment recognition involves both localization of the vehicle and environment mapping. Therefore, in this case, Simultaneous Localization and Mapping (SLAM) algorithms can be used. In turn, at the stage of realization of motion using path tracking control module localization of the vehicle is also an important problem and in some cases also detection of obstacles in vehicle’s surroundings.

One of the examples of works devoted to the problems of planning and control of automatic parking maneuver is the paper [2]. The authors present the solution of segmental path planning with back-and-forth shuttling maneuvers in a narrow parking lot. They discuss the flatness-based steering control algorithm and the longitudinal motion control strategy. The proposed method was verified by hardware-in-the-loop test results with a driving simulator. This system considers also a case when no parking place is found. The solution involves also web-based driver-to-vehicle interfaces.

Another example includes paper [4], in which the authors proposed an idea of a general autonomous parking system. This system detects that a driver gets out of a car, searches for an empty parking space as fast and safely as possible, autonomously drives a car towards the parking lot and later, on demand, goes back to a human driver.
In some solutions in the field of automatic parking, artificial intelligence methods are used. An example of such work is [1], in which a fuzzy PD+I controller was applied for parallel parking. In paper [12], a reinforcement learning-based end-to-end parking algorithm was proposed to achieve automatic parking. In turn, in [10] a method which use TensorFlow training Recurrent Neural Network (RNN) to predict parking trajectory curves and determine the optimal parking trajectory was used.

It should be noted that for successful realization of parking maneuver it is not sufficient to track only the selected point of the vehicle. It is also necessary to guarantee an appropriate course angle of the vehicle. This angle can be determined on the basis of the velocity vector of selected point, which should be tangential to the motion path of that point. In the general case, however, the position of a selected point can be achieved for different values of the course angle of the vehicle. Therefore, it may be more appropriate to use the pose of the vehicle, that is, the position of the selected point and the course angle of the vehicle. Then the problem of parking of a car can be reduced to the problem of planning and realization of motion to successive desired poses. These poses can include characteristic locations of the vehicle resulting from the motion planning process. The number of these poses may be arbitrarily high, but it should be chosen in such a way so as to enable collision-free movement of the vehicle in a known environment.

The described approach is the subject of this work and it will be carried out on the example of the automatic parallel parking of a car.

2. Kinematic Modeling of a Car

For simulation research purposes, the bicycle model of a car will be adopted [3, 5]. The use of such a model is justified in case of motion of a car with low velocity. In this instance, one can assume lack of slip of wheels as well as neglect dynamics of a vehicle and properties of drive units for driving and steering of wheels.

Kinematic structure of a car is illustrated in Fig. 1. The geometric center of the front equivalent wheel is designated as $F$, whilst the rear one as $R$. It is assumed that vehicle motion is realized in $\mathcal{O}x\mathcal{O}y$ plane of the fixed coordinate system $\{\mathcal{O}\}$. The moving coordinate system, considered as rigidly connected to the center of the rear wheel, is denoted with symbol $\{R\}$. Actual position and course of a vehicle, that is its pose, can be described by the vector of generalized coordinates:

$$\dot{\mathbf{q}} = [\dot{x}_R, \dot{y}_R, \dot{\varphi}_3]^T$$

(1)

where: $\dot{x}_R$, $\dot{y}_R$ are coordinates describing position vector $\dot{R}_R$ of characteristic point $R$ belonging to the vehicle, and $\dot{\varphi}_3$ denotes angle of spin of that vehicle about $z$ axis with respect to fixed coordinate system $\{\mathcal{O}\}$, which is also named the course angle.

![Fig. 1. Kinematic structure of a car, desired and actual poses as well as pose errors](image)

In turn, vectors of generalized velocities respectively in $\{\mathcal{O}\}$ and $\{R\}$ coordinate systems can be written as:

$$\dot{\mathbf{q}} = [\dot{x}_R, \dot{y}_R, \dot{\varphi}_3]^T, \dot{\mathbf{R}} = [\dot{R}_{x}, \dot{R}_{y}, \dot{R}_{z}]^T$$

(2)

where: $\dot{R}_{x} = \dot{x}_R$, $\dot{R}_{y} = \dot{y}_R$, $\dot{R}_{z} = \dot{\varphi}_3$.

It is assumed that $\dot{R}_{y} = 0$, which means the vehicle does not move in the lateral direction.

In case of steady turning of the vehicle, between velocity of the point $R$, angular velocity of vehicle body $\dot{\varphi}_3$, and turning radius $R$, the following dependencies take place:

$$\dot{R}_{x} = \dot{R}_{x} \Rightarrow \dot{\varphi}_3 = \dot{R}_{x}/R_z.$$ (3)

From these relationships follow that vehicle rotation is impossible for $\dot{R}_{x} = 0$. Moreover, for $\delta = \pi/2$ longitudinal motion of the vehicle is impossible, i.e. $\dot{R}_{x} = 0$.

As it is known, in case of classical car, the turning radius $R_z$ depends on turning angle of steerable wheel $\delta$ and wheelbase $L$, i.e.:

$$R_z = L/\tan(\delta) \quad \text{and} \quad R_{z_{\text{min}}} = L/\tan(\delta_{\text{max}}).$$ (4)

One can notice that wheelbase $L$ and maximum absolute value of turning angle of steerable wheel $\delta_{\text{max}}$ determine geometry of possible maneuvers during parking of a car, for example minimum radius of turning $R_{z_{\text{min}}}$.

According to work [7], one can define curvature of path and curvature derivative using the following relationships:

$$\kappa = 1/R_z = \tan(\delta) / L$$ (5)

which should satisfy their limitations:

$$|\kappa| \leq \kappa_{\text{max}} \quad \text{and} \quad |\kappa'| \leq \sigma_{\text{max}}.$$ (6)

The vector of generalized velocities $\dot{\mathbf{q}}$ can be defined on the basis of kinematic equations of motion in the form:
where vector $\mathbf{v} = [v_x, v_y, \omega_z]^T$ contains respectively longitudinal velocity of the point $R$ of the vehicle and yaw rate of the vehicle body, both in $\{R\}$ coordinate system.

Moreover, in this case the following equation of non-holonomic constraints (non-integrable) can be written:

$$\phi - \phi = \cos(\delta) \sin(\delta) = 0.$$

Between velocities of points $F$ and $R$ of the vehicle, that is geometric centers of wheels, the following dependencies take place:

$$v_F = v_R, \quad \omega_F = \omega_R \tan(\delta), \quad \omega_R = \frac{|v_F| \cos(\delta)}{r}.$$

Assuming that both wheels of the vehicle roll without slip and that their radius is equal to $r$ one can write relationships:

$$v_R = v_F, \quad \omega_R = \omega_F \tan(\delta), \quad \omega_F = \frac{v_R}{r} \frac{\cos(\delta)}{r}.$$

One can also notice that particular pose of the vehicle can be achieved for various values of turning angle of steerable wheel $\delta$. Therefore, the state of the vehicle can be described by the state vector:

$$x = [\phi, \delta, \omega_x, \omega_y, \omega_z]^T.$$ (11)

For the analyzed vehicle, assuming as previously lack of slip of wheels, both forward and inverse kinematics problems can be solved.

Forward kinematics problem consists in determination of velocities of the vehicle $\mathbf{v} = [v_x, v_y, \omega_z]^T$ on the basis of known turning angle of the front steerable wheel $\delta$ and angular velocity of spin of the front or rear wheel.

Forward kinematics problem can be solved on the basis of the following equations:

$$v_R = \omega_x r, \quad \omega_R = \omega_x r \tan(\delta)/L.$$ (12)

Inverse kinematics problem consists in determination of turning angle of the front steerable wheel $\delta$ and angular velocity of spin of the front or rear wheel on the basis of known velocities of the vehicle $\mathbf{v} = [v_x, v_y, \omega_z]^T$.

This task can be solved based on relationships:

$$\omega_R = \frac{v_R}{r}, \quad \delta = \tan(\omega_x r / v_R) = \tan(L / R) .$$ (13)

These dependencies will be used at the stage of control of a pose of the vehicle.

### 3. Automatic Parking of a Car

In this work, parallel parking maneuver will be analyzed. It is assumed that surroundings of a car is known and was determined suitable parking place. Moreover, it is assumed that vehicle’s surroundings is static, that is, it does not change during parking maneuver. Therefore, the use of the surroundings sensors is not necessary.

The problem of automatic parking of a car will be decomposed into two following subproblems:

- maneuver planning, that is planning of characteristic poses of the vehicle,
- realization of motion between successive poses using pose controller.

#### 3.1. Maneuver Planning

As it was mentioned before, planning maneuver will consist in planning of characteristic poses of the vehicle. These poses should be selected in such places to provide collision-free motion between them, assuming sufficient accuracy of motion realization by pose controller.

Detailed planning process of poses of the vehicle is not a subject of this work, therefore only general assumptions in this respect will be provided. According to Fig. 2, it is assumed that parking place is behind a car. With such a parking place, absolute coordinate system $\{O\}$ is associated.

On the basis of known geometric parameters of the vehicles (first of all its length $L_R$, width $W_R$ and wheelbase $L$), dimensions of a parking place ($L_S$ and $W_S$) and initial pose $D_0$ it is possible to determine the following successive desired poses of the vehicle ($p = 1, \ldots, n$) during parallel parking maneuver:

1. pose $D_1$ in the area of a corner $C_1$ of a parking place (the vehicle is situated in such a way to assure that

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Fig. 2. Characteristic poses of a car determined as a result of motion planning process
3.2. Pose Control

Considerations concerning pose controller will be based on Fig. 1. With desired pose of the vehicle will be associated additional coordinate system \( \{D\} \). The desired pose of the vehicle in absolute coordinate system \( \{O\} \) can be described by the following vector of generalized coordinates:

\[
q_i = [a_x, a_y, \theta_i]^T,
\]

which includes the following errors:

\[
e_p = [e_x, e_y, e_\theta]^T,
\]

where:

\[
e_{\xi} = \sqrt{(\rho_{\xi} e_{\theta})^2 + (\rho_{\xi} e_{\psi})^2},
\]

\[
e_{\alpha} = \begin{cases} \alpha \cos(\alpha) \sin(\alpha) + \rho_{\xi} e_{\theta} \sin(\alpha) & \text{for } d = 1, \\ \alpha \cos(\alpha) \sin(\alpha) - \rho_{\xi} e_{\theta} \sin(\alpha) & \text{for } d = -1. \end{cases}
\]
\[
| \mathbf{u} | \leq \mathbf{u}_{\text{max}} = [v_{\text{max}}, \delta_{\text{max}}] \quad \text{and} \\
| \mathbf{a} |_{\text{max}} = [a_{\text{max}}, \omega_{\text{max}}] \ .
\]

It means that during parking maneuver there will be limited values of linear velocity and acceleration of the vehicle as well as values of turning angle of the front steerable wheel and angular velocity of turning that wheel.

The above limitations cause the following limitations on maximum angular velocity and acceleration of the vehicle:

\[
\begin{align*}
\omega_{\text{max}} &= \frac{v_{\text{max}} \tan(\delta_{\text{max}})}{L} , \\
\epsilon_{\text{max}} &= a_{\text{max}} \tan(\delta_{\text{max}}) / L + v_{\text{max}} (1 + \tan^2(\delta_{\text{max}})) \omega_{\text{max}} / L .
\end{align*}
\]

In this work, kinematic model of a car is considered, that is, dynamics of the vehicle as well as properties of drive units driving and steering the wheels are neglected. Therefore, in simulation, the velocity of characteristic point \( R \) of the vehicle and turning angle of the front steerable wheel are equal to corresponding control signals of a pose controller, that is \( \dot{\mathbf{v}}_{R} = u_{v} \) and \( \dot{\delta} = u_{\delta} \).

4 Simulation Research

The subject of this paper is automatic parallel parking maneuver of a car. For this purpose, the simulation investigations in Matlab/Simulink environment were performed. The scheme of developed simulation model is shown in Fig. 3.

Based on desired poses \( \dot{\mathbf{q}}_{1d}, \ldots, \dot{\mathbf{q}}_{nd} \), which are the result of maneuver planning process, the actual desired pose \( \dot{\mathbf{q}}_{d} = \dot{\mathbf{q}}_{kd} \) is selected, where \( p \) is the number of selected desired pose \( (p = 1, \ldots, n) \). The next desired pose is chosen after achieving actual desired pose with defined accuracy, that is taking into account parameters \( \epsilon_{\text{max}} \) and \( v_{\text{max}} \). For actual desired and real poses of the vehicle there are determined errors \( \dot{\mathbf{q}}_{e} \) and \( \mathbf{q}_{p} \) as well as direction of motion \( d \). Based on the

error vector \( \mathbf{q}_{p} \) and direction of motion \( d \) pose controller calculates the control vector \( \mathbf{u} \), which takes into consideration limitations \( \mathbf{u}_{\text{max}} \) and \( \mathbf{v}_{\text{max}} \). Finally, using kinematic model of a car there is determined actual pose \( \mathbf{q}_{d} \) and turning angle of the front wheel \( \delta \) (that is state vector \( \mathbf{x} \) of the vehicle) and another motion parameters like \( \omega_{\delta} \) and \( \dot{a}_{d} \).

For simulation research the following parameters were adopted:

- dimensions of a car: \( L = 2.9 \text{ m}, L_s = 4.7 \text{ m}, W_s = 2 \text{ m}, \)
  \( l_r = 0.9 \text{ m}, l_l = 0.9 \text{ m}, r = 0.3 \text{ m} ; \)
- limitations of a car: \( \delta_{\text{max}} = \pi/4 \text{ rad}, \omega_{\text{max}} = \pi/4 \text{ rad/s} , \)
  \( v_{\text{max}} = 1.2 \text{ m/s}, a_{\text{max}} = g/6 , \) where \( g = 9.81 \text{ m/s}^2 \) is gravity acceleration;
- initial state of a car: \( \mathbf{x}_0 = [0 0 \alpha \beta \gamma]^\top = [12, 3.9, 0, 0]^\top , \)
- gains of a pose controller: \( k_v = 3 \text{ s}^{-1}, k_\alpha = 8, k_\delta = 3 \),
- dimensions of a parking spot: \( L_s = 7 \text{ m}, W_s = 2.2 \text{ m} ; \)
- accuracy parameters of parking maneuver: \( \epsilon_{\text{max}} = 0.08 \text{ m}, v_{\text{max}} = 0.05 \text{ m/s} \).

As a result of planning process of parallel parking maneuver, the desired poses of the vehicle presented in Tab. 1 were determined.

<table>
<thead>
<tr>
<th>( \dot{\mathbf{q}}_{1d} )</th>
<th>( \dot{\mathbf{q}}_{2d} )</th>
<th>( \dot{\mathbf{q}}_{3d} )</th>
<th>( \dot{\mathbf{q}}_{4d} )</th>
<th>( \dot{\mathbf{q}}_{5d} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi_{x} ) (m)</td>
<td>6.90</td>
<td>6.45</td>
<td>3.10</td>
<td>1.10</td>
</tr>
<tr>
<td>( \chi_{y} ) (m)</td>
<td>3.42</td>
<td>3.25</td>
<td>1.50</td>
<td>1.15</td>
</tr>
<tr>
<td>( \chi_{\phi} ) (rad)</td>
<td>0.36</td>
<td>0.46</td>
<td>0.46</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Fig. 4 illustrates subsequent phases of parking maneuver obtained as a result of simulation, that is, from \( t = 0 \text{ s} \) until the end of the movement. The figure shows: initial pose, subsequent desired and actual poses during motion of the vehicle, and final pose.

Fig. 3. Scheme of simulation model developed in Matlab/Simulink environment.

![Fig. 3. Scheme of simulation model developed in Matlab/Simulink environment.](image-url)
In turn, in Fig. 5 there are presented detailed results of simulation research covering:

- desired positions of characteristic point \( R \) of a car and actual motion path of that point (Fig. 5a),
- desired and actual generalized coordinates describing desired and actual poses of the vehicle (Fig. 5b-d),
- pose errors in absolute coordinate system \( \{ O \} \) (Fig. 5e),
- control signals for longitudinal velocity of the vehicle and turning angle of the front steerable wheel (Fig. 5f).

5. Conclusion and Future Works

In the present work, the method of automatic parking of a car was described. This method was used for planning and performing parallel parking maneuver. The characteristic poses of a car for that maneuver were determined within planning process. Bicycle kinematic model was used as a model of a car. To control a car, the pose controller in a form, which enables changing the direction of motion of the vehicle, was adopted. The simulation research of parallel parking maneuver in Matlab/Simulink environment was performed.

The most important conclusions of the work are summarized below.

- At the stage of motion planning it is not required to accurately determine the desired motion path of the selected point of the vehicle taking into account the suitable curvatures of that path and its derivative.
- The proposed method, despite its simplicity and required small number of desired poses of a car,
enables accurate realization of planned parking maneuver.

- It allows realization of even complex parking maneuvers, that is, involving back-and-forth several trials.
- The applied pose controller generates a variable longitudinal velocity of the vehicle, that is, the closer to the desired position the car is, the lower its velocity is.
- The pose controller takes into consideration limitations of a car during parking maneuver, that is, linear velocity and acceleration of the vehicle as well as turning angle of the front steerable wheel and angular velocity of turning of that wheel.
- The parallel parking maneuver is carried out efficiently and takes slightly longer than 13 s.
- It should be noted however that the key factor for successful completion of the parking maneuver is in this case the accuracy of achieving the desired poses of the vehicle. Here, duration of this maneuver is less important.

Directions of future works can cover among others:

- the inclusion of information from surrounding sensors and modification of structure of the pose controller in order to allow for avoiding obstacles or emergency stopping the vehicle,
- the studies of other parking maneuvers, which would require planning of the desired poses of a car for successive phases of the maneuver.

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